

**MAT 2377 A (Winter 2017)**  
**Prof : Rachid Bentoumi**

**Assignment 3**

*Deadline : Please submit in the drop box at 585 King Edward before 7 pm on Thursday, February 16, 2017*  
*There are 3 questions*

Please solve the following problems using a calculator permitted by the Faculty of Science (TI30, TI34, Casio fx-260 and Casio fx-300) :

1. Consider the following joint probability mass function :

$x$	$y$	$p_{XY}(x, y)$
-2	-5	$7/50$
-2	0	$5/50$
-2	5	$3/50$
0	-5	$9/50$
0	0	$6/50$
0	5	$12/50$
2	-5	$1/50$
2	0	$4/50$
2	5	$3/50$

- (a) Determine the covariance of  $X$  and  $Y$ .
- (b) Are  $X$  and  $Y$  independent? (please provide a justification for your answer)
- (c) Determine the variance of  $X + 2Y$  and the variance of  $7X - Y$

2. Consider the following joint probability mass function :

$x$	$y$	$p_{XY}(x, y)$
-1	-1	$1/20$
-1	0	$4/20$
0	0	$2/20$
0	1	$7/20$
0	3	$2/20$
1	1	$3/20$
2	-2	$1/20$

- (a) Determine  $P(X = -1)$  and  $P(X = -1|Y = -1)$ .
- (b) Determine the marginal probability mass functions of  $X$  and of  $Y$ .
- (c) Determine the covariance of  $X$  and  $Y$ .
- (d) Are  $X$  and  $Y$  independent? (please provide a justification for your answer)

3. Consider the joint distribution  $p_{XY}(x, y) = c \times (\frac{1}{2})^x \times (\frac{1}{2})^y$  for all pairs  $(x, y)$  such that  $x \in \{0, 1, 2, \dots\}$  and  $y \in \{0, 1, 2, \dots\}$ , and  $p_{XY}(x, y) = 0$  otherwise. ( $c$  is a fixed constant).

- (a) Find  $c$ . (hint : use the geometric series formula  $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$  for  $0 < r < 1$ )
- (b) Find the marginal distributions of  $X$  and of  $Y$ . (hint : the probability over the entire range must add up to 1)
- (c) Are  $X$  and  $Y$  independent? (please provide a justification for your answer)
- (d) Determine the covariance of  $X$  and  $Y$ .