Assignment II Solutions – Ravi Mateti

FINA 455

<u>You can thank me later</u>

Use the following discount factors to answer questions 1 and 2.

T	$Z(0,T)$
0.50	0.9940
1.00	0.9880
1.50	0.9740
2.00	0.9620
2.50	0.9460
3.00	0.9330
3.50	0.9170
4.00	0.8950
4.50	0.8770
5.00	0.8580
5.50	0.8340
6.00	0.8130
6.50	0.7990
7.00	0.7760
7.50	0.7570
8.00	0.7360

1. What are the forward discount factor $F(0,3,5)$, the semi-annually compounded forward rate $f_2(0,3,5)$, and the continuously compounded forward rate $f(0,3,5)$?

   A. 0.9098, 4.3126%, 4.2995%

   B. 0.9113, 4.2739%, 4.2167%

   C. 0.9196, 4.2343%, 4.1901%

   D. 0.9217, 4.1887%, 4.1123%

   E. 0.9286, 4.0991%, 4.1773%

   Solution:

   $F(0,3,5)$ *= Z(0,5)/Z(0,3) = 0.8580/0.9330 = 0.9196*

   $$\frac{1}{\left(1+\frac{f_2(0,3,5)}{2}\right)^4} = F(0,3,5)$$ *We have used the definitions of semi-annually compounded forward rate f2(0, 3, 5) and the forward discount factor F (0, 3, 5) to arrive at this equation.Note that the exponent in the denominator is 4 because there are 4 half-year periods in 2 years. Use the value of* $F(0,3,5)$ *and solve for*

$f_2(0,3,5)$ = 4.2342%.

$e^{-f(0,3,5)\times(5-3)} = F(0,3,5)$ *We have used the definitions of continuously compounded forward rate $f(0,3,5)$ and the forward discount factor $F(0,3,5)$. Solve for $f(0,3,5)$ = 4.1901%.*

2. What are $F(0,5,6)$, $f2(0,5,6)$ and $f(0,5,6)$?

   A. 0.9476, 5.4605%, 5.3873%

   B. 0.9552, 5.4355%, 5.4217%

   C. 0.9689, 5.3186%, 5.3013%

   D. 0.9713, 5.2792%, 5.2694%

   E. 0.9720, 5.2573%, 5.2431%

   Solution:

   Proceed along the same lines as in the above question.

3. Which of the following statements is true?

   i. The greater the discount factor Z(0, T), the greater is the corresponding spot rate.

   ii. The greater the forward discount factor F(0, t, T), the lower is the corresponding forward rate.

   A. i only

   B. ii only

   C. Both

   D. None

   Solution:

   (i) We use the spot rate $r(0,T)$ to discount a dollar to be received in the future at time T to arrive at its present value, which is the discount factor Z(0, T). The higher the spot rate, the lower is the discount factor, and vice-versa. Thus, the spot rate and the discount factor are inversely related.

   (ii) The same relationship applies to forward rate and forward discount factor. The spot rate is replaced with the forward rate and the discount factor is

replaced with the forward discount factor.

4. Consider a swap where the cash flows are exchanged every six months. Assume that the 6-month spot rate is 3.8%, the 1-year spot rate is 4.1%, and the 1.5-year spot rate 4.5%. If the 2-year swap rate is 5%, what is the 2-year spot rate? All rates are continuously compounded.

   A. 4.6728%

   B. 4.7133%

   C. 4.8506%

   D. 4.9651%

   E. None of the above
   Solution:
   Since the 2-year swap rate is 5%, this can be interpreted as the 2-year par bond coupon rate. The coupons are paid half-yearly. Let the 2-year spot rate be denoted by r(0,2).
   100 = 2.5exp(-0.038 x 0.5) + 2.5exp(-0.041 x 1) + 2.5exp(-0.045 x 1.5) + 102.5exp{-r(0,2) x 2}
   Solve for r(0,2) = 0.049651

5. Continuing with the above question, if the 2.5-year swap rate is 5.25%, what is the 2.5-year spot rate?

   A. 5.219%

   B. 5.286%

   C. 5.348%

   D. 5.893%

   E. None of the above
   Solution:
   100 = 2.625exp(-0.038 x 0.5) + 2.625exp(-0.041 x 1) + 2.625exp(-0.045 x 1.5) + 2.625exp(-0.049651 x 2) + 102.625exp{-r(0,2.5) x 2.5}
   Solve for r(0,2.5) = 0.05219

6. Suppose that the 6-month, 1-year, 1.5-year, and 2-year continuously compounded spot rates are 4.0%, 4.3%, 4.7%, and 5.2%, respectively. What is the 2-year swap rate where payments are made semi-annually?

   A. 4.8216%

B. 4.9733%

C. 5.0609%

D. 5.1121%

E. 5.2382%
Solution:
Let the swap rate be K. Then,
100 = 0.5K{exp(-0.04 x 0.5)} + 0.5K{exp(-0.043 x 1)} +0.5K{exp(-0.047 x 1.5)} + (100 + 0.5K)exp(-0.052 x 2)

Solve for K = 5.2382%

7. The following rates are posted by a swaps dealer:

Maturity (years)	Bid	Ask
3	5.28%	5.32%
5	5.42%	5.48%
10	6.17%	6.26%

A company has a 5-year floating rate loan. The interest paid is LIBOR + 15bp. Using a swap, this can be converted to ________ fixed rate loan.

A. 5.57%

B. 5.63%

C. 5.72%

D. 5.78%

E. None of the above
Solution:
The company enters into a 5-year swap in which it pays fixed (5.48%) and receives floating (LIBOR). The net rate paid is:
(LIBOR + 15bp) – (LIBOR) + (5.48%) = 0.0015 + 0.0548 = 0.0563

8. Using the table in the above question, determine which of the following statements is true.
   i. A 10-year floating rate <u>investment</u> with LIBOR + 8bp can be converted to a fixed rate investment of 6.09%.
   ii. A 3-year fixed rate <u>loan</u> of 5.43% can be converted to a floating loan of LIBOR + 15bp.

A. i only

B. ii only

C. Both of the above

D. None of the above

Solution:

(i) *Enter into a 10-year swap paying LIBOR and receiving fixed rate of 6.17%. The net rate received is:*
(LIBOR + 8bp) – (LIBOR) + (6.17%) = 0.0008 + 0.0617 = 0.0625

(ii) *Enter into a 3-year swap paying LIBOR and receiving fixed rate of 5.28%. The net rate paid is:*
(5.43%) – (5.28%) + LIBOR = LIBOR + 15bp

9. Use the following discount factors to determine the swap rate for the following maturities: 0.50, 1.00, and 2.00 years. Note that the swap rates will have to be expressed as an annual rate, though the payments in this question are made quarterly.

T	$Z(0, T)$
0.25	0.9840
0.50	0.9680
0.75	0.9520
1.00	0.9360
1.25	0.9190
1.50	0.9040
1.75	0.8880
2.00	0.8730

A. 6.118%, 6.228%, and 6.614%
B. 6.223%, 6.333%, and 6.518%
C. 6.335%, 6.445%, and 6.626%
D. 6.446%, 6.556%, and 6.718%
E. 6.557%, 6.667%, and 6.843%

Solution:
Swap rate for 0.50-year maturity:
Note that the payments are made quarterly.
100 = 0.9840(C/4) + 0.9680 (C/4 + 100)
Solve for C = 6.557. Therefore, the swap rate is 6.5557%.

Swap rate for 1.00-year maturity:

100 = 0.9840(C/4) + 0.9680 (C/4) + 0.9520(C/4) + 0.9360(C/4 + 100)
Solve for C = 6.667. Therefore, the swap rate is 6.667%.

Swap rate for 2.00-year maturity:
Proceed along the same lines as above.

10. The 1-year to 7-year spot rates are 4.2%, 4.4%, 4.7%, 5.1%, 5.6% , 6.3%, and 6.6%, respectively. These rates are annually compounded rates. Compute the forward rate yield curve at the end of three years (i.e., forward rates at a certain time in the future, 3 years in this question, for various maturities). What are the forward rates for 1-year to 4-year maturities?
    a. 6.102%, 6.558%, 7.670%, and 7.881%
    b. 6.109%, 6.665%, 7.780%, and 7.948%
    c. 6.208%, 6.774%, 7.796%, and 7,980%
    d. 6.253%, 6.838%, 7,859%, and 8.005%
    e. 6.309%, 6.965%, 7.924%, and 8.048%

    Solution:
    To find the forward yield curve after year t, we are asking what is the 1-year forward rate after t years F(0, t, t+1), what is the 2-year forward rate after t years F(0, t, t+2) and so on. Therefore, to plot the forward yield curve, we just have to find the forward rates starting after t years for various maturities.

    Let $f_1(0,t,T)$ *be the forward rate starting at time t and ending at time T. This forward rate is for the length of time (T-t). The subscript 1 of* f_1 *indicates that the rate is compounded once a year. More generally, if it is* $f_n(0,t,T)$, *the rate is compounded n times a year.*

    $r_1(0,T)$ *is the spot rate for T-year maturity. The subscript 1 shows, just like above, that the rate is compounded once a year.*

    When the rates are discretely compounded (once a year in the question), the relationship between the spot rates and the forward rates is as follows:

    $$\left(1+r(0,T)\right)^T = \left(1+r(0,t)\right)^t \times \left((1+f_1(0,t,T)\right)^{(T-t)}$$

    Or

    $$f_1(0,t,T) = \left(\frac{\left(1+r(0,T)\right)^T}{\left(1+r(0,t)\right)^t}\right)^{\frac{1}{T-t}} - 1$$

$$f_1(0,3,4) = \left\{ \frac{[1+r_1(0,4)]^4}{[1+r_1(0,3)]^3} \right\}^{\frac{1}{4-3}} - 1$$

This will give you the forward rate for a length of 1 year

$$f_1(0,3,5) = \left\{ \frac{[1+r_1(0,5)]^5}{[1+r_1(0,3)]^3} \right\}^{\frac{1}{5-3}} - 1$$

This will give you the forward rate for a length of 2 years

$$f_1(0,3,6) = \left\{ \frac{[1+r_1(0,6)]^6}{[1+r_1(0,3)]^3} \right\}^{\frac{1}{6-3}} - 1$$

This will give you the forward rate for a length of 3 years

$$f_1(0,3,7) = \left\{ \frac{[1+r_1(0,7)]^7}{[1+r_1(0,3)]^3} \right\}^{\frac{1}{7-3}} - 1$$

This will give you the forward rate for a length of 4 years

Substitute the values of the spot rates in the above equations and plot the forward yield curve.

11. For the above spot rates, compute the forward rate yield curve after four years. What are the forward rates for 1-year to 3-year maturities?
    a. 7.519%, 8.687%, and 8.552%
    b. 7.624%, 8.741%, and 8.633%
    c. 7.709%, 8.898%, and 8.722%
    d. 7.782%, 8.911%, and 8.806%
    e. 8.105%, 8.944%, and 8.873%

    Solution:
    Proceed along the same lines as in the above question.

12. For the spot rates in problem 10, compute the par coupon yield curve. What are the par coupon yields for maturities of 3 and 4 years? (Par coupon yield curve is the ytm of par coupon bonds of different maturities. Since the coupon rate is equal to ytm for par coupon bonds, par coupon yield curve gives the coupon rates of par coupon bonds of different maturities).

a. 4.683% and 5.057%
b. 4.701% and 4.508%
c. 4.792% and 4.566%
d. 4.815% and 4.645%
e. 4.904% and 4.712%

Solution:
This is the same as question 9. In question 11 we found the swap rates, which are nothing but par coupon yields. There, the given data was discount factors. Here, the given data is spot rates (compounded yearly). The discount factors $Z(0,T)$ *can be determined from the yearly compounded spot rates* $r_1(0,T)$ *as*

$$Z(0,T) = \frac{1}{1 + r_1(0,T)^T}$$

13. An investor in an *n*-year coupon paying bond reinvests coupons until end of year *n* at forward rates prevailing today. What is the realized rate of return?
a. *n*-year yield to maturity
b. *n*-year par yield
c. *n*-year forward rate
d. *n*-year zero coupon rate
e. None of the above

See your class notes.

14. Which of the following statements is true?
i. When the yield curve is rising, the forward rate curve is above it.
ii. When the yield curve is falling, the forward rate curve is below it.
a. i only
b. ii only
c. Both i and ii
d. Neither i nor ii

Solution:
Both bonds earn the 7-year spot rate.

15. Quite often the forward rates are readily available (as from Eurodollar futures contracts, which are quarterly compounded nominal rates but can easily be converted to continuously compounded rates) but not the spot rates. We saw that

the <u>continuously</u> compounded spot rate, $r(0,\mathrm{T}_n)$, is given by the following equation when the maturities of the <u>continuously</u> compounded forward rates are constant (Δ):

$$r(0,\mathrm{T}_n) = \frac{\Delta}{T_n}\sum_{i=i}^{n} f(0,T_{i-1},T_i)$$

where $T_i - T_{i-1} = \Delta$ (a constant) for all i

The above equation tells us that the continuously compounded spot rate is the simple average of the continuously compounded forward rates. For example, if

$f(0,0,0.25) = r(0,0.25) = 4.8\%$

$f(0,0.25,0.50) = 5.2\%$

$f(0,0,50,0.75) = 5.6\%$

we see that the constant maturity of the forward rates is $\Delta = 0.25$ and $T_n = 0.75$ because the last forward rate given above starts at 0.50 years and ends at 0.75 years. Therefore,

$$r(0,\mathrm{T}_n) = r(0,0.75) = \frac{0.25}{0.75}(0.048+0.052+0.056)$$

$$= \frac{1}{3}(0.048+0.052+0.056) = 0.052 = 5.2\%$$

This is just the simple average of three numbers.

Now assume that the maturities of the continuously compounded forward rates are not constant and the forward rates are:

$f(0,0,0.25) = 4\%, f(0,0.25,0.75) = 4.5\%, f(0,0.75,2.00) = 5\%$

What is the continuously compounded 2-year spot rate, $r(0,2)$?

A. 4.00%
B. 4.20%
C. 4.50%
D. 4.60%
E. 4.75%

Solution:

We see that the maturities of the given forward constant rates are not constant – they are 0.25, 0.50, and 1.25 years, respectively. The sum of the maturities is 2 years. We find the weighted average of forward rates to find the 2-year spot rate:

$$r(0,2) = \frac{0.25}{2}(4\%) + \frac{0.50}{2}(4.5\%) + \frac{1.25}{2}(5\%) = 4.75\%$$

16. For the above problem, what is the spot rate expressed in terms of quarterly compounding?
   A. 4.0319%
   B. 4.2421%
   C. 4.5263%
   D. 4.6377%
   E. 4.7783%

   Solution:
   {exp(0.0475 X 0.25) – 1} X 4 = 4.7783%
   Or do by any other method you are comfortable with.

17. If $f(0,0,0.5) = 4\%$, $f(0,0.5,2.0) = 4.5\%$, $f(0,2,5) = 5\%$, and $f(0,5,6) = 5.8\%$, what is $r(0,6)$? All rates are continuously compounded?
   A. 4.875%
   B. 4.885%
   C. 4.925%
   D. 5.135%
   E. 5.215%

   Solution:
   Note that the maturities of the given forward rates are not constant. Therefore, we have to use the weighted average of these rates to find the six-year spot rate.
   $$r(0,6) = \frac{0.5}{6}(4\%) + \frac{1.5}{6}(4.5\%) + \frac{3}{6}(5\%) + \frac{1}{6}(5.8\%) = 4.925\%$$

18. Spot rates can be derived from Treasury bond prices, and this is called the Treasury yield curve. As we have seen above, spot rates can also be derived from swap rates. This is called the LIBOR yield curve because LIBOR is the underlying floating rate of swaps. Which of the following statements is true above the relationship between the two curves?
   i. Only when the curves are upward sloping, the LIBOR yield curve is above the Treasury yield curve.
   ii. Only when the curves are downward sloping, the Treasury yield curve is above the LIBOR yield curve.
   iii. The LIBOR yield curve is always above the Treasury yield curve.
   A. i only
   B. ii only
   C. iii only
   D. None of the above

See your class notes.

19. Which of the following statements is true?
    i. Financial institutions use the LIBOR yield curve, and not the Treasury yield curve, to determine the time value of money.
    ii. TED spread is the difference between the return on 3-month Eurodollar deposit and return on 3-month Treasury bills. If the TED spread is larger than usual, it is a sign of the economy weakening. (Search in Google for TED spread)

    A. i only
    B. ii only
    C. Both of the above
    D. None of the above

20. Which of the following statements are true about Eurodollar futures contracts?
    i. The asset underlying Eurodollar futures contracts is $1 million, 3-month Eurodollar time deposit.
    ii. The price of Eurodollar futures is quoted as 100 – R, where R is 3-month LIBOR rate with quarterly compounding expressed as percentage.
    iii. For every 0.01 decrease in price leads to $25 loss in long position.

    A. i and ii only
    B. ii and iii only
    C. i and iii only
    D. All of the above
    E. None of the above

See your class notes.

21. Today is March 1, 2015. Your company plans to invest $5 million after 1-year for a period of 3 months. You are entrusted with managing the interest rate risk. You decide to trade in March 2016 Eurodollar futures when the price is 96.82. Suppose that in March 2016 when you close your position in the contracts, the price is 95.79. What is your profit or loss in the futures market?

    A. Profit of $12,875
    B. Loss of $12,875
    C. Profit of $10,125
    D. Loss of $10,125
    E. None of the above

Solution:
Since your company plans to invest in the future, you should buy Eurodollar futures contracts. How many? 5miilion/1million = 5 contracts. Since you are going to buy Eurodollar futures, you gain $25 for 1 basis point increase in the price and lose $25 for 1 basis point decrease in the price.
Initial price of Eurodollar futures = 96.82, and the final price = 95.79
The change in price = 95.79 – 96.82 = -1.03% = -103 basis points
In basis points, the calculation is (95.79 – 96.82) x 100 = -103 basis points
*Since the price has decreased, you make a **loss** of 103 x 25 x 5 = $12,875.*

22. Which of the following are the tools of monetary policy of central banks?
    i. Open market operations
    ii. Reserve requirements
    iii. Federal discount rate
    A. i and ii only
    B. ii and iii only
    C. i and iii only
    D. All of the above
    E. None of the above

    See your class notes.

23. When the central bank sells Treasury securities to financial institutions, it tends to increase the federal funds rate.
    A. True
    B. False

    See your class notes.

24. You are given the following discount factors:

T	$Z(0, T)$
0.50	0.9940
1.00	0.9880
1.50	0.9740
2.00	0.9620
2.50	0.9460
3.00	0.9330
3.50	0.9170
4.00	0.8950

You are told that the price of a European Put option on a 1-year zero coupon bond, with T = 0.5 (maturity of the option) and K = 99.35 is 0.11. What is the price of a European Call option with the exact same specification?

A. 0.1561

B. 0.1629

C. 0.1713

D. 0.1827

E. None of the above

Solution:

The put-call parity for bonds is as follows:

Put + present value of forward price of bond

= Call + present value of strike price

The 0.50-year forward price of the 1-year zero coupon bond is:

100 x Z(0,1) / Z(0,0.5) = 100 x 0.9880/0.9940 = $99.396

The present value of the forward price is 99.396 x 0.9940 = $98.80

The present value of the strike price is 99.35 x 0.9940 = $98.754

Now, using the put-call parity, the price of the European call = 0.11 + 98.80 – 98.754 = $0.156

25. You are given the following discount factors:

T	$Z(0, T)$
0.50	0.9940
1.00	0.9880
1.50	0.9740
2.00	0.9620
2.50	0.9460
3.00	0.9330
3.50	0.9170
4.00	0.8950

You are told that the price of a European Put option on a 4-year fixed rate bond paying 5% semiannually, with T = 2 and K = 101 is 3.05. What is the price of a European Call option with the exact same specification?

A. 4.4275

B. 4.6155

C. 4.8295

D. 5.0115

E. None of the above

Solution:

We first have to find the 2-year forward price of the bond and then its present value. After 2 years, the cash flows remaining form the bond are 2.50, 2.50, 2.50, and 102.50.

2-year forward price = (2.50 x 0.946/0.962) + (2.50 x 0.933/0.962) + (2.50 x 0.917/0.962) + (102.5 x 0.895/0.962) = $102.627

PV of 2-year forward price = 102.627 x 0.962 = $98.727

PV of strike price = 101 x 0.962 = $97.162

Call price = 3.05 + 98.727 – 97.162 = $4.6155