Problem 1. (20 marks)[1]

Data for this month's advertising problem for a certain company are described in the table below:

Media Type	Number of Customers Reached	Cost ($) per Advertisement	Maximum Time Available per Month	Exposure Quality Units
Daytime TV (DTV)	1000	1500	15	65
Evening TV (ETV)	2000	3000	10	90
Daily newspaper (DN)	1500	400	25	40
Sunday newspaper magazine (SN)	2500	1000	4	60
Radio (R)	300	100	30	20

At most $30,000 can be spent on advertisements this month. There must be at least 10 TV ads. At most $18,000 may be spent on TV ads. At least 50,000 customers must be reached. The objective is to maximize total exposure quality.

The SOLVER formulation and output is shown below.

	DTV	ETV	DN	SN	R			
	10	0	25	2	30			
	65	90	40	60	20	2370		
Available DTV	1					10	<=	15
Available ETV		1				0	<=	10
Available DN			1			25	<=	25
Available SN				1		2	<=	4
Available R					1	30	<=	30
Budget	1500	3000	400	1000	100	30000	<=	30000
TV restriction 1	1	1				10	>=	10
TV restriction 2	1500	3000				15000	<=	18000
Customers reached	1000	2000	1500	2500	300	61500	>=	50000

[1] This problem is taken from Anderson, Sweeney and Williams, An Introduction to Management Science, 11th edition, Thompson, Mason, Ohio, 2005.

The optimal value of the objective function is 2370.

Microsoft Excel 9.0 Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C8	DTV	10	0	65	25	65
D8	ETV	0	-65	90	65	1E+30
E8	DN	25	0	40	1E+30	16
F8	SN	2	0	60	40	16.66666667
G8	R	30	0	20	1E+30	14

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H10	Available DTV	10	0	15	1E+30	5
H11	Available ETV	0	0	10	1E+30	10
H12	Available DN	25	16	25	5	5
H13	Available SN	2	0	4	1E+30	2
H14	Available R	30	14	30	20	20
H15	Budget	30000	0.06	30000	2000	2000
H16	TV restriction 1	10	-25	10	1.333333333	1.333333333
H17	TV restriction 2	15000	0	18000	1E+30	3000
H18	Customers reached	61500	0	50000	11500	1E+30

a. What is the new optimal value of the objective function, and what are the optimal values of the decision variables, if the objective function coefficient for DN were to decrease from 40 to 30? (10 marks)

The new optimal value would be 2370-10(25) = 2120.

The new objective function would be:

Maximize Exposures = 65DTV + 90ETV + 30DN + 60SN + 20R

Notice that we are changing the coefficient for DN from 40 to 30. Because this is within the allowable decrease (16 from the 1st table above), our optimal solution does not change. Our maximized exposures would however change. We have to plug the optimal solution back into the objective function.

DTV=10, ETV=0, DN=25, SN=2, R=30. No change from original solution.

Maximize Exposures = 65(10) + 90(0) + 30(25) + 60(2) + 20(30) = 2120

b. If the right hand side of the second constraint (Available ETV) were to increase from 10 to 13, what would be the new optimal value of the objective function? (5 marks)

The final value from the 2nd table above for ETV constraint is 0. Meaning we are not using any of this constraint in the optimal solution. So increase the RHS or the availability of this constraint would not affect our optimal solution or maximized cost. The increase is within the allowable increase of infinity. The shadow price is "0" which tells us that a 1 unit increase in the RHS of this constraint will have "0" effect on the maximized exposures.

The new optimal value would be 2370+3(0) = 2370.

c. If the right hand side of the seventy constraint (TV restriction 1) were decreased from 10 to 9.5, what would be the new optimal value of the objective function? (5 marks)

This decrease is within the allowable decrease of 1.333333 found in the 2nd table above. The shadow price of -25 tells us that a 1 unit increase in the RHS of this constraint would actually decrease exposures by "25".

The new optimal value would be 2370 +(-25)(-0.5) = 2382.5

The formula is calculated at ->

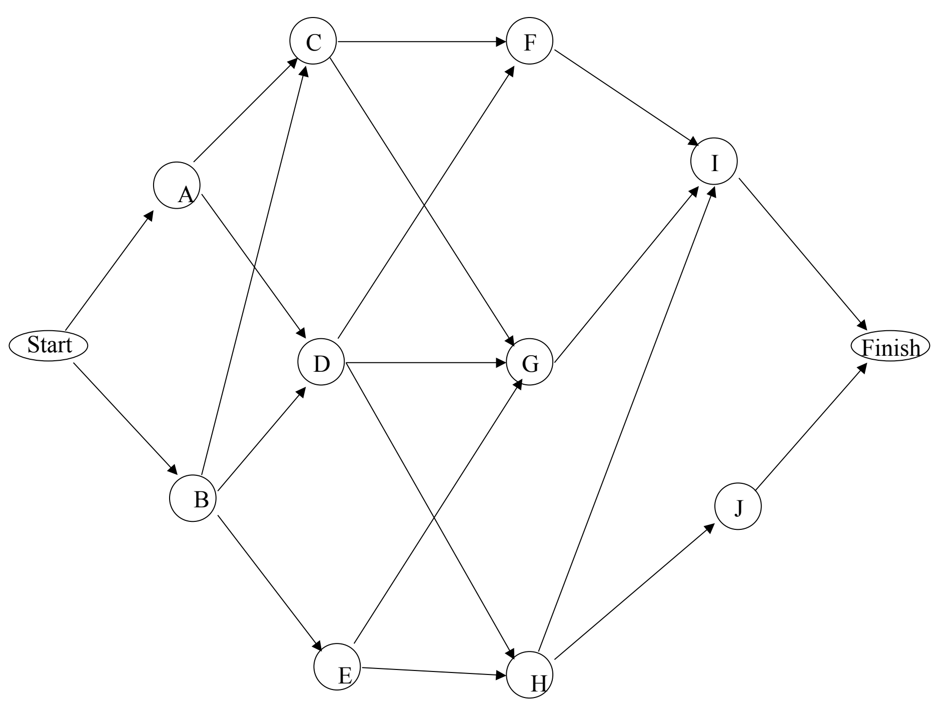
current optimized exposures + (shadow price) *(change in RHS of constraint)

Problem 2. (16 marks)

Andrews Machine Parts, Ltd., has to reinforce its second floor manufacturing area in order to be able to install the latest machinery, which is considerably heavier than the current machinery. The Andrews project is described in the following table.

Activity	Immediate predecessors	Time (weeks)
A	--	5
B	--	3
C	A,B	4
D	A,B	6
E	B	9
F	C,D	8
G	C,D,E	4
H	D,E	5
I	F,G,H	4
J	H	3

a. Draw the project network below. (4 marks)



Give the values of ES, EF, LS, LF, slack for each activity in the table below. (8 marks)

Activity	ES	EF	LS	LF	Slack
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					

Duration	**Activity**	**ES**	**EF**	**LS**	**LF**	**Slack**	**Critical?**
5	**A**	**0**	**5**	**0**	**5**	**0**	**Y**
3	**B**	**0**	**3**	**2**	**5**	**2**	**N**
4	**C**	**5**	**9**	**7**	**11**	**2**	**N**
6	**D**	**5**	**11**	**5**	**11**	**0**	**Y**
9	**E**	**3**	**12**	**5**	**14**	**2**	**N**
8	**F**	**11**	**19**	**11**	**19**	**0**	**Y**
4	**G**	**12**	**16**	**15**	**19**	**3**	**N**
5	**H**	**12**	**17**	**14**	**19**	**2**	**N**
4	**I**	**19**	**23**	**19**	**23**	**0**	**Y**
3	**J**	**17**	**20**	**20**	**23**	**3**	**N**

b. What are the critical path and its length (duration)? (2marks)

Only activities with a slack="0" are on the critical path. The total duration can be calculated by adding up the durations of the activities on the critical path. It is also equal to the earliest completion time for the last activity in the critical path.

Critical Path: A-D-F-I

Length of critical path = earliest completion time = 23 weeks (5+6+8+4)

c. If you wanted to shorten the duration of the project by one week, which activities would you definitely NOT consider shortening? (2 marks)

You only want to crash activities on the critical path(s). So any activities that do not fall on the critical path(s) should not be crashed.

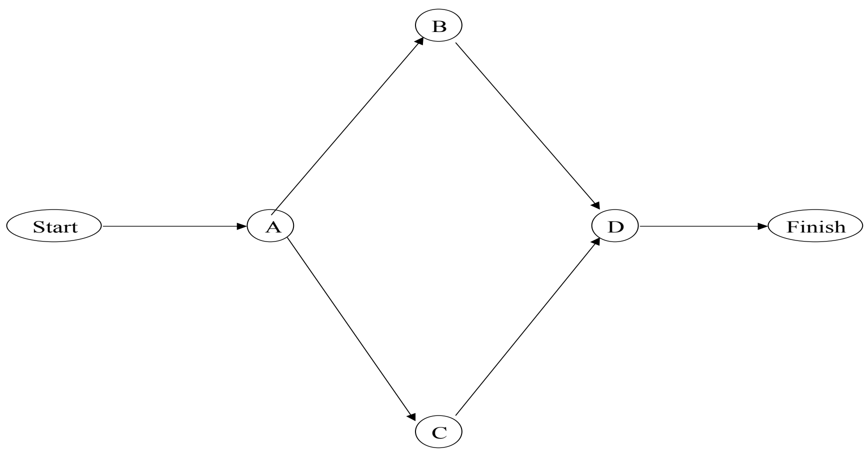
B, C, E, G, H and J should NOT be shortened.

Problem 3. (14 marks)

A four-activity PERT project is described by the table below.

Activity	Immediate Predecessors	Optimistic duration	Most likely duration	Pessimistic duration
A	--	6	9	12
B	A	9	9	9
C	A	9	15	21
D	B,C	6	9	30

The network is shown below:



a. Develop the information you need to answer the probability questions in parts b and c. (4 marks)

Activity	**Expected Duration**	**Standard Dev. of Duration**
A	**=(6+4*9+12)/6 = 9**	**=SQRT[{(12-6)/6}^{2}] = 1**
B	**9**	**0**
C	**15**	**2**
D	**12**	**4**

PATH 1 = ABD -> Length = 9+9+12 = 30

PATH 2 = ACD -> Length = 9+15+12 = 36

Critical path is ACD

Expected length of critical path is 36.

Standard deviation of critical path = SQRT($1^2+2^2+4^2$)=4.58

b. What is the probability that the project will be completed by time 29? (5 marks)

Use you z formula -> z = (d-tp)/SD = (29-36)/4.58 = -1.53

This is negative because we are dealing with the left had side of the curve, because we would like the probability of it being complete in under 29 days, we are dealing with the outer tail. We find a value of 0.437 in the z-table. Because we are dealing with the outer tail, we must subtract this from 0.5.

P[X ≤ 29] = 0.5-0.437 = 0.063

c. The probability is .60 that the project will be finished by a given time. What is that time? (5 marks)

Use you z formula -> z = (d-tp)/SD

In this question we are given a probability and must work backwards to find "d". So in order to calculate "z" we have to convert 0.6 to a z-score. 0.6 is 0.1 above the centerline of 0.5 (50% of data), so we need to look a value of 0.1 within the z-table (not along the top and side). We then look at the side and top value that this corresponds to. Doing this we find a side value of 0.2 and a top value of 0.05. So our z-score is 0.25. We can now solve for "d".

0.25 = (d-36)/4.58 d = 37.145

Problem 4. (13 marks)

High-End Construction Company has to decide which of four sites to develop over the forthcoming planning period. If High-End decides to build on a site, an infrastructure preparation cost is incurred. The infrastructure preparation budget is $530,000. At each site there is a maximum number of units that can be built, and a profit contribution, i.e., (revenue – variable cost), for each unit built. High-End has decided that if it does NOT build at site D, it will definitely build at both sites A and B. High-End's objective is to maximize profit, which is total revenue less total fixed costs less total variable costs. Relevant data is given in the table below:

Site	Infrastructure preparation cost ($1,000)	Profit contribution per unit ($1,000)	Maximum number of units that can be built on site
A	150	120	20
B	170	70	30
C	190	50	40
D	200	100	60

Build a model that will help to determine the number of units to build on each site to maximize profit. (define the decision variables, objective function and constraints). FORMULAT ONLY. DO NOT SOLVE.

Define your variables:

Let xi = number of units to produce on site i (i=1,2,3,4)

yi = 1 if site i is developed, 0 otherwise (i=1,2,3,4) (these are binary variables)

Optimization Function: (we take profit/unit and must subtract the preparation costs)

Maximize 120x1 + 70x2 + 50x3 + 100x4 – 150y1 - 170y2 – 190y3 – 200y4

Constraints:

subject to:

150y1 + 170y2 + 190y3 + 200y4 ≤ 530 (budget constraint)

x1 ≤ 20y1 (maximum unit on site constraint)

x2 ≤ 30y2 (maximum unit on site constraint)

x3 ≤ 40y3 (maximum unit on site constraint)

x4 ≤ 60y4 (maximum unit on site constraint)

2(1-y4) ≤ y1+y2 (if y4=0, y1+y2 must be 2)

x1, x2, x3, x4 ≥ 0 (non-negativity constraints)

y1, y2, y3, y4 = 0 or 1 (let us know these are binary)

Problem 5. (12 marks)

Jabsidlanski Inc. manufactures smafises and framises. Labour requirements and contribution to profit per unit produced are shown in the table below:

	Smafis	Framis
Labour hours required	3	1
Contribution to profit	\$4	\$2

Jabsidlanski has a goal of a <u>minimum</u> of \$50 profit. A second goal is to use all 30 hours of labour available (Jabsidlanski will not be happy if it uses either less or more than 30 hours). Demand for smafises is 7, and Jabsidlanski will be most unhappy if fewer than 7 smafises are produced. Jabsidlanski has contracted to produce 10 framises for its best customer – and Jabsidlanski is committed to NOT defaulting on this contract.

Formulate the above problem as a goal programming problem. (Define the decision variables, objective function and constraints.) FORMULATE ONLY. DO NOT SOLVE.

Variables:

Let S be the number of smafises to produce

Let F be the number of Framis to produce

Goal Definition:

Let d^+_i and d^-_i be the positive and negative deviations from the goal for the ith constraint (i=1,2,3)

Objective Function:

Minimize $d_1^- + d_2^+ + d_2^- + d_3^-$

Since anything under \$50 would be a deviation from our goal, we only need to define d_1^-

Since anything under or over 30 hours would be a deviation from our goal, we must include both d_2^+ and d_2^-

Since anything under 7 framies would be a deviation from our goal, we only need to define d_3^-

Constraints:

$4S + 2F = 50 + d^+_1 - d^-_1$ (profit constraint)

$3S + F = 30 + d^+_2 - d^-_2$ (labour constraint)

$S = 7 + d^+_3 - d^-_3$ (demand constraint)

$F \geq 10$ (demand constraint)

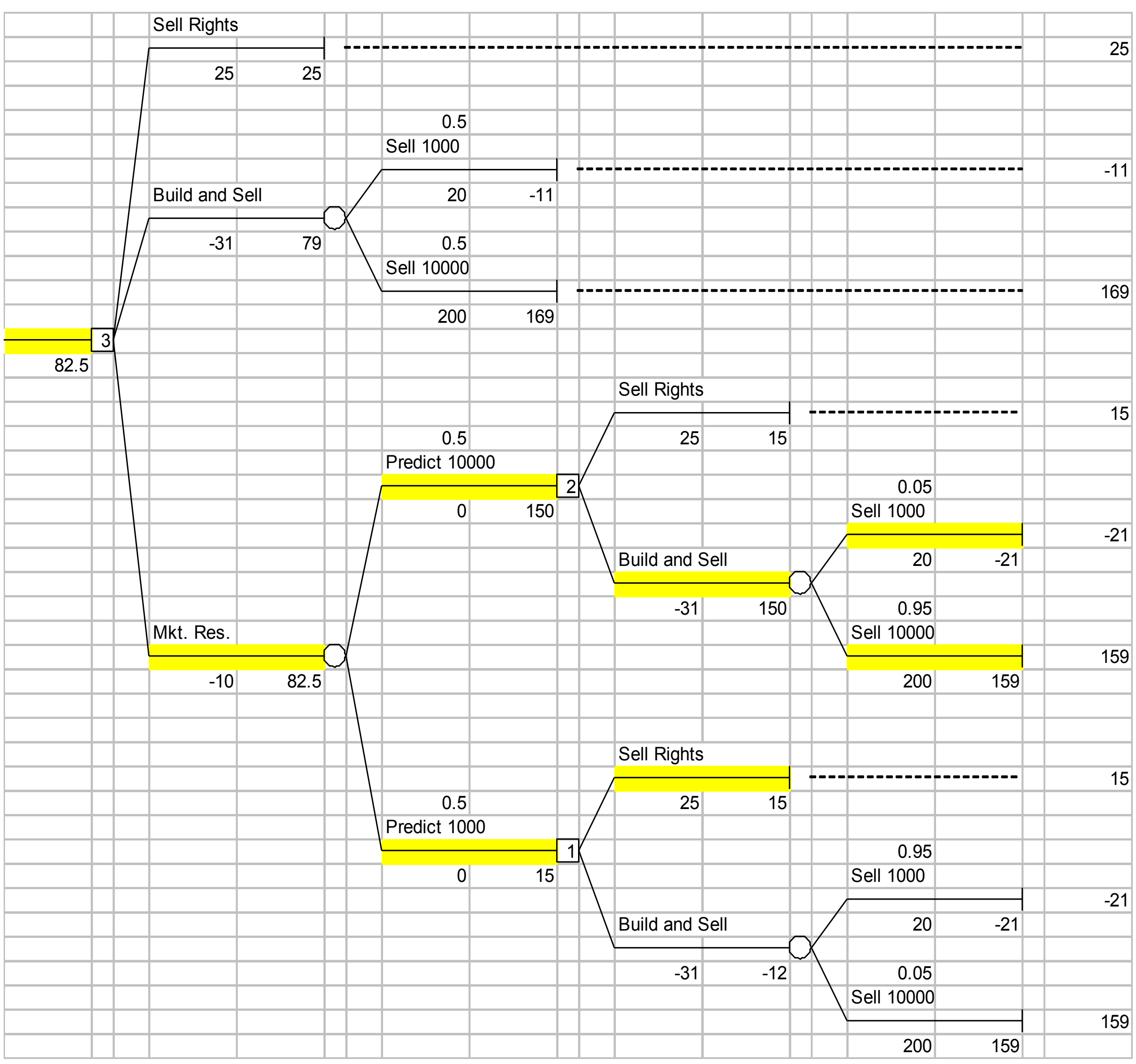
$S, F \geq 0$ (non-negativity constraint)

Notice that we must include both positive and negative deviations in the constraints, because it is possible to deviate in either direction. The objective function defines which of these deviations in a problem.

Problem 6. (25 marks)

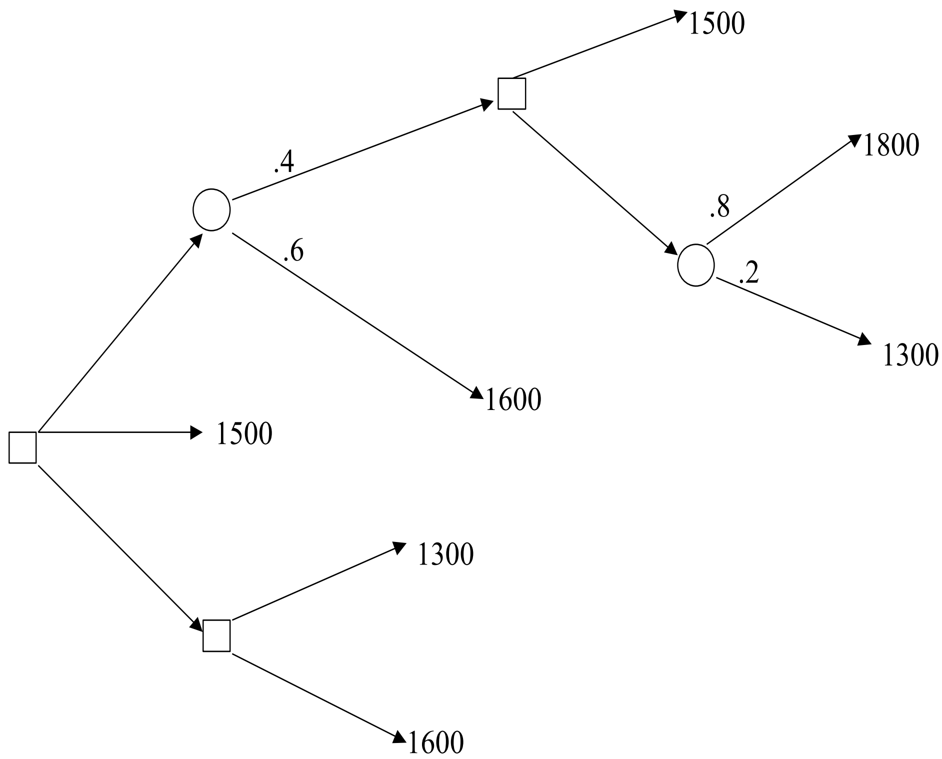
a. Working in your garage, you have developed a new chip that would allow you to enter the home computer market. Alternatively, you could sell all rights to the chip to Intel for $25,000. If you decide to build and sell the home computers yourself, the profitability of your venture depends on your ability to market the home computer during the first year. Through your brother-in-law at Computerland, you can be certain of selling 1,000 computers. If however, your computer catches on, 10,000 will be sold. Your brother-in-law convinces you that either sales level is equally likely, and that the probability of other sales levels is negligible. The cost of setting up your assembly line is $31,000. Because the home computer market is so competitive, the difference between the selling price and the variable cost of production is only $20. Market research, which is 95% accurate, can be undertaken at a cost of $10,000 to determine which of the two sales levels is more realistic. Thus, there is a 50% chance the market survey will predict high sales, and if so, there is a 95% chance that sales are actually high. Similarly, there is a 50% chance the market survey will predict low sales, and if so, there is a 95% chance that sales are actually low.

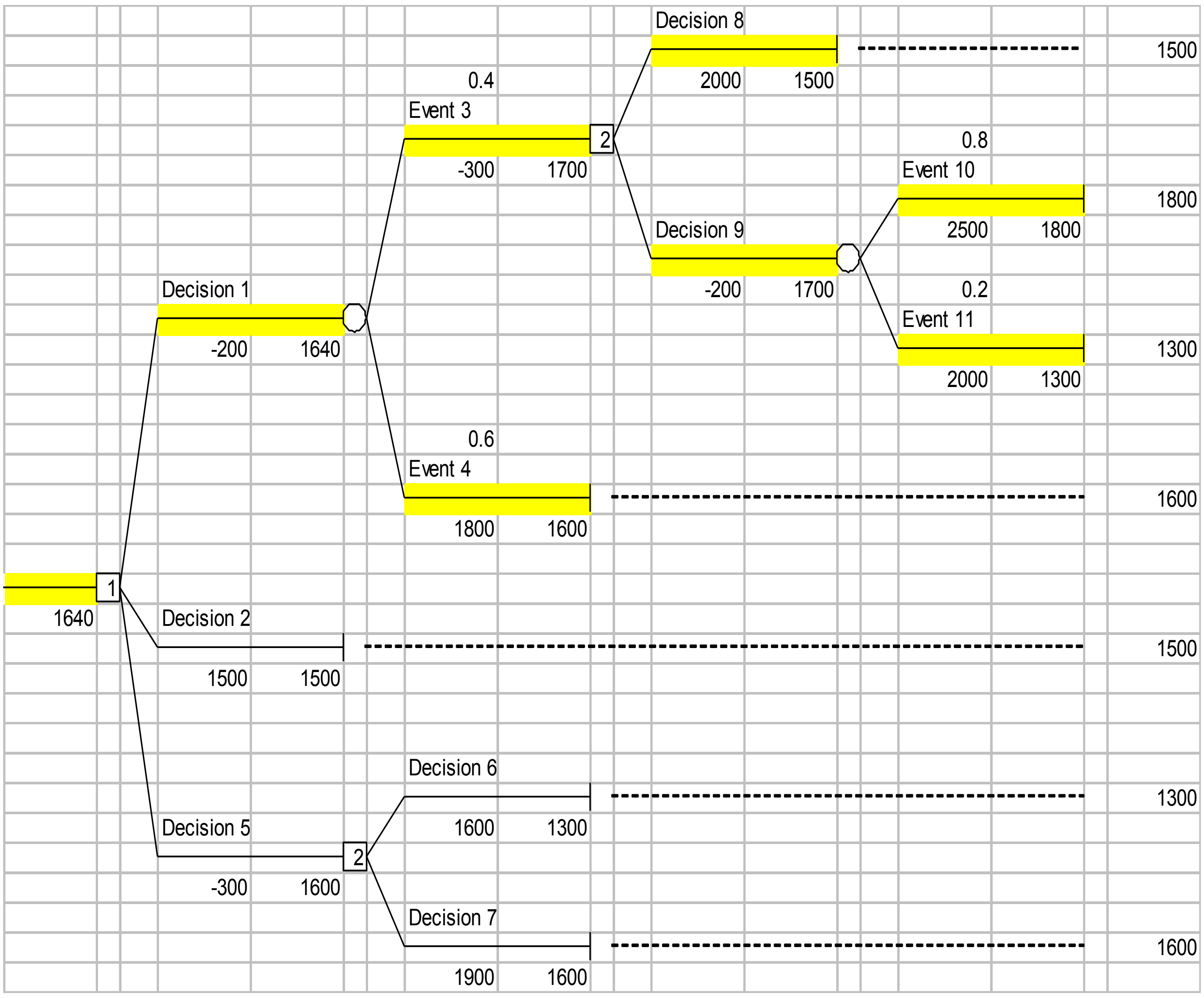
Draw the decision tree for this problem. Make sure it is well labeled, and that you include probabilities and cash flows (profits) as appropriate. FORMULATE ONLY. DO NOT SOLVE for the optimal strategy. (15 marks)



b.

The following is the decision tree for a maximization problem. Probabilities and payoffs are shown. Determine the optimal expected payoff (EP or EMV) and the optimal strategy. Darken all the branches on the tree which can occur in the optimal strategy. (10 marks)





Decision Strategy: Go with decision 1:

- **If event 3 occurs then choose decision 9 (if event 10 occurs it will result in a payoff of 1800, otherwise the payoff is 1300).**
- **Otherwise if event 4 occurs then the payoff is 1600**

The expected payoff of such a decision strategy is 1640.