MAT 2777 (Hiver 2017)

Devoir 2 - solutionnaire
Il y a 5 questions

[**1**] 1. Il nous faudra la fonction masse de probabilité:

$$p_X(x) = \begin{cases} 0{,}17; & x = 0 \\ 0{,}36; & x = 1 \\ 0{,}31; & x = 2 \\ 0{,}13; & x = 3 \\ 0{,}03; & x = 4 \end{cases}$$

[**1**] (a) On veut $P(X < 2) = P(X = 0) + P(X = 1) = 0{,}17 + 0{,}36 = 0{,}53$.
[**1**] (b) On veut $P(X > 3) = P(X = 4) = 0{,}03$.
[**1**] (c) On veut $P(X = 1) = 0{,}36$
[**1**] (d) On veut

$$E(X) = \sum_{x \in R_X} x\, p_X(x) = 0(0{,}17) + 1(0{,}36) + \cdots + 5(0{,}03) = 1{,}49.$$

N.B. Vous auriez pu utiliser F_X pour répondre aux parties (a) à (c) :
(a) $P(X < 2) = P(X \leq 1) = 0{,}53$.
(b) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0{,}97 = 0{,}03$
(c) $P(X = 1) = P(X \leq 1) - P(X \leq 0) = 0{,}53 - 0{,}17 = 0{,}36$

2. (a) $P(X = 6) = 0{,}3$, $\quad P(X \leq 7) = P(X = 4) + P(X = 6) = 0{,}7$,
et $P(X > 5) = 1 - P(X \leq 5) = 1 - P(X = 4) = 0{,}6$
(b) On veut

$$E[7 + 10\,X] = \sum_{x \in R_X} (7 + 10\,x)\, p_X(x) = (7 + 10(4))(0{,}4) + \cdots + (7 + 10(10))(0{,}1) = 67.$$

En alternatif: On peut calculer

$$E[X] = 4(0{,}4) + 6(0{,}3) + 8(0{,}2) + 10(0{,}1) = 6.$$

Alors, $E[7 + 10\,X] = 7 + 10\,E[X] = 7 + 10(6) = 67$.
(c) On veut

$$E[1/X] = \sum_{x \in R_X} (1/x) p_X(x) = (1/4)(0{,}4) + \cdots + (1/10)(0{,}1) = 0{,}185.$$

3. (a) On veut

$$f_X(x) = \frac{d}{dx} F_X(x) = \frac{(4)81}{x^5} = \frac{324}{x^5}, x > 3.$$

(b) On veut
(i) $P(5 < X < 10) = F_X(10) - F_X(5) = \left(1 - \frac{81}{10^4}\right) - \left(1 - \frac{81}{5^4}\right) = 0{,}1215$
(ii) $P(5 < X < 10) = \int_5^{10} f_X(x)\, dx = \int_5^{10} (324/x^5)\, dx = \left[324/(-4x^4)\right]_5^{10} = 0{,}1215$
(c) On veut

$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx = \int_3^{\infty} x\, (324/x^5)\, dx = \int_3^{\infty} 324/x^4\, dx = \left.\frac{324}{-3x^3}\right|_3^{\infty} = 0 - \frac{324}{-81} = 4.$$

(d) Calculons

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_3^{\infty} x^2\,(324/x^5)\,dx = \int_3^{\infty} 324/x^3\,dx = \left.\frac{324}{-2x^2}\right|_3^{\infty} = 0 - \frac{324}{-18} = 18.$$

Alors, l'écart type de X est $\sigma_X = \sqrt{V[X]} = \sqrt{E[X^2] - \mu_X^2} = \sqrt{18 - 4^2} = 1{,}4142$.

[**1**] 4. (a)

$$1 = \sum_{x \in R_X} p_X(x) = 11\,p \quad \Rightarrow \quad p = \frac{1}{11}.$$

[**1**] (b) La moyenne de X est

$$\mu_X = \sum_{x \in R_X} x\,p_X(x) = 0(1/11) + 1(2/11) + 2(3/11) + 3(5/11) = \frac{23}{11} = 2{,}0909.$$

[**2**] (c) Calculons

$$E(X^2) = \sum_{x \in R_X} x^2\,p_X(x) = 0^2(1/11) + 1^2(2/11) + 2^2(3/11) + 3^2(5/11) = \frac{59}{11} = 5{,}3636.$$

Alors, l'écart type de X est $\sigma_X = \sqrt{V[X]} = \sqrt{E[X^2] - \mu_X^2} = \sqrt{5{,}3636 - (2{,}0909)^2} = 0{,}99586$.

[**1**] (d) $E[100 - 10\,X] = 100 - 10\,E[X] = 100 - 10\,(2{,}0909) = 79{,}091$

[**1**] (e) On a $V[100 - 10\,X] = (-10)^2\,V[X]$. Alors, l'écart type de $100 - 10\,X$ est

$$\sqrt{V[100 - 10\,X]} = \sqrt{(-10)^2\,V[X]} = 10\,\sigma_X = (10)\,(0{,}99586) = 9{,}9586.$$

[**1**] 5. (a)

$$1 = \int_{-\infty}^{\infty} f_X(x)\,dx = \int_1^{\infty} \frac{c}{x^3} dx = \left.\frac{c}{-2x^2}\right|_1^{\infty} = \frac{1}{2} \quad \Rightarrow \quad c = 2.$$

[**1**] (b) On veut

$$P(X \leq 10) = \int_1^{10} \frac{2}{x^3}\,dx = \left.\frac{2}{-2x^2}\right|_1^{10} = 1 - \frac{1}{100} = 0{,}99.$$

[**1**] (c) On veut

$$P(X \leq 2{,}5) = \int_1^{2{,}5} \frac{2}{x^3}\,dx = \left.\frac{2}{-2x^2}\right|_1^{2{,}5} = 1 - \frac{1}{(2{,}5)^2} = 0{,}84.$$

[**1**] (d) On veut

$$P(X \leq 2{,}5 | X \leq 10) = \frac{P(\{X \leq 2{,}5\} \cap \{X \leq 10\})}{P(X \leq 10)} = \frac{P(X \leq 2{,}5)}{P(X \leq 10)} = \frac{0{,}84}{0{,}99} = 0{,}8485.$$

[/**15**]