Mid term Exam review questions

Problem 1:

American Mining Company is interested in obtaining quick estimates of the supply and demand curves for coal. The firm' s research department informs you that the elasticity of supply is approximately 1.7, the elasticity of demand is approximately - 0.85, and the current price and quantity are $41 and 1,206, respectively. Price is measured in dollars per ton, quantity the number of tons per week.

a. Estimate linear supply and demand curves at the current price and quantity.

b. What impact would a 10% increase in demand have on the equilibrium price and quantity?

c. If the government refused to let American raise the price when demand increased in (b) above, what shortage is created?

Problem 2:

Consider following production function of DVDs $q = L^{0.5}K^{0.5}$.

In the short run, the capital units are fixed at K=100 and the cost of capital and labor are equal to $ 4 and $ 2 respectively.

1. What is the equation of the total and average total cost as a function of quantity?
2. What is the equation of the marginal cost in the short term as a function of quantity?
3. At what output level the firm is minimizing its cost on the short term?
4. How many units of labor are required for the firm to minimize its cost?

In the long term, both capital and labor are changing.

5. What is the optimal input ratio of labor (L) and capital (K) for the production?
6. What is the cost of producing 500 DVDs at the optimal level?

Problem 3:

The Montreal Press produces memo pads in its local shop. The company can rent its equipment and hire workers at competitive rates. Equipment needed for this operation can be rented at $48 per hour, and labor can be hired at $12 per worker hour. The company has allocated $200,000 for the initial run of memo pads. The production function using available technology can be expressed as:

$$Q = 0.25K^{0.2}L^{0.8}$$

where Q represents memo pads (boxes per hour), K denotes capital input (units per hour), and L denotes labor input (units of worker time per hour).

a) Find the optimal input ratio in the long run.

b) Determine the appropriate input mix to get the greatest output for a cost of $200,000 for a production run of memo pads. Compute the level of output.

c) Calculate the new input mix in the short run (keeping capital fixed) if production were increased by 200 units per hour.

d) Calculate the new cost of production in the short run.

e) Draw isoquents and isocost line, show the long term mix and the short term one and show the short term expansion path. Label your graph.

Solutions to review problems for the mid term:

Problem 1:

a. First we estimate the demand curve
$Q = a_0 - b_0P$
$b_0 = 25$
$Q = a_0 - b_0P$
$1206 = a_0 - 25(41)$
$1206 = a_0 - 1025$
$a_0 = 2231$
$Q_0 = 2231 - 25P$
Next, we estimate the supply curve
$Q = a_1 + b_1P$
$b_1 = 50$
$Q = a_1 + b_1P$
$1206 = a_1 + 50(41)$
$a_1 = -844$
$Qs = -844 + 50P$
b. Multiply demand equation by 1.10
$1.10\,(2231 - 25P)$
Qd' = Qs and solve
$Qs = -844 + 50P$
Set Qd' = Qs and solve.
$2454.1 - 27.5P = -844 + 50P$
$3298.1 = 77.5P$
$P = 42.56$
Substitute P into Qd' to find quantity demanded
$Qd' = 2454.1 - 27.5(42.56)$
$Qd' = 1283.7$ or 1284
c. Since price cannot rise, the shortage will be the quantity demanded with the new demand minus the quantity supplied with the unchanged supply
Quantity demanded: $Q = 2454.1 - 27.5(41) = 1326.6$
Quantity supplied: $Q = -844 + 50(41) = 1206.0$
Shortage = $1326.6 - 1206.0 = 120.6$ tons per week.

Problem 2:

1. The total cost is defined as Cost=FC+VC

FC=4*100=400
We substitute the fixed capital into the production function equation: $q=L^{0.5}(100)^{0.5} \Leftrightarrow L=q^2/100$
Hence,
$VC=2/100\, q^2$
$TC=400+0.02\, q^2$

2. The first derivative function of the total cost function is: $MC=0.04q$

3. The minimum cost is achieved at: MC=ATC
$0.04q=400/q+0.02q \Leftrightarrow q=141.42$

4. The labor needed to achieve the lowest level of cost in the short tem is: $L=141.42^2/100=200$

5. The optimal ratio in the long run is: $MRTS=w/r \Leftrightarrow K/L=w/r=2/4$

   $K=0.5L$
6. $500=L^{0.5}\,(0.5L)^{0.5}=0.5^{0.5}L$

$L = 500/0.5^{0.5} = 707.10$
$K = 0.5*707.10 = 353.55$
Cost $= 707.10*2 + 353.55*4 = 2828.4$

Problem 3:

(a)

$$MP_L = 0.25 \times 0.8 \times K^{0.2} L^{-0.2}$$
$$MP_K = 0.25 \times 0.2 \times K^{-0.8} L^{0.8}$$

$$\frac{MP_L}{MP_K} = \frac{0.25 \times 0.8 \times K^{0.2} L^{-0.2}}{0.25 \times 0.2 \times K^{-0.8} L^{0.8}} = 4 \frac{K}{L}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$
$$4\frac{K}{L} = \frac{12}{48}$$
$$\underline{L = 16K}$$

(b)

$$200{,}000 = 12L + 48K \iff 200{,}000 = 12(16K) + 48K$$
$$200{,}000 = 240K \iff K = 833.33$$

$$L = 16 \times 833.33 = 13\,333.33$$

$$q = 0.25(833.33)^{0.2}(13333.33)^{0.8} = 1914$$

(c) New output $q' = 1914 + 200 = 2114$

$$2114 = 0.25(833.33)^{0.2} L^{0.8}$$

$$L = \left(\frac{2114}{0.25(833.33)^{0.2}}\right)^{1/0.8} = 15092.17$$

(d)

$$\text{Cost} = 48 \times 833.33 + 12 \times 15092.17 = 221105.99$$

(e) Check the graph in the book, page: 253.

Problems for Chapter 5

Problem 1

The production function of widgets can be expressed as: $Q = 50K^{1.25}L^{0.5}$, where Q = units of widgets per month, K = capital input (units of machine hour), and L = labour input (units of worker hour). The company can rent its equipment and hire workers at competitive rates. Equipment needed for this operation can be rented at $100 per hour, and labour can be hired at $25 per worker hour. The

a) Find the marginal rate of technical substitution.
b) Determine the optimal input mix to produce an output of 20,000 units. Compute the cost of production.
c) Suppose the rate of worker hour increases to $40. Determine the optimal input mix to produce 20,000 units. Calculate the cost of production.
d) Suppose the cost of production remains constant as in part (b) and the worker hour rate is $40. Determine the greatest output that the firm can produce.
e) Calculate the substitution effect and the scale effect in the employment of labour to the increase in the worker hour rate.
f) Draw the isoquants and isoexpenditure lines in parts (b) to (d), and illustrate the substitution and scale effect.

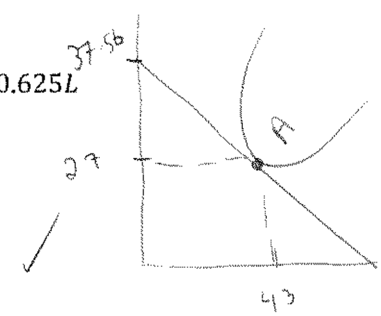
Answer:

a) $MP_L = 50*0.5*K^{1.25}L^{-0.5}$

$MP_K = 50*1.25*K^{0.25}L^{0.5}$

$\text{MRTS} = \frac{MP_L}{MP_K} = \frac{50*0.5*K^{1.25}L^{-0.5}}{50*1.25*K^{0.25}L^{0.5}} = \frac{2K}{5L}$

b) Optimal K/L ratio: $\frac{MP_L}{MP_K} = \frac{w}{c} = \frac{25}{100} \rightarrow \frac{2K}{5L} = \frac{1}{4} \rightarrow \frac{K}{L} = \frac{5}{8} \rightarrow K = \frac{5L}{8} = 0.625L$



$Q = 20000 = 50*(0.625L)^{1.25}*L^{0.5} = 50*0.625^{1.25}*L^{1.75}$

$L = [20000/(50*0.625^{1.25})]^{1/1.75} = 42.9231$ units

$K = 0.625*42.9231 = 26.8270$ units

Cost of production = 25L + 100K = 25*42.9231 + 100*26.8270 = $3,755.7775

3756 = 25(6)+100K

c) The worker hour rate = $40

$$\frac{MP_L}{MP_K} = \frac{w}{c} = \frac{40}{100} \rightarrow \frac{2K}{5L} = \frac{2}{5} \rightarrow K = L$$

$Q = 20000 = 50K^{1.25}L^{0.5} = 50L^{1.25}L^{0.5} = 50L^{1.75}$

$L = 20000/50L^{1.75} = 30.6825$ units = K

New cost of production = 40*30.6825 + 100*30.6825 = $4,295.5567

d) Production cost = $3,755.7775 and w = $40

3755.7775 = 40L + 100K = 40L + 100L = 140L

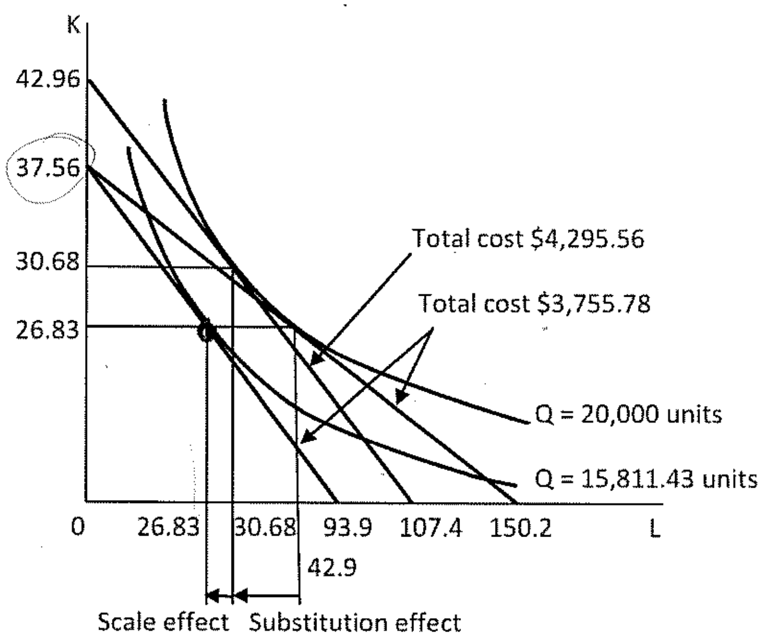
L = 3755.7775/140 = 26.8270 units = K

The greatest output $Q = 50*26.8270^{1.25}*26.8270^{0.5} = 15{,}811.4317$ units

e) The substitution effect in the employment of labour = 42.9231 – 30.6825 = 12.2406 units

The scale effect in the employment of labour = 30.6825 – 26.8270 = 3.8555 units

f)



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