Université d'Ottawa
Faculté de genie

Départment de
Génie Méchanique

University of Ottawa
Faculty of Engineering

Department of
Mechanical Engineering

MCG 4303
Mechanical Vibration Analysis

MIDTERM EXAMINATION
February 13, 2007

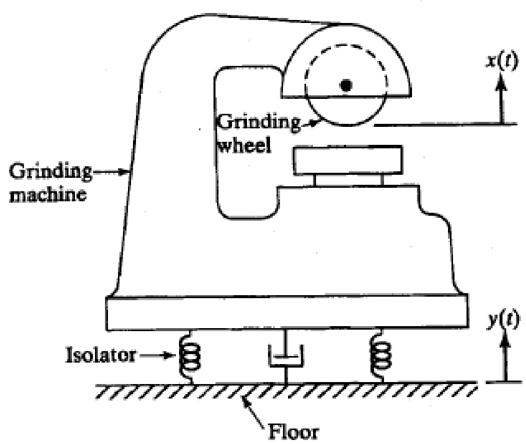
Comments and general instructions:

1. PUT YOUR NAME AND STUDENT NUMBER ON YOUR EXAM BOOKLET!
2. There are ***three problems*** of ***equal weight***.
3. Potentially useful mathematical formulae are on the last two pages.
4. Be neat and clear in your presentation, and show all work for full credit.
5. This exam is closed book.
6. Time: **75 minutes**.

Question 1

A precision griding machine is supported on an isolator consisting of two springs and a damper. The floor, on which the machine is mounted, is subjected to a harmonic disturbance due to the operation of an unbalanced pump that is located nearby.

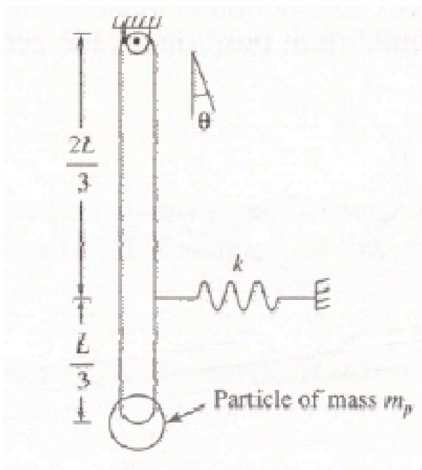
1. Write the equation of motion for this system.
2. Calculate the displacement transmissibilty of the base motion.
3. It is discovered that the operating frequency of the unbalanced pump is close to the natural frequency of the grinding mchine so that the amplitude of the motion of the grinding machine is too high. Which of the following techniques can be used, with confidence, to reduce the motion of the machine? Choose ALL that apply and explain.
    a. Reduce the mass of the grinding machine
    b. Increase the mass of the grinding machine
    c. Reduce the stifffness of the isolator (springs)
    d. Increase the stiffness of the isolator (springs)
    e. Reduce the damping of the isolator
    f. Increase the damping of the isolator



Question 2

Consider the system shown in the figure. Use θ, the clockwise angular displacement of the bar from the system's equilibrium position, as the generalized coordinate. The moment of intertia of the bar about its centre of mass is given by $I_G = \frac{1}{12}mL^2$ where m is the mass of the bar. For this sytem find:

1. The equivalent mass
2. The equivalent stiffness
3. The natural frequency



Question 3

The motion of a sprimg-mass-damper system is described by the following equation:

$$\ddot{x} + 2\dot{x} + 10x = 20 + 37\cos(3t)$$

If this system is initially located at a displacement of 3 with zero initial velocity, find:

1. The total solution to the problem
2. The steady-state solution to the problem
3. What is the damping ratio for this system?

Useful formulas

Quadratic formula: roots of ax^2+bx+c are given by $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Integration by parts: $\int udv = uv - \int vdu$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$\sin(u) = \frac{e^{iu} - e^{-iu}}{2i} = \text{Im}\left\{e^{iu}\right\} \qquad \cos(u) = \frac{e^{iu} + e^{-iu}}{2} = \text{Re}\left\{e^{iu}\right\}$$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right]$$

$$a_0 = \frac{2}{T}\int_0^T x(t)dt = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} x(t)dt$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos\left(\frac{2n\pi t}{T}\right)dt = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} x(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin\left(\frac{2n\pi t}{T}\right)dt = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} x(t)\sin(n\omega t)dt$$

Small angle approximations: $\begin{cases} \sin\theta \approx \theta \\ \cos\theta \approx 1 - \dfrac{\theta^2}{2} \end{cases}$

Trigonometric Identities

Reciprocal

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$
$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$
$$\cot x = \frac{1}{\tan x} \qquad \tan x = \frac{1}{\cot x}$$

Pythagorean

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Negative Angle

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\tan(-x) = -\tan x$$

Addition and Subtraction

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Periodicity

$$\sin(x+2\pi) = \sin x \qquad \cos(x+2\pi) = \cos x$$
$$\csc(x+2\pi) = \csc x \qquad \sec(x+2\pi) = \sec x$$
$$\tan(x+\pi) = \tan x \qquad \cot(x+\pi) = \cot x$$

Cofunction

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \cos x = \sin\left(\frac{\pi}{2} - x\right)$$
$$\tan x = \cot\left(\frac{\pi}{2} - x\right) \qquad \cot x = \tan\left(\frac{\pi}{2} - x\right)$$
$$\sec x = \csc\left(\frac{\pi}{2} - x\right) \qquad \csc x = \sec\left(\frac{\pi}{2} - x\right)$$

Product

$$\sin x \cos y = \frac{1}{2}\left(\sin(x+y) + \sin(x-y)\right)$$
$$\sin x \sin y = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$$
$$\cos x \cos y = \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)$$
$$\cos x \sin y = \frac{1}{2}\left(\sin(x+y) - \sin(x-y)\right)$$

Factoring

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Double Angle

$$\sin(2x) = 2\sin x \cos x \qquad \cos(2x) = 1 - 2\sin^2 x$$
$$\cos(2x) = \cos^2 x - \sin^2 x \qquad \tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$
$$\cos(2x) = 2\cos^2 x - 1$$

Half-Angle

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$
$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} \qquad \tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$