

MAT 2377A (Winter 2017)
Prof: Rachid Bentoumi

Assignment 6

Deadline : Please submit in the dropbox at 585 King Edward before 7:00pm on Thursday 6 April, 2017.

There are 4 questions.

Please solve the following problems with a calculator authorized by the Faculty of Science (TI30, TI34, Casio fx-260 or Casio fx-300) :

1. Let X , Y , and Z be independent normal random variables, where $X \sim N(\mu_X = 1, \sigma_X^2 = 3)$, $Y \sim N(\mu_Y = 0, \sigma_Y^2 = 2)$, and $Z \sim N(\mu_Z = 5, \sigma_Z^2 = 1)$.
Let $W = 3X - 6Y + 2Z$.
 - (a) Find $E[W]$ and $V[W]$.
 - (b) What is the distribution of W ?
 - (c) What is $P(7 < W < 9)$?

2. Let X_1, X_2, \dots, X_n be a random sampling of size $n = 60$ from a population with the following distribution: For every i , $P(X_i = x) = \frac{1}{6}$ if $x = 0, 1, 2, 3, 4, 5$ and $P(X_i = x) = 0$ otherwise. Let $Y = X_1 + X_2 \dots + X_n$.
 - (a) What is the population mean μ and population variance σ^2 ?
 - (b) What is $E[Y]$ and $V[Y]$?
 - (c) Find the approximate distribution of Y .
 - (d) Find the approximate value of $P(Y > 160)$

3. Let X_1, X_2, \dots, X_n be a random sampling of size $n = 100$ from a population with unknown mean μ but **known** variance $\sigma^2 = 0.5$. The sample mean is $\bar{x} = 8.3$.
 - (a) Find a (two-sided) 95% confidence interval for the population mean μ .
 - (b) Find a 99.9% upper-confidence bound for the population mean μ .
 - (c) Find a 90% lower-confidence bound for the population mean μ .
 - (d) Suppose we can now choose the number n of samples to collect. How many samples in total should we collect so that the error $E = |\bar{x} - \mu|$ is such that we will be 90% confident that the error will satisfy $E \leq 0.007$?

4. Let X_1, X_2, \dots, X_n be a random sampling of size $n = 10$ from a normal population with unknown mean μ and **unknown** variance σ^2 . The sample mean is $\bar{x} = -6$ and the *sample* variance is $s^2 = 0.3$. Find a (two-sided) 95% confidence interval for the population mean μ .