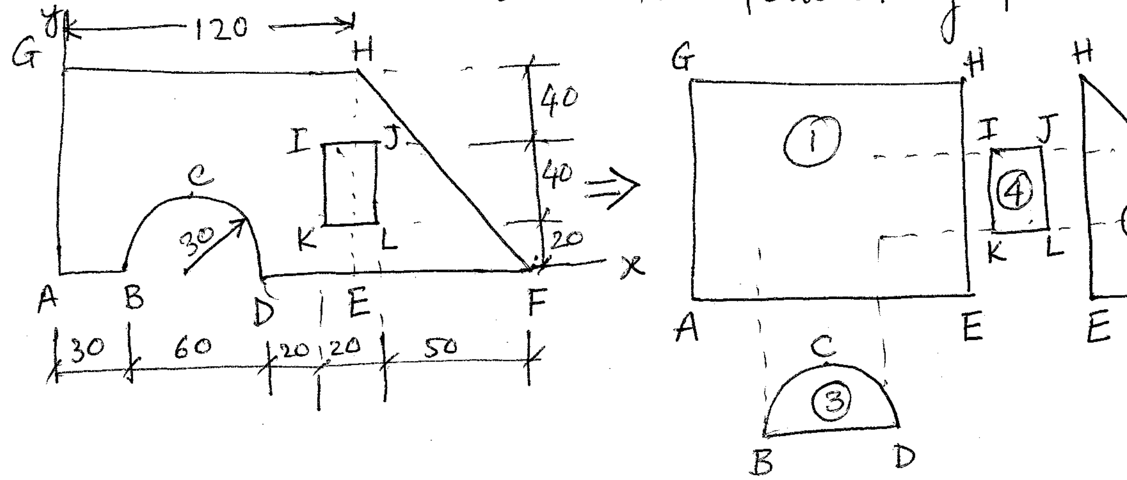
Prob 1.

Break the area into the following parts



Part	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	12000	60	50	720 000	600 000
2	3000	140	$\frac{100}{3}$	420 000	100 000
3	−1413.72	60	$\frac{40}{\pi}$	−84823.2	−18 000
4	−800	120	40	−96000	−32000
	12786.3			959176.8	650 000

Notes:

① Areas of segments 1 and 2 are positive and the areas of segments 3 and 4 are negative

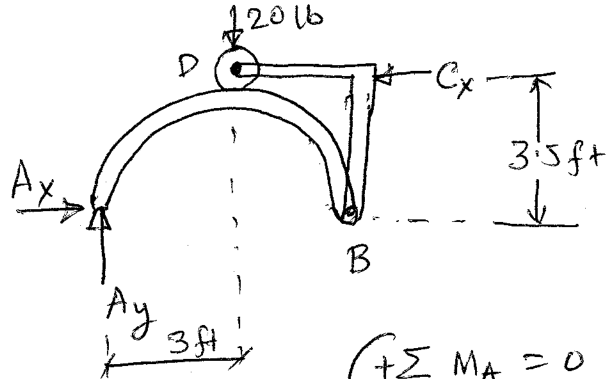
② The distances of the centroids of each segments are determined with respect to the x and y axes as shown.

③ The area of the semi circular part (part 3) is $\frac{1}{2}\pi r^2 = -1413.72$ and $\bar{y} = \frac{4r}{3\pi} = \frac{40}{\pi}$

Now, $\bar{X} = \frac{\Sigma A_i \bar{X}_i}{\Sigma A_i} = \frac{959176.8}{12786.3} = 75$, $\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{650000}{12786.3} = 50.8$

Prob 2 :

① Draw the FBD of the whole frame and find the reactions



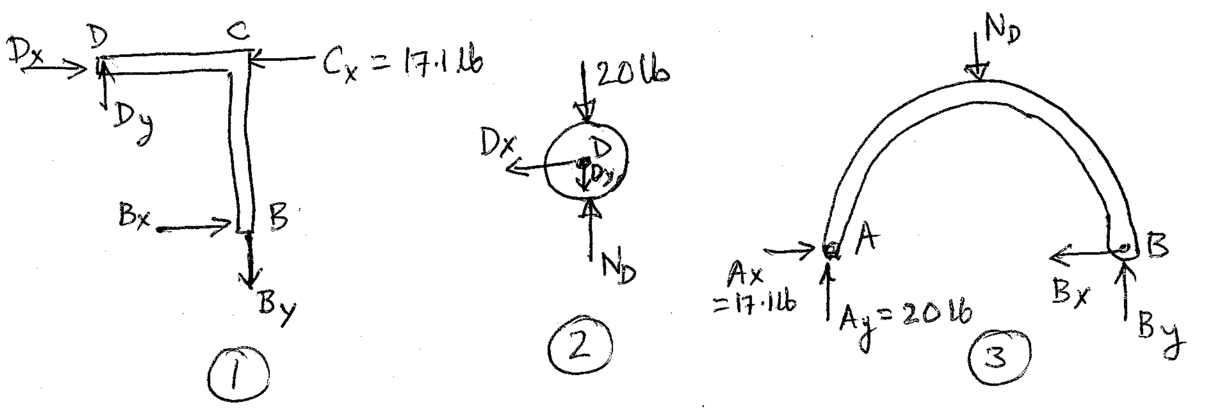
$\circlearrowleft + \Sigma M_A = 0$ gives,

$-20(3) + C_x(3.5) = 0 \quad \Rightarrow C_x = 17.1$ lb

$\overset{+}{\rightarrow} \Sigma F_x = 0$ gives, $A_x - 17.1 = 0 \Rightarrow A_x = 17.1$ lb

$+\uparrow \Sigma F_y = 0$ gives, $A_y - 20 = 0 \Rightarrow A_y = 20$ lb

② Now draw the FBDs of the individual frame elements and the disk.



Considering FBD 3 (member AB)

$\overset{+}{\rightarrow} \Sigma F_x = 0$ gives, $\overset{A_x}{17.1} - B_x = 0 \quad \Rightarrow \quad B_x = 17.1 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0$ gives, $-\overset{A_y}{20}(6) + N_D(3) = 0 \quad \Rightarrow \quad N_D = 40 \text{ lb}$

$+\uparrow \Sigma F_y = 0$ gives $\overset{A_y}{20} - \overset{N_D}{40} + B_y = 0 \quad \Rightarrow \quad B_y = 20 \text{ lb}$

Considering FBD 2 (the disk)

$\overset{+}{\rightarrow} \Sigma F_x = 0$ gives, $D_x = 0$

$+\uparrow \Sigma F_y = 0$ gives, $\overset{N_D}{40} - 20 - D_y = 0 \quad \Rightarrow \quad D_y = 20 \text{ lb}$

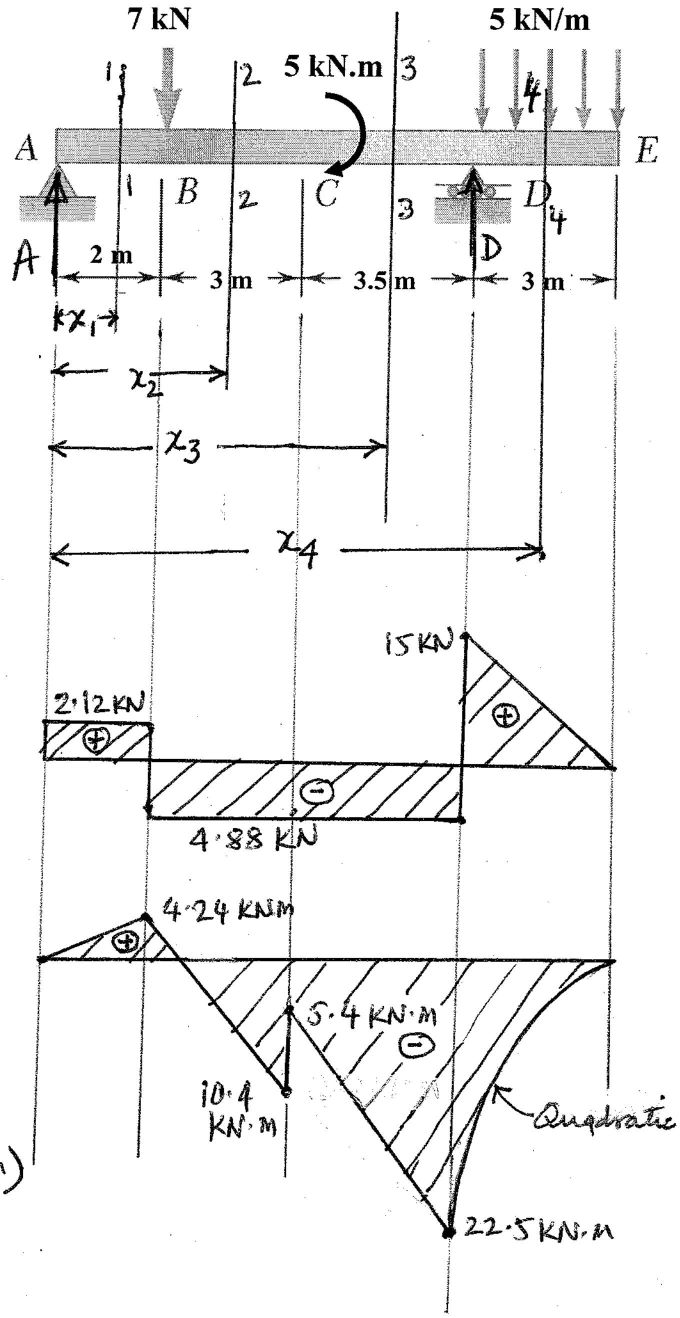
In this case,
FBD 1 is not used for the solution; however,
it can be used for checking the results.

Ans: Reactions at D: $D_x = 0$, $D_y = 20 \text{ lb} \downarrow$
(considering the disk)

Reactions at B: $B_x = 17.1 \text{ lb} \leftarrow$
$B_y = 20 \text{ lb} \uparrow$
(considering member AB)

Q3

(3) For the beam shown, (a) draw the shear force and bending moment diagrams, and (b) determine the maximum absolute values of shear force and bending moment.



Find the support Reactions

$$\circlearrowleft + \Sigma M_A = -(2)(7) - 5 + (8.5)(D) - (10)(5\times3) = 0$$

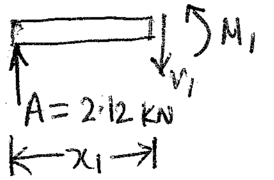
$$D = 19.88 \text{ KN}$$

$$\Sigma F_y = A + D - 7 - (5\times3) = 0$$

$$A = 7 + 15 - 19.88$$
$$= 2.12 \text{ KN}$$

a) Shear Force and Bending moment diagrams

1) Consider the left side of section 1-1 $(0 \le x_1 \le 2m)$



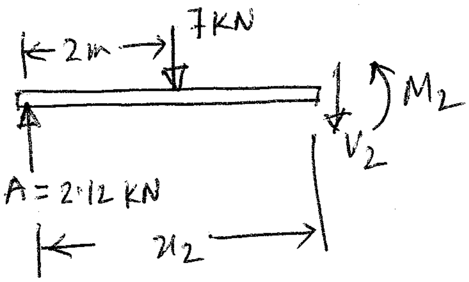
$$\Sigma F_y = A - V_1 = 0 \rightarrow V_1 = A = 2.12 \text{ KN}$$

$$\circlearrowleft + \Sigma M_1 = M_1 - (x_1)A = 0 \rightarrow M_1 = 2.12x$$

at $x_1 = 0$: $V_1 = 2.12$ KN and $M_1 = 0$

at $x_1 = 2m$: $V_1 = 2.12$ KN and $M_1 = 4.24$ KN.m

2) consider the left side of section 2-2 ($2m \le x_2 \le 5m$)



$\Sigma F_y = A - 7 - V_2 = 0$

$\Rightarrow V_2 = A - 7$

$= 2.12 - 7$

$= -4.88 KN$

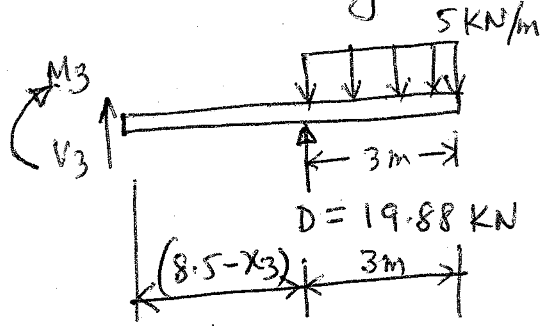
$\circlearrowleft + \Sigma M_2 = M_2 - (x_2)(A) + (x_2 - 2)(7) = 0$

$M_2 = 2.12\, x_2 - 7(x_2 - 2)$

at $x_2 = 2m$: $V_2 = -4.88 KN$, $M_2 = 4.24\ KN.m$

at $x_2 = 5m$: $V_2 = -4.88 KN$, $M_2 = -10.4\ KN.m$

3) Consider the right side of section 3 ($5m \le x_3 \le 8.5m$)



$\Sigma F_y = V_3 + D - (3)(5) = 0$

$V_3 = 15 - 19.88 = -4.88\ KN$

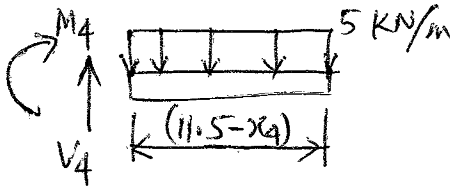
$\circlearrowleft + \Sigma M_3 = -M_3 + (8.5 - x_3) D - (8.5 - x_3 + 1.5) 15 = 0$

$M_3 = 19.88(8.5 - x_3) - (10 - x_3)(15)$

at $x = 5m$; $V_3 = -4.88\ KN$, $M_3 = -5.4\ KN.m$

at $x = 8.5m$; $V_3 = -4.88\ KN$, $M_3 = -22.5\ KN.m$

(4) Consider the right side of section 4 $(8.5\,m \le x_4 \le 11.5\,m)$



$$\Sigma F_y = V_4 - 5(11.5 - x_4) = 0$$

$$V_4 = 5(11.5 - x_4)$$

$$\curvearrowleft + \Sigma M_4 = -M_4 - \left(\frac{11.5 - x_4}{2}\right)[5(11.5 - x_4)$$

$$M_4 = -2.5(11.5 - x_4)^2$$

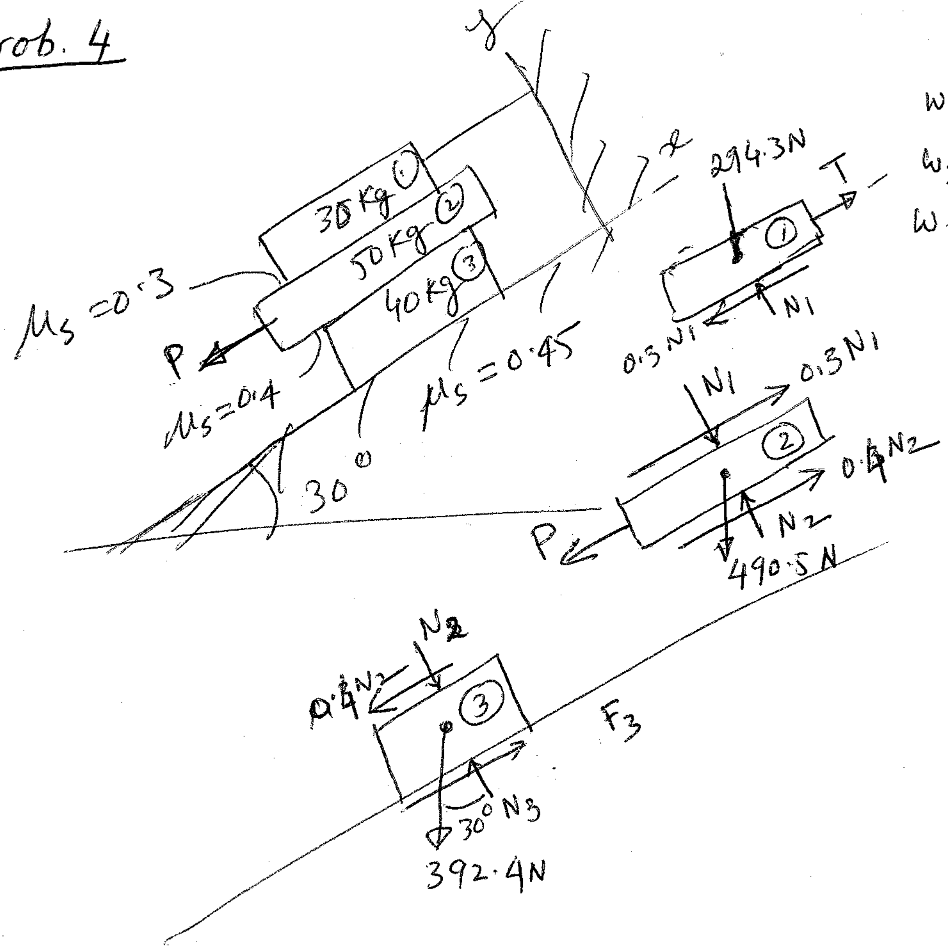
at $x_4 = 8.5\,m$: $V_4 = 15$ KN , $M_4 = -22.5$ KN.m

at $x_4 = 11.5\,m$; $V_4 = 0$, $M_4 = 0$

(b) Maximum shear force $= 15$ KN at D

Maximum bending moment $= 22.5$ KN.m at D

Prob. 4



$w_1 = 30 \times 9.81 = 294.3$ N
$w_2 = 50 \times 9.81 = 490.5$ N
$w_3 = 40 \times 9.81 = 392.4$ N

Case 1. Assume slippage on both faces of block 2.

FBD (1) $\Sigma F_x = -294.3 \sin 30° + T - 0.3 N_1 = 0$ (1)

$\Sigma F_y = -294.3 \cos 30° + N_1 = 0 \rightarrow N_1 = 254.87$ N

substituting N_1 in Eqn. (1) $\rightarrow$

$T = 0.3 N_1 + 294.3 \sin 30° = 223.61$ N

FBD (2)

$\Sigma F_x = -P + 0.3 N_1 + 0.4 N_2 - 490.5 \sin 30° = 0$ — (2)

$\Sigma F_y = -N_1 + N_2 - 490.5 \cos 30° = 0$

or, $N_2 = N_1 + 490.5 \cos 30° = 679.66$ N

substituting N_2 in Eqn (2)

$P = 0.3 N_1 + 0.4 N_2 - 490.5 \sin 30° = 103.1$ N

FBD (3) $\Sigma F_y = N_3 - N_2 - 392.4 \cos 30° = 0 \rightarrow N_3 = 679.66 + 392.4 \cos 30°$

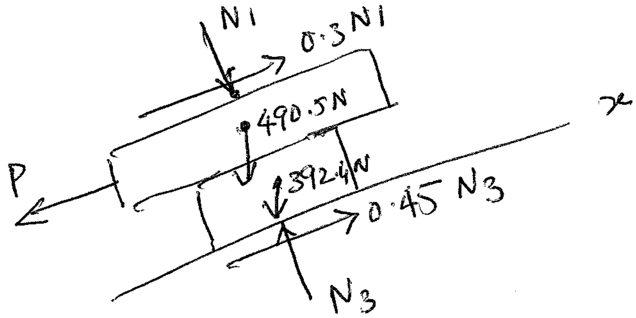
$= 1018.83$ N

$\Sigma F_x = -0.4 N_2 - 392.4 \sin 30° + \cancel{0.45} F_3 = 0$

$F_3 = 0.4(679.66) + 392.4 \sin 30° = 468$ N

So, block 3 moves as well

Max possible friction $F_{3\,max} = \mu_s N_3 = 0.45(1018.83) = 458.5$ N < 468 N case 2 Assume No slippage between middle and bottom blocks, block 2 and 3 move together.



$N_1 = 254.87$ N
from case 1

$$\Sigma F_x = 0.3N_1 + 0.45N_3 - P - 490.5\sin 30^\circ - 392.4\sin 30^\circ = 0 \quad (3)$$

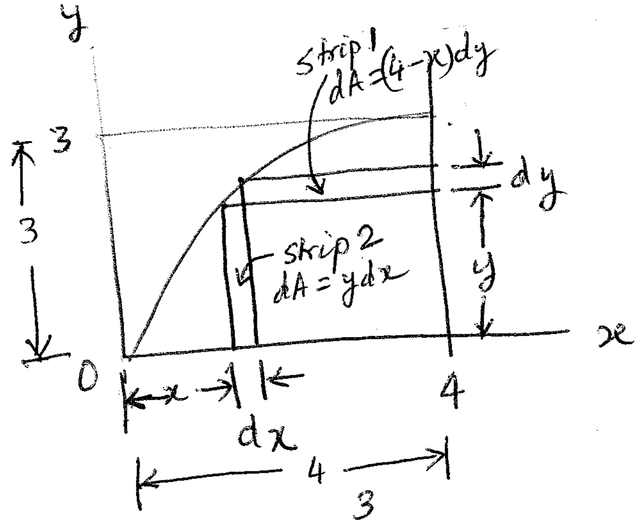
$$\Sigma F_y = -N_1 + N_3 - (490.5 + 392.4)\cos 30^\circ = 0$$

$$N_3 = N_1 + (490.5 + 392.4)\cos 30^\circ = 1019.48\text{ N}$$

Substituting N_3 in 3

$$P = 0.3(254.87) + 0.45(1019.48) - (490.5 + 392.4)\sin 30^\circ$$
$$= 93.78\text{ N}$$

Prob. 5



Find the value of k

$x = ky^2$

for $x = 4,\ y = 3$

$4 = k\,9$

$k = 4/9$

then $x = \frac{4}{9}y^2 \rightarrow y = \frac{3}{2}\sqrt{x}$

$$I_x = \int_A y^2\,dA = \int_0^3 y^2(4-x)\,dy = \int_0^3 y^2\left(4-\frac{4}{9}y^2\right)dy$$

consider strip 1

$$= \int_0^3 \left(4y^2 - \frac{4}{9}y^4\right)dy = 4\left[\frac{y^3}{3}\right]_0^3 - \frac{4}{9}\left[\frac{y^5}{5}\right]_0^3$$

$$= \frac{4}{\cancel{3}} \times \cancel{27}^{\,9} - \frac{4}{\cancel{9}_{\,1}} \times \frac{\cancel{9}^{\,1} \times 27}{5}$$

$$= 36 - 21.6 = 14.4$$

$$I_y = \int_A x^2\,dA = \int_0^4 x^2 \cdot y\,dx = \int_0^4 x^2 \cdot \frac{3}{2}\sqrt{x}\,dx$$

consider strip 2

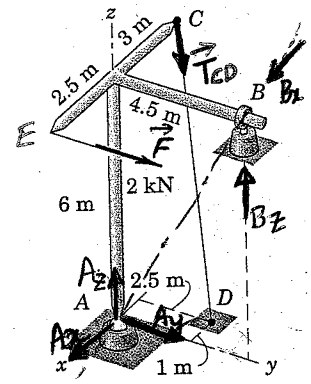
$$= \frac{3}{2}\int_0^4 x^{5/2}\,dx = \frac{3}{2}\cdot\left[\frac{x^{5/2+1}}{\frac{5}{2}+1}\right]_0^4$$

$$= \frac{3}{2}\cdot\left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^4 = \frac{3}{2}\times\frac{2}{7}\times(4)^{7/2}$$

$$= \frac{3}{7}\times 2^7 = 54.86$$

prob 6

① Draw the FBD of the frame



6 unknown quantities

a) Cable force T

b) Reactions B_x and B_z at B

c) Reactions A_x, A_z and A_z at A

② To solve the problem, get the coordinates of the key points and set up the equilibrium equations $\Sigma \vec{M}_A = 0$ and $\Sigma \vec{F} = 0$.

A (0, 0, 0) m C (−3, 0, 6) m E (2.5, 0, 6) m

B (0, 4.5, 6) m D (−1, 2.5, 0) m

$$\vec{CD} = (-1+3)\bar{i} + (2.5-0)\bar{j} + (0-6)\bar{k} = 2\bar{i} + 2.5\bar{j} - 6\bar{k} \text{ m}$$

$$|\vec{CD}| = \sqrt{2^2 + 2.5^2 + 6^2} = 6.8 \text{ m}$$

unit vector $\vec{\lambda}_{CD} = \dfrac{\vec{CD}}{|\vec{CD}|} = \dfrac{2i + 2.5j - 6k}{6.8}$

$$\vec{T}_{CD} = T_{CD}\vec{\lambda}_{CD} = \frac{T_{CD}}{6.8}(2\bar{i} + 2.5\bar{j} - 6\bar{k})$$

③ Moment Equation $\Sigma \vec{M}_A = 0$

$$\vec{M}_A = \vec{r}_{C/A} \times \vec{T}_{CD} + \vec{r}_{E/A} \times \vec{F} + \vec{r}_{B/A} \times \vec{B}$$

$\vec{r}_{C/A} = -3\bar{i} + 6\bar{k}$ m $\vec{F} = 2\bar{j}$

$\vec{r}_{E/A} = 2.5\bar{i} + 6\bar{k}$ m $\vec{B} = B_x\bar{i} + B_z\bar{k}$

$\vec{r}_{B/A} = 4.5\bar{j} + 6\bar{k}$ m

$$\vec{M}_A = (-3\bar{i} + 6\bar{k}) \times \frac{T_{CD}}{6.8}(2\bar{i} + 2.5\bar{j} - 6k)$$

$$+ (2.5\bar{i} + 6\vec{k}) \times 2\bar{j}$$

$$+ (4.5\bar{j} + 6\bar{k}) \times (B_x\bar{i} + B_z\bar{k})$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -3 & 0 & 6 \\ 2 & 2.5 & -6 \end{vmatrix} \frac{T_{CD}}{6.8} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2.5 & 0 & 6 \\ 0 & 2 & 0 \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 4.5 & 6 \\ B_x & 0 & B_z \end{vmatrix}$$

$$= \frac{T_{CD}}{6.8}(-15\bar{i} - 6\bar{j} \quad 7.5\bar{k}) + (-12\bar{i} + 5\bar{k})$$

$$+ (4.5B_z\bar{i} \quad 6B_x\bar{j} - 4.5B_x\bar{k})$$

$$= \bar{i}\left[-\frac{15T_{CD}}{6.8} - 12 + 4.5B_z\right] + \bar{j}\left[-\frac{6T_{CD}}{6.8} + 6B_x\right]$$

$$+ k\left[\frac{-7.5T_{CD}}{6.8} + 5 - 4.5B_x\right]$$

With $\vec{M}_A = 0$

$$-\frac{15T_{CD}}{6.8} - 12 + 4.5B_z = 0 \quad — (1)$$

$$-\frac{6T_{CD}}{6.8} + 6B_x = 0 \quad — (2)$$

$$\frac{-7.5T_{CD}}{6.8} + 5 - 4.5B_x = 0 \quad — (3)$$

From Eqn (2) $\quad B_x = \frac{T_{CD}}{6.8}$

Substituting B_x to Eqn (3) $\rightarrow \frac{-7.5T_{CD}}{6.8} + 5 - 4.5\frac{T_{CD}}{6.8} = 0$

$T_{CD} = \underline{2.83 \text{ kN}}$

Then $B_y = \frac{T_{CD}}{6.8} = \frac{2.83}{6.8} = 0.417 \text{ kN} \simeq 0.42 \text{ kN}$

From Eqn (1) $\quad 4.5 B_z = 12 + \frac{15 T_{CD}}{6.8} = 12 + \frac{15(2.83)}{6.8}$

$$B_z = 4.05 \text{ kN} \qquad \vec{B} = 0.417\bar{i} + 4.05\bar{k} \text{ kN}$$

$$\vec{T}_{CD} = \frac{2.83}{6.8}(2\bar{i} + 2.5\bar{j} - 6\bar{k}) = (0.83\bar{i} + 1.04\bar{j} - 2.49k) \text{ kN}$$

(4) The force equation $\quad \Sigma\vec{F} = 0$

$$\Sigma\vec{F} = \vec{A} + \vec{B} + \vec{F} + \vec{T}_{CD} = 0$$

$$\vec{A} = A_x\bar{i} + A_y\bar{j} + A_z k$$

Then $\left(A_x\bar{i} + A_y\bar{j} + A_z\bar{k}\right) + \left(0.417\bar{i} + 4.05\bar{k}\right) + 2\bar{j} + \left(0.83\bar{i} + 1.04\bar{j} - 2.49\bar{k}\right) = 0$

which gives, (isolating the $\bar{i}$, $\bar{j}$ and $\bar{k}$ components)

$$A_x + 0.417 + 0.83 = 0 \quad \Rightarrow \quad A_x = -1.25 \text{ kN}$$

$$A_y + 2 + 1.04 = 0 \quad \Rightarrow \quad A_y = -3.04 \text{ kN}$$

$$A_z + 4.05 - 2.49 = 0 \quad \Rightarrow \quad A_z = -1.56 \text{ kN}$$

$$\left.\begin{aligned} \vec{A} &= -1.25\bar{i} - 3.04\bar{j} - 1.56\bar{k} \text{ kN} \\ \vec{B} &= 0.417\bar{i} + 4.05\bar{k} \text{ kN} \\ \vec{T}_{CD} &= 0.83\bar{i} + 1.04\bar{j} - 2.49\bar{k} \text{ kN} \end{aligned}\right\} \text{Ans.}$$