MAT 2377 (Winter 2017)

Assignment 4
Solution

Remark: *Please only grade questions 1 and 2.* ***Total=12 points***

1.(a) **1** X follows a binomial distribution with $n = 25$ and $p = 0.04$.

(b) **1.5** i) **Without Poisson approximation:**

From a) we have $X \sim b(n = 25, p = 0.04)$. So that, $P(X = 3) = \binom{25}{3}(0.04)^3(1 - 0.04)^{22} = 0.05996$.

1.5 ii) **With Poisson approximation:**

We have $\mu = n\,p = 1$. So that, $P(X = 3) \approx e^{-\mu}\,\mu^3/3! = 0.0613$.

`Remark`: since $n \geq 20$ and $\mu \geq 0.05$, we have a good approximation.

(c)**2** We need to find

$$P(X > 2) = 1 - P(X \leq 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]\,.$$

Therefore,

$$P(X > 2) = 1 - \left[\binom{25}{0}(0.04)^0(0.96)^{25} + \binom{25}{1}(0.04)^1(0.96)^{24} + \binom{25}{2}(0.04)^2(0.96)^{23}\right] = 0.0765$$

and

$$P(4 \leq X < 6) = P(X = 4) + P(X = 5) = \binom{25}{4}(0.04)^4(0.96)^{21} + \binom{25}{5}(0.04)^5(0.96)^{20} = 0.01615.$$

2.(a) **2** We need to find

$$P(T = 5) = (1 - p)^4\,p = 0.03397, \quad \text{where } T \text{ follows a geometric distribution with } p = 0.04.$$

(b) **2** We need to find

$$P(T_2 = 6) = \binom{5}{1}p^2(1-p)^4 = 0{,}00679, \text{ where } T_2 \text{ follows a Negative binomial distribution with } r = 2 \text{ and } p = 0.04.$$

(c) **2** Since T_2 follows a Negative binomial distribution with $r = 2$ and $p = 0.04$,

$$E[T_2] = \frac{r}{p} = \frac{2}{0.04} = 50 \quad \text{and} \quad \sigma_{T_2} = \sqrt{V[T_2]} = \sqrt{\frac{r(1-p)}{p^2}} = \sqrt{\frac{2(0.96)}{(0.04)^2}} = 34.641.$$

3. (a) Let X be the number of defective components in box. X follows a binomial distribution with $n = 18$ and $p = 0.1$. We have to find

$$P(X = 0) = \binom{18}{0}p^0(1 - p)^{18} = 0.1501.$$

(b) Let X be the number of boxes among $n = 12$ boxes which contain only non-defective components. X follows a binomial distribution with $n = 12$ and $p = 0.1501$. We want to find

$$P(X \geq 3) = 1 - [P(X < 3)] = 1 - [P(X = 0) + P(X = 1) + P(X = 2)] \,.$$

It follows that,

$$P(X \geq 3) = 1 - \left[\binom{12}{0} 0.1501^0 (1 - 0.1501)^{12} + \binom{12}{1} 0.1501^1 (1 - 0.1501)^{11} + \binom{12}{2} 0.1501^2 (1 - 0.1501)^{10} \right] = 0.2645.$$

(c) Let T be the number of boxes required in order to have a box containing only non-defective components. T follows a geometric distribution with $p = 0.1501$.

4. Let X_t be the number of geomagnetic storms in a period of t **years**. X_t follows a Poisson distribution with parameter $\mu = \lambda\, t$.

(a) We want to find

$$P[X_t \geq 4] = 1 - e^{-\mu} \frac{\mu^0}{0!} - e^{-\mu} \frac{\mu^1}{1!} - e^{-\mu} \frac{\mu^2}{2!} - e^{-\mu} \frac{\mu^3}{3!} = 0.2424,$$

where $\mu = \lambda\, t = 2.5(1) = 2.5$. (Here, the time interval is $[0, 1]$ (year))

(b) We have $\mu = \lambda\, t = 2.5(1) = 2.5$ in the time interval $[0, 1]$ (year) or equivalently in the time interval $[0, 365]$ (days) .

Now, let X_t be the number of geomagnetic storms in a period of t **days**. X_t follows a Poisson distribution with parameter $\mu = \lambda\, t$, where λ denotes the parameter of the Poisson distribution in the unit time interval $[0, 1]$ (day).

It follows that, $\lambda = 2.5(\frac{1}{365})$ in the unit interval $[0, 1]$ (day). Hence, in $[0, 120]$ (days), X_t follows a Poisson distribution with parameter $\mu = \lambda\, t = 2.5(\frac{1}{365})120$. Consequently, $E[X_t] = \lambda\, t = (2.5)(120/365) = 0,8219$ geomagnetic storms.

(c) From b) $\lambda = 2.5(\frac{1}{365})$ in the unit interval $[0, 1]$ (day). Hence, in $[0, 153]$ (days), X_t follows a Poisson distribution with parameter $\mu = \lambda\, t = 2.5(\frac{1}{365})153 = 1.04795$. Now, we need to find $P[X_t = 0] = e^{-\mu}\, \mu^0 / 0! = e^{-\mu} = 0.35066$, where $\mu = 1.04795$.