MAT 2377 A (Winter 2017)
Prof : Rachid Bentoumi

Assignment 2

Deadline : Please submit in the drop box at 585 King Edward before 7 pm on Thursday, February 9, 2017
There are 5 questions

Please solve the following problems using a calculator permitted by the Faculty of Science (TI30, TI34, Casio fx-260 and Casio fx-300) :

1. The output of a chemical process is continually monitored to ensure that the concentration remains within acceptable limits. Whenever the concentration drifts outside the limits, the process is shut down and recalibrated. Let X be the number of times in a given week that the process is recalibrated. Its cumulative distribution function is

$$F_X(x) = \begin{cases} 0, & x < 0 \\ 0.17, & 0 \le x < 1 \\ 0.53, & 1 \le x < 2 \\ 0.84, & 2 \le x < 3 \\ 0.97, & 3 \le x < 4 \\ 1, & x \ge 4 \end{cases}$$

(a) What is the probability that the process is recalibrated less than two times during the week ?
(b) What is the probability that the process is recalibrated more than three times during the week ?
(c) What is the probability that the process is recalibrated exactly once during the week ?
(d) What is the expected number times that the process is recalibrated during the week ?

2. Suppose that the probability mass function $p_X(x)$ for the discrete random variable X is given by the following table :

$$p_X(x) = \begin{cases} 0.4, & x = 4 \\ 0.3, & x = 6 \\ 0.2, & x = 8 \\ 0.1, & x = 10 \end{cases}$$

(a) Find $P(X = 6)$, $P(X \le 7)$, and $P(X > 5)$
(b) What is the expected value of $7 + 10\,X$?
(c) What is the expected value of $1/X$?

3. The waiting time, in minutes, between successive speeders spotted by a radar unit is a continuous random variable X with cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \le 3, \\ 1 - \frac{81}{x^4} & \text{if } x \ge 3. \end{cases}$$

(a) Find the probability density function of X.
(b) Find the probability that the waiting time between successive speeders is more than 5 minutes but less than 10 minutes
 (i) using the cumulative distribution function of X,
 (ii) using the probability density function of X.
(c) Find the expected waiting time between successive speeders.
(d) Find the standard deviation of X.

4. Suppose that the probability mass function $p_X(x)$ for the discrete random variable X is given by the following table :

$$p_X(x) = \begin{cases} p, & x = 0 \\ 2\,p, & x = 1 \\ 3\,p, & x = 2 \\ 5\,p, & x = 3 \end{cases}$$

(a) Find the value of p. (Hint : what is the total probability ?) (b) Find the mean of X.
(c) Find the standard deviation of X.
(d) What is the expected value of $100 - 10\,X$?
(e) What is the standard deviation $100 - 10\,X$?

5. Particles are a major component of air pollution in many areas. It is of interest to study the sizes of contaminating particles. Let X represent the diameter, in micrometers, of a randomly chosen particle. Assume that in a certain area, the probability density function of X is inversely proportional to the volume of the particle ; that is, assume that

$$f_X(x) = \frac{c}{x^3}, \quad x > 1,$$

where c is a constant.
(a) Find the value of c so that f_X is a probability density function.
(b) The term PM_{10} refers to particles 10 μm or less in diameter. What proportion of contaminating particles are PM_{10} ?
(c) The term $\mathrm{PM}_{2.5}$ refers to particles 2.5 μm or less in diameter. What proportion of contaminating particles are $\mathrm{PM}_{2.5}$?
(d) What proportion of the PM_{10} particles are $\mathrm{PM}_{2.5}$?