Midterm 2 Examination

ECSE 330: Introduction to Electronics

March 10, 2017

11:35 AM – 12:55 PM

Professor: David V. Plant

Pertinent Information:

1) This is a closed-book examination, no notes permitted.

2) The examination consists of 4 problems and 36 total possible points.

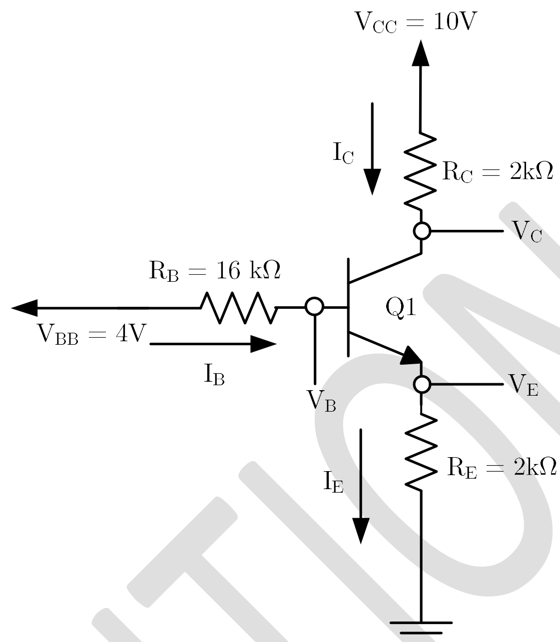
3) Only the Faculty Standard Calculator is permitted.

4) This paper will NOT be collected. Write all answers in the answer booklet only.

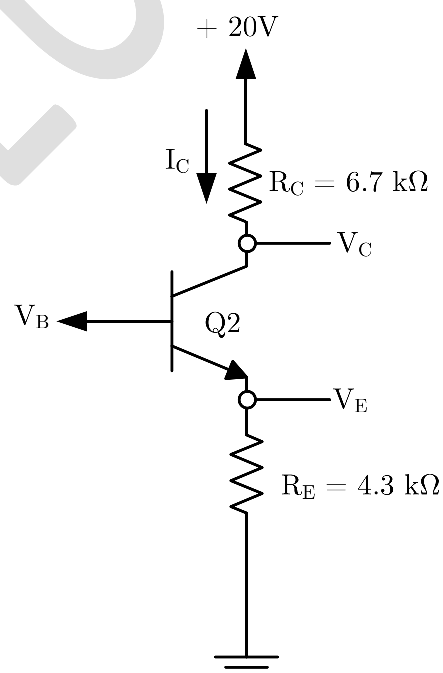
5) You must show all your work. An answer by itself is worth zero points.

Question #1 [8 points]

(a) [6 points] In the circuit shown below, find the values of the following voltages and currents (1 point per answer): V_B, V_E, V_C, I_B, I_C, and I_E. Clearly state your assumptions and check their validity. The transistor has $\beta = 100$ and $V_{BCon} = 0.4$ V.



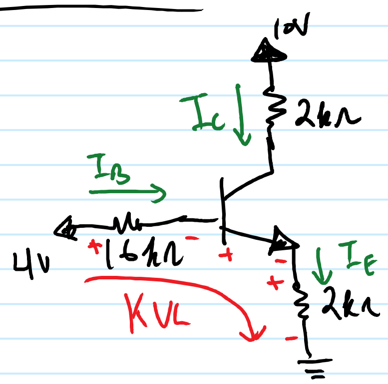
(b) [2 points] For the circuit shown below, find the highest voltage to which the base can be raised while the transistor remains in active mode. The transistor has $\alpha = 1$ ($\beta \to \infty$) and $V_{BCon} = 0.4$ V.



Question # 1

*Assume active mode of operation ($V_{BC} < 0.4V$)

(a)



$4V - I_B 16k\Omega - 0.7V - I_E 2k\Omega = 0$ ← 1eqn, 2 unknowns

$$I_B = \frac{I_E}{\beta+1} = \frac{I_E}{101}$$

$4V - \frac{I_E}{101}16k\Omega - 0.7V - I_E 2k\Omega = 0$ ← now, 1eqn 1 unknown

$$-I_E\left(\frac{16k\Omega}{101} + 2k\Omega\right) = -4V + 0.7V$$

$$I_E = \frac{4V - 0.7V}{\frac{16k\Omega}{101} + 2k\Omega} = \frac{3.3V}{2.1584k\Omega} = 1.53\,mA$$

$\boxed{I_E = 1.53\,mA}$

$$I_B = \frac{I_E}{\beta+1} = \frac{1.53\,mA}{101} = 0.015\,mA$$

$\boxed{I_B = 0.015\,mA}$

$$I_C = \beta I_B = 100(0.015\,mA) = 1.514\,mA$$

$\boxed{I_C = 1.514\,mA}$

$V_E = I_E 2k\Omega = \boxed{V_E = 3.06V}$

$V_B = 4V - I_B 16k\Omega = 4V - (0.015\,mA)(16k\Omega) = \boxed{V_B = 3.76V}$

$V_C = 10V - I_C(2k\Omega) = 10V - 1.514\,mA(2k\Omega) = \boxed{V_C = 6.97V}$

$V_B < V_C + 0.4V$

$3.76V < 6.97V$

∴ Q_1 is indeed in active mode ✓

b) for active mode, need transistor on AND $V_{BC} < V_{BCon}$ ← 0.4V

$V_{BC} < V_{BCon}$

$V_B < V_{BCon} + V_C$

⇓

at edge of saturation & active

$$V_B = V_{BCon} + V_C$$

↳ ohm's law

$$V_C = V_{CC} - I_C R_C$$

V_{CC} ← 20V

↳ Since $\alpha = 1$ ($\beta \to \infty$) $I_C = \alpha I_E$ [and $I_B = 0$]

$I_C = I_E$

V_{BE} ← 0.7V

$$I_C = I_E = \frac{V_B - V_{BE}}{R_E}$$

↓ substitute in V_C

$$V_C = V_{CC} - \left[\frac{V_B - V_{BE}}{R_E}\right]R_C = V_{CC} - \frac{R_C}{R_E}(V_B - V_{BE})$$

$$V_B = V_{Bcon} + V_C$$

$$V_B = V_{Bcon} + V_{CC} - \frac{R_C}{R_E}(V_B - V_{BE})$$

$$V_B = V_{Bcon} + V_{CC} - \frac{R_C}{R_E}V_B + \frac{R_C}{R_E}V_{BE}$$

$$V_B\left(1 + \frac{R_C}{R_E}\right) = V_{Bcon} + V_{CC} + \frac{R_C}{R_E}V_{BE}$$

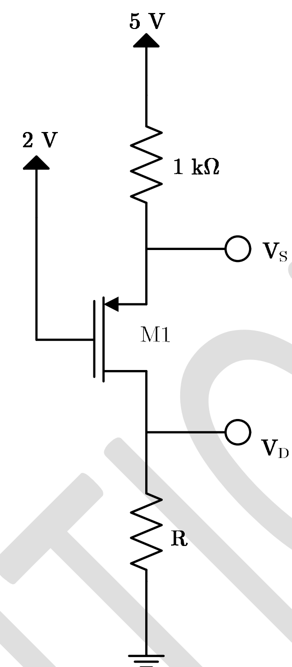
$$V_B = \frac{V_{Bcon} + V_{CC} + \frac{R_C}{R_E}V_{BE}}{\left(1 + \frac{R_C}{R_E}\right)}$$

$$V_B = \frac{0.4V + 20V + \frac{6.7k\Omega}{4.3k\Omega}0.7V}{\left(1 + \frac{6.7k\Omega}{4.3k\Omega}\right)} \qquad \left(\frac{R_C}{R_E} = \frac{6.7k\Omega}{4.3k\Omega} = 1.558\right)$$

$$\boxed{V_B = 8.40V}$$

Question #2 [9 points]

Consider the transistor biasing arrangement shown below. For all parts of this question, the transistor has $k_p' = 100$ µA/V^2, $V_{tp} = -1$ V, and assume that the body effect and the channel length modulation effect can be ignored, i.e., $\lambda = 0$. The voltages V_S and V_D, the current, resistor R, and the other parameters of the transistors are distinct for each subpart of the question.



(a) If the transistor is in the saturation region and given that (W/L) = 20 and R= 1 kΩ, determine:
   i. [1 point] the source voltage V_S
   ii. [1 point] the drain current I_D
   iii. [1 point] the drain voltage V_D

(b) [2 points] Consider another biasing scheme for the circuit above, where the width of the transistor, the ratio W/L, and the resistor R are unknown. The desired drain current I_D is 200 µA and drain voltage V_D is 1 V. Given that L = 0.2 µm, determine
   i. [1 point] the value required for R and the mode of operation of the transistor, and
   ii. [1 point] the value required for the width of the transistor W

(c) Given that (W/L) = 20 and V_S = 4 V.
   (i) [2 points] Find the condition on the value of R for the transistor to be in the triode regime.
   (ii) [2 points] Given R = 3.5 kΩ, find the drain current I_D in the triode region.

Question #2 $\quad k'_p = 100\,\mu A/V^2 \quad V_{tp} = -1\,V$

a) in saturation mode $\therefore$ $I_D = \frac{1}{2}k'_p \underbrace{\frac{W}{L}}_{20}(V_{GS}-V_{tp})^2 = \frac{1}{2}k'_p\frac{W}{L}(V_G - V_S - V_{tp})^2$

$W/L = 20$
$R = 1k\Omega$

$= \frac{1}{2}(100\times10^{-6})(20)(2V - V_S - (-1V))^2$

$I_D = 1\times10^{-3}\,A/V^2(3V - V_S)^2 \rightarrow$ 2 unknowns

MOSFET $\therefore$ $I_D = I_S = \frac{5 - V_S}{1k\Omega}$

$\frac{5V - V_S}{1k\Omega} = 1\times10^{-3}\,A/V^2\,(3V - V_S)^2 \rightarrow$ now, 1 unknown

$5 - V_S = 1\,A/V^2\,(3V - V_S)^2$

$5 - V_S = 9V - 6V_S + V_S^2$

$0 = V_S^2 - 5V_S + 4V \Rightarrow$ factors of $+4$ that add up to -5 are -4 & -1

$0 = (V_S - 4)(V_S - 1)$

$V_S = 4 \qquad V_S = 1$

$V_{GS} = 2V - 4V = -2V$ good solution $\qquad V_{GS} = 2V - 1V = +1V \Rightarrow$ PMOS would be off $\rightarrow$ bad solution

(i) $\boxed{V_S = 4V}$

(ii) $I_D = I_S = \frac{5V - 4V}{1k\Omega} = 1mA \quad \boxed{I_D = 1mA}$

(iii) $V_D = I_D R = 1mA(1k\Omega) = 1V \quad \boxed{V_D = 1V}$

b) $I_D = 200\,\mu A,\ V_D = 1V,\ L = 0.2\,\mu m$

(i) $R = \frac{V_D}{I_D} = \frac{1V}{200\,\mu A} = 5k\Omega \quad \boxed{R = 5k\Omega}$

for saturation mode $\quad V_{DS} \leq V_{GS} - V_{tp}$

$V_D - \cancel{V_S} \leq V_G - \cancel{V_S} - V_{tp}$

$V_D \leq V_G - V_{tp}$

$\leq 2V - (-1V)$

$1V \leq 3V$

(Note: V_D & V_G same as in part (a) $\therefore$ if (a) is in saturation, so is (b))

$\therefore$

$\boxed{\text{transistor is in saturation mode}}$

(ii) Since transistor in saturation mode: $\underbrace{I_D}_{200\,\mu A} = \frac{1}{2}\underbrace{k'_p}_{100\,\mu A/V^2}\frac{W}{\underbrace{L}_{L=0.2\,\mu m}}(\overbrace{V_{GS}}^{V_G=2V} - \underbrace{V_{tp}}_{-1V})^2$

need V_S, $V_S = 5V - \overset{I_D = 200uA}{I_S}(1k\Omega) = 5V - 0.2mA(1k\Omega) = 4.8V$

$V_{GS} = 2V - 4.8V = -2.8V$

$$I_D = \frac{1}{2}k'_p \frac{W}{L}(V_{GS} - V_{tp})^2$$

$$W = \frac{L \cdot I_D \cdot 2}{k'_p (V_{GS} - V_{tp})^2} = \frac{0.2um \cdot 200uA \cdot 2}{100uA/V^2(-2.8V-(-1V))^2} = \frac{80um \cdot \cancel{uA}}{100\cancel{uA}/V^2 \; 3.24V^2} = 0.247\ um$$

$$\boxed{W = 0.25um}$$

(c) $W/L = 20$ & $V_S = 4V$

(i) transistor in triode when $V_{DS} \geq V_{GS} - V_{tp}$

$V_{DS} \geq V_{GS} - V_{tp}$

$V_D - \cancel{V_S} \geq V_G - \cancel{V_S} - V_{tp}$

$V_D \geq V_G - V_{tp}$

$V_D \geq 2V - (-1V)$

need $V_D \geq 3V$ for triode mode

$V_D = I_D R$

$I_D R \geq 3V$

$R \geq \frac{3V}{I_D}$

$I_D = I_S = \frac{5V - V_S}{1k\Omega} = \frac{5-4V}{1k\Omega} = 1mA$

$\therefore\ R \geq \frac{3V}{1mA} = 3k\Omega$

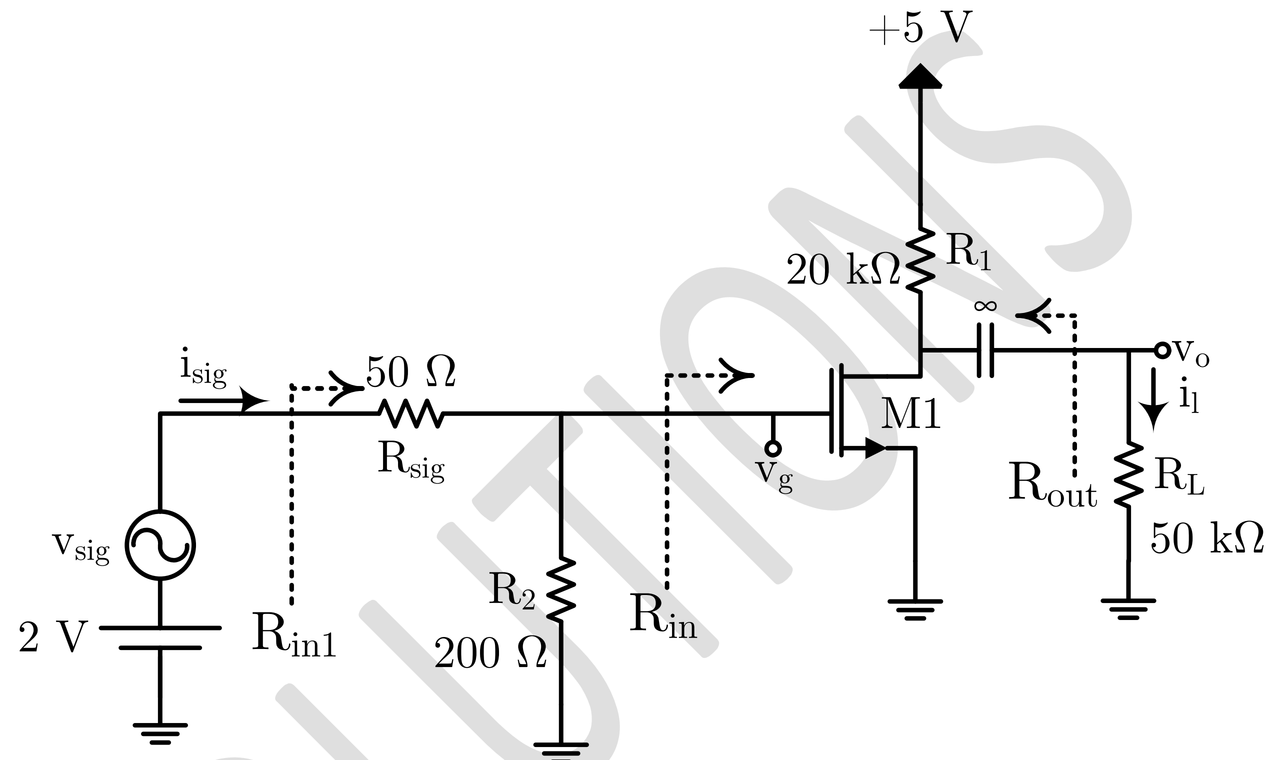
want $\boxed{R > 3k\Omega}$

(ii)

if $V_S = 4V$, then $I_D = I_S = \frac{5V - 4V}{1k\Omega} = 1mA$ (* ⇒ 2 easy points * ☺)

Question #3 [9 points]

The following figure depicts an amplifier circuit with the resistances indicated in the figure, infinitely large capacitances, and where v_{sig} is an AC-only signal source. The transistor is in saturation mode and has $k_n = 200\ \mu A/V^2$, $V_{tn} = 0.5$ V, an output resistance r_O of 413.2 kΩ due to the channel-length modulation (CLM) effect and a DC drain current I_D (including the CLM effect) of 127.24 µA.



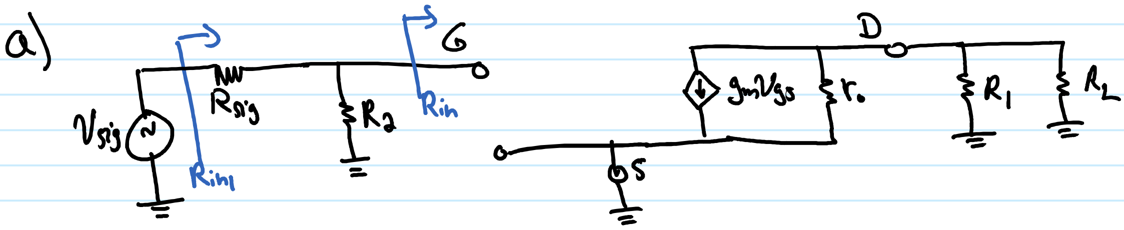
(a) [2 points] Draw the small-signal equivalent circuit and determine the small-signal parameter g_m.

(b) [3 points] Find the resistance seen at the voltage source (R_{in}), at the gate (R_{in1}), and at the output (R_{out}). Provide both the expression and the numerical value.

(c) [4 points] Determine the following small-signal voltage gains (i) $\frac{v_g}{v_{sig}}$, (ii) $\frac{v_o}{v_g}$, (iii) the overall voltage gain $\frac{v_o}{v_{sig}}$, and (iv) the overall current gain $\frac{i_l}{i_{sig}}$. Provide both the expression and the numerical value.

Question 3 ⇒ has channel-length modulation, must include effect of r_o
[M1 has source at AC ground, r_o is trivial to include]

a)



$$g_m = \frac{2 I_D}{V_{GS} - V_{tn}}$$ ⇒ need to find V_{GS}, quick DC analysis
voltage divider

$$V_{GS} = \frac{R_2}{R_{sig}+R_2}(2V) = 1.6V$$

$$g_m = \frac{2(123.24\mu A)}{1.6V - 0.5V} = \boxed{g_m = 0.23134\ mA/V}$$ $[231.34\ \mu A/V]$

b) because of oxide (insulator) at gate

$\boxed{R_{in} = \infty}$

at source:

$$R_{in1} = R_{sig} + R_2 \| \underbrace{R_{in}}_{\infty}$$
$$= R_{sig} + R_2$$
$$= 50\Omega + 200\Omega$$
$$\boxed{R_{in1} = 250\Omega}$$

* Note: typo in question sheet R_{in} & R_{in1} answers could be flipped

R_{out} ⇒ turn off signal source, $v_{sig}=0$ ⇒ $V_{gs}=0$ ⇒ $g_m V_{gs}=0$ [VDCS is open circuit]

$$R_{out} = R_1 \| r_o = 20\,k\Omega \| 413.2k\Omega = \boxed{19.08k\Omega = R_o}$$

c) (i) $\frac{V_g}{V_{sig}}$ ⇒ voltage divider $= \frac{R_2}{R_{sig}+R_2} = \frac{200\Omega}{200\Omega+50\Omega} = 4/5 = 0.8$

$$\boxed{\frac{V_o}{V_{sig}} = 0.8}$$

(ii) $\frac{V_o}{V_g}$ ⇒ $V_o = -g_m V_{gs}(r_o \| R_1 \| R_L)$

$V_{gs} = V_g$ because source terminal is at AC ground

$$\frac{V_o}{V_g} = -g_m(r_o \| R_1 \| R_L)$$

$$r_o \| R_1 \| R_L = \left(\frac{1}{413.2k\Omega} + \frac{1}{20k\Omega} + \frac{1}{50k\Omega}\right)^{-1} = 13.808\,k\Omega$$

$$\frac{V_o}{V_g} = (-0.231\ mA/V)(13.808\ V/mA) = -3.19\ V/V$$ $$\boxed{\frac{V_o}{V_g} = -3.19\ V/V}$$

(iii) $\frac{v_o}{v_{sig}} = \frac{v_o}{v_g} \times \frac{v_g}{v_{sig}} = (-3.19\ V/V)(0.8\ V/V) = -2.56\ V/V$

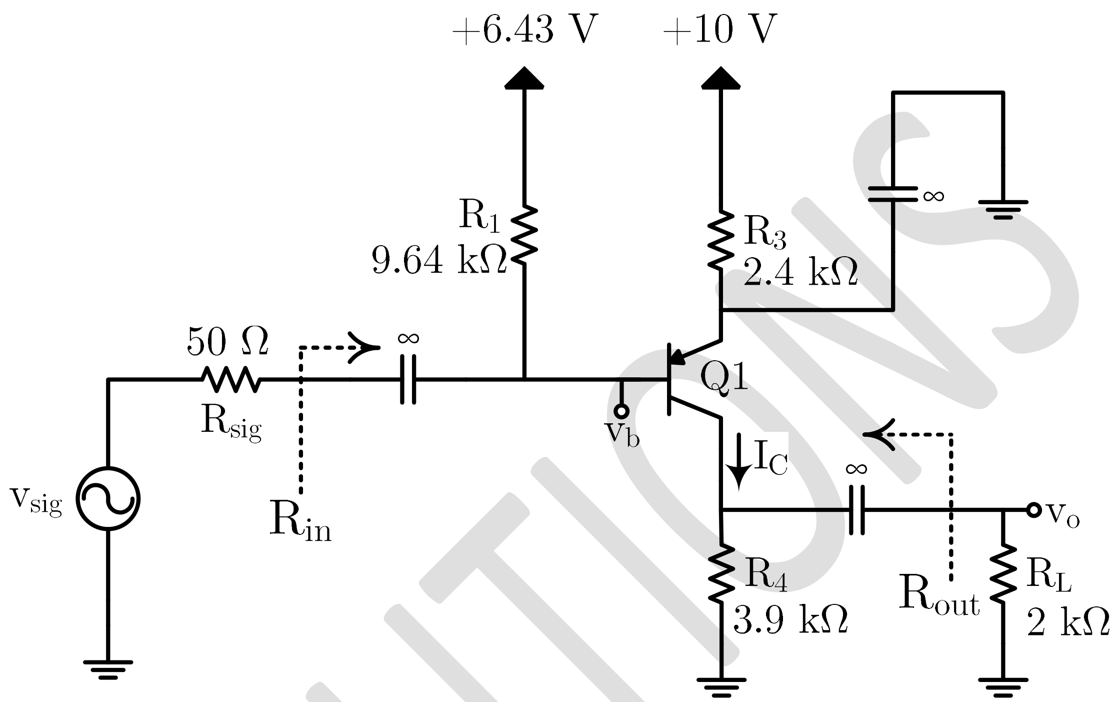
$$\boxed{\frac{v_o}{v_{sig}} = -2.56\ V/V}$$

(iv) $\frac{v_o}{v_{sig}} = \frac{i_L R_L}{i_{sig} R_{in1}} \Rightarrow \frac{i_L}{i_{sig}} = \frac{R_{in1}}{R_L}\left(\frac{v_o}{v_{sig}}\right) = \frac{250\Omega}{50\,000\Omega}(-2.56\ V/V)$

$$\boxed{\frac{i_L}{i_{sig}} = -0.0128\ A/A}$$

Question #4 [10 points]

The following figure depicts an amplifier circuit with the resistances indicated in the figure, infinitely large capacitances, and where v_{sig} is an AC-only signal source. The transistor is in active mode and has $\beta = 100$.



(a) [2 points] Perform a DC analysis and determine the current flowing through the collector (I_c).

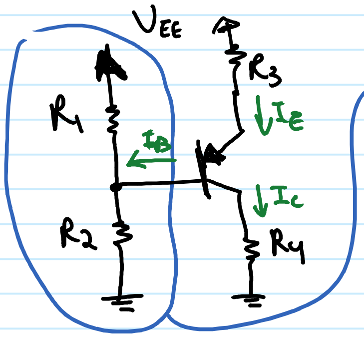
(b) [3 points] Draw the small-signal equivalent circuit, and determine the small-signal parameters g_m and r_π or r_e. Provide both the expression and the numerical value.

(c) [2 points] Calculate the small-signal resistance seen at the input (R_{in}) and at the output (R_{out}), as drawn in the schematic above. Provide both the expression and the numerical value.

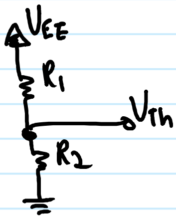
(d) [3 points] Determine the following small-signal voltage gains (i) $\frac{v_b}{v_{sig}}$, (ii) $\frac{v_o}{v_b}$, and (iii) the overall voltage gain $\frac{v_o}{v_{sig}}$. Provide both the expression and the numerical value.

Question 4

a) DC analysis : caps → o/c, AC sources off

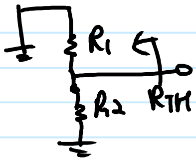


Thevenin equivalent

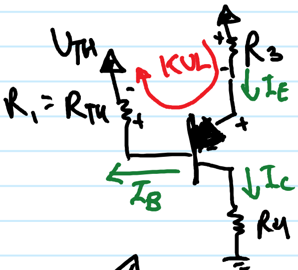


$$V_{Th} = \frac{R_2}{R_1+R_2} V_{EE} = \frac{27k\Omega}{27k\Omega + 15k\Omega}(10V)$$

$$V_{TH} = 6.4286\ V$$



$$R_{TH} = R_1 \| R_2 = \frac{15k\Omega \cdot 27k\Omega}{15k\Omega + 27k\Omega} = 9.6429\,k\Omega$$



note:
to save time
changed Q4 to start
from this point

KVL: $V_{EE} - I_E R_3 - V_{EB} - I_B R_{TH} = V_{TH}$

(or put both I_B & I_E in terms of I_C and solve directly for I_C)

$$I_B = \frac{I_E}{\beta+1} = \frac{I_E}{101}$$

$$V_{EE} - I_E R_3 - V_{EB} - \frac{I_E}{\beta+1} R_{TH} = V_{TH}$$

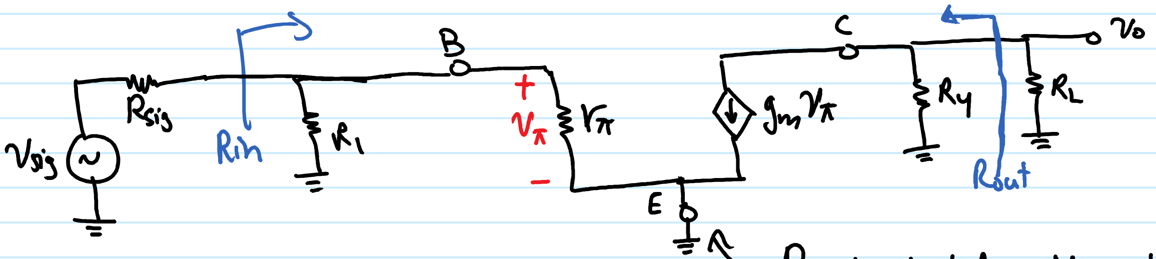
$$-I_E\left(R_3 + \frac{R_{TH}}{\beta+1}\right) = V_{TH} + V_{EB} - V_{EE}$$

$$I_E = \frac{V_{EE} - V_{TH} - V_{EB}}{\left(R_3 + \frac{R_{TH}}{\beta+1}\right)} = \frac{10V - 6.4286V - 0.7V}{2.4k\Omega + \frac{9.64286k\Omega}{101}} = \frac{2.8714V}{2.49555k\Omega}$$

$$I_E = 1.15\,mA$$

$$I_C = \alpha I_E = \left(\frac{\beta}{\beta+1}\right) I_E = 1.139\,mA = 1.14\,mA \Rightarrow \boxed{I_C = 1.14\,mA}$$

b)



R_3 is shorted out by path to AC ground via capacitor

from (a)

$$g_m = \frac{I_C}{V_T} = \frac{1.14\,mA}{25\,mV} \Rightarrow \boxed{g_m = 0.0456\ A/V}$$

$$r_\pi = \beta \frac{V_T}{I_C} = (100)\frac{25\,mV}{1.14\,mA} = 2192.98\,\Omega = 2.193\,k\Omega$$

$$\boxed{r_\pi = 2.19\,k\Omega}$$

c) $$R_{in} = R_1 \| r_\pi = \frac{2.19\,k\Omega \cdot 9.64\,k\Omega}{2.19\,k\Omega + 9.64\,k\Omega} = 1.78\,k\Omega$$

$$\boxed{R_{in} = 1.78\,k\Omega}$$

R_{out} ⇒ turn off signal source ∴ $v_{sig} = 0$ ⇒ $v_\pi = 0$ ⇒ $g_m v_\pi = 0$, ie, VDCS is off open-circuit

$$\boxed{R_{out} = R_4 = 3.9\,k\Omega}$$

d)

(i) $$v_b = \frac{r_\pi \| R_1}{r_\pi \| R_1 + R_{sig}} v_{sig} = \frac{R_{in}}{R_{in} + R_{sig}} v_{sig} \Rightarrow \frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.78\,k\Omega}{1.78\,k\Omega + 0.05\,k\Omega}$$

$$\boxed{\frac{v_b}{v_{sig}} = 0.97\,V/V}$$

(ii) $$\frac{v_o}{v_b} \Rightarrow v_o = -g_m v_\pi (R_4 \| R_L) \Rightarrow \frac{v_o}{v_b} = -g_m (R_4 \| R_L)$$

$v_b = v_\pi$ because emitter at AC ground

$$R_4 \| R_L = 1.322\,k\Omega$$

$$\frac{v_o}{v_b} = -(0.0456\,A/V)(1322.03\,V/A) = -60.285\,V/V$$

$$\boxed{\frac{v_o}{v_b} = -60.29\,V/V}$$

(iii) overall voltage gain

$$\frac{v_o}{v_{sig}} = \frac{v_o}{v_b} \times \frac{v_b}{v_{sig}} = -60.285\,V/V * 0.9728\,V/V = -58.645\,V/V$$

$$\boxed{\frac{v_o}{v_{sig}} = -58.65\,V/V \approx -58.48\,V/V}$$