

MAT 2377 (Winter 2017)

Assignment 2  
Solution

There are 5 questions

**Remark:** Please only grade questions 1, 4 and 5.

- [1] 1. First, we determine the probability mass function

$$p_X(x) = \begin{cases} 0,17; & x = 0 \\ 0,36; & x = 1 \\ 0,31; & x = 2 \\ 0,13; & x = 3 \\ 0,03; & x = 4 \end{cases}$$

- [1] (a) We want to find  $P(X < 2) = P(X = 0) + P(X = 1) = 0,17 + 0,36 = 0,53$ .  
[1] (b) We want to find  $P(X > 3) = P(X = 4) = 0,03$ .  
[1] (c) We want to find  $P(X = 1) = 0,36$   
[1] (d) We want to find

$$E(X) = \sum_{x \in R_X} x p_X(x) = 0(0,17) + 1(0,36) + \dots + 5(0,03) = 1,49.$$

**N.B.** For parts (a), (b) and (c) we may use the cumulative distribution function  $F_X$  as follows:

- (a)  $P(X < 2) = P(X \leq 1) = F_X(1) = 0,53$ .  
(b)  $P(X > 3) = 1 - P(X \leq 3) = 1 - F_X(3) = 1 - 0,97 = 0,03$   
(c)  $P(X = 1) = P(X \leq 1) - P(X \leq 0) = F_X(1) - F_X(0) = 0,53 - 0,17 = 0,36$
2. (a)  $P(X = 6) = 0,3$ ,  $P(X \leq 7) = P(X = 4) + P(X = 6) = 0,7$ ,  
and  $P(X > 5) = 1 - P(X \leq 5) = 1 - P(X = 4) = 0,6$   
(b) We want to find

$$E[7 + 10X] = \sum_{x \in R_X} (7 + 10x) p_X(x) = (7 + 10(4))(0,4) + \dots + (7 + 10(10))(0,1) = 67.$$

**Alternatively:** one can calculate

$$E[X] = 4(0,4) + 6(0,3) + 8(0,2) + 10(0,1) = 6.$$

Hence,  $E[7 + 10X] = 7 + 10E[X] = 7 + 10(6) = 67$ .

- (c) We want to find

$$E[1/X] = \sum_{x \in R_X} (1/x) p_X(x) = (1/4)(0,4) + \dots + (1/10)(0,1) = 0,185.$$

3. (a) The probability density function,  $f_X(x)$ , of the continuous random variable  $X$  can be found as follows:

$$f_X(x) = \frac{d}{dx} F_X(x) = \frac{(4)81}{x^5} = \frac{324}{x^5}, \quad \text{for } x > 3.$$

- (b) We need to evaluate

(i)  $P(5 < X < 10) = F_X(10) - F_X(5) = \left(1 - \frac{81}{10^4}\right) - \left(1 - \frac{81}{5^4}\right) = 0,1215$

- (ii)  $P(5 < X < 10) = \int_5^{10} f_X(x) dx = \int_5^{10} (324/x^5) dx = [324/(-4x^4)]_5^{10} = 0,1215$   
(c) The definition of the expected value of the continuous random variable  $X$  leads to

$$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx = \int_3^{\infty} x (324/x^5) dx = \int_3^{\infty} 324/x^4 dx = \frac{324}{-3x^3} \Big|_3^{\infty} = 0 - \frac{324}{-81} = 4.$$

- (d) First, we compute

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x) dx = \int_3^{\infty} x^2 (324/x^5) dx = \int_3^{\infty} 324/x^3 dx = \frac{324}{-2x^2} \Big|_3^{\infty} = 0 - \frac{324}{-18} = 18.$$

Therefore, the standard deviation of  $X$  is  $\sigma_X = \sqrt{V[X]} = \sqrt{E[X^2] - \mu_X^2} = \sqrt{18 - 4^2} = 1,4142$ .

- [1] 4. (a)

$$1 = \sum_{x \in R_X} p_X(x) = 11p \Rightarrow p = \frac{1}{11}.$$

- [1] (b) By definition, the expected value of the discrete random variable  $X$  is

$$\mu_X = E[X] = \sum_{x \in R_X} x p_X(x) = 0(1/11) + 1(2/11) + 2(3/11) + 3(5/11) = \frac{23}{11} = 2.0909.$$

- [2] (c) First, we calculate

$$E(X^2) = \sum_{x \in R_X} x^2 p_X(x) = 0^2(1/11) + 1^2(2/11) + 2^2(3/11) + 3^2(5/11) = \frac{59}{11} = 5.3636.$$

Therefore, the standard deviation of  $X$  is  $\sigma_X = \sqrt{V[X]} = \sqrt{E[X^2] - \mu_X^2} = \sqrt{5.3636 - (2.0909)^2} = 0.99586$ .

- [1] (d)  $E[100 - 10X] = 100 - 10E[X] = 100 - 10(2.0909) = 79.091$

- [1] (e) We have  $V[100 - 10X] = (-10)^2 V[X]$ . Hence, the standard deviation of  $100 - 10X$  is  $\sqrt{V[100 - 10X]} = |-10| \sqrt{V[X]} = 10\sigma_X = 10(0.99586) = 9.9586$ .

- [1] 5. (a)

$$1 = \int_{-\infty}^{\infty} f_X(x) dx = \int_1^{\infty} \frac{c}{x^3} dx = \frac{c}{-2x^2} \Big|_1^{\infty} = \frac{1}{2} \Rightarrow c = 2.$$

- [1] (b) We need to find

$$P(X \leq 10) = \int_1^{10} \frac{2}{x^3} dx = \frac{2}{-2x^2} \Big|_1^{10} = 1 - \frac{1}{100} = 0,99.$$

- [1] (c) We need to find

$$P(X \leq 2,5) = \int_1^{2,5} \frac{2}{x^3} dx = \frac{2}{-2x^2} \Big|_1^{2,5} = 1 - \frac{1}{(2,5)^2} = 0,84.$$

- [1] (d) We need to find

$$P(X \leq 2,5 | X \leq 10) = \frac{P(\{X \leq 2,5\} \cap \{X \leq 10\})}{P(X \leq 10)} = \frac{P(X \leq 2,5)}{P(X \leq 10)} = \frac{0,84}{0,99} = 0,8485.$$