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MCG 4308
Mechanical Vibration Analysis

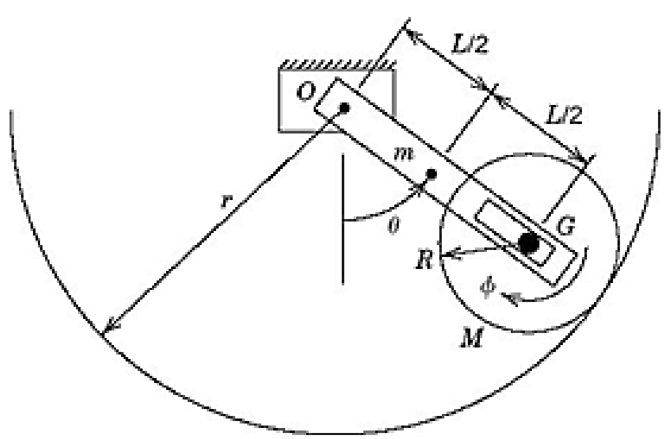
MIDTERM EXAMINATION
February 12, 2015

There are three questions: all questions carry equal weight.

Question 1

Consider the cylinder in the figure with radius R, mass M and moment of inertia I_C about its mass centre. It rolls without slipping in the *vertical* plane on a semicircular track. The distance between the centre of rotation of the bar and the centre of rotation of the cylinder is L, as shown in the figure. The uniform thin link has mass m and inertia I_L about its mass centre. Use θ, the angular displacement from the vertical, as your generalized coordinate. Assume small oscillations, so θ is assumed small.

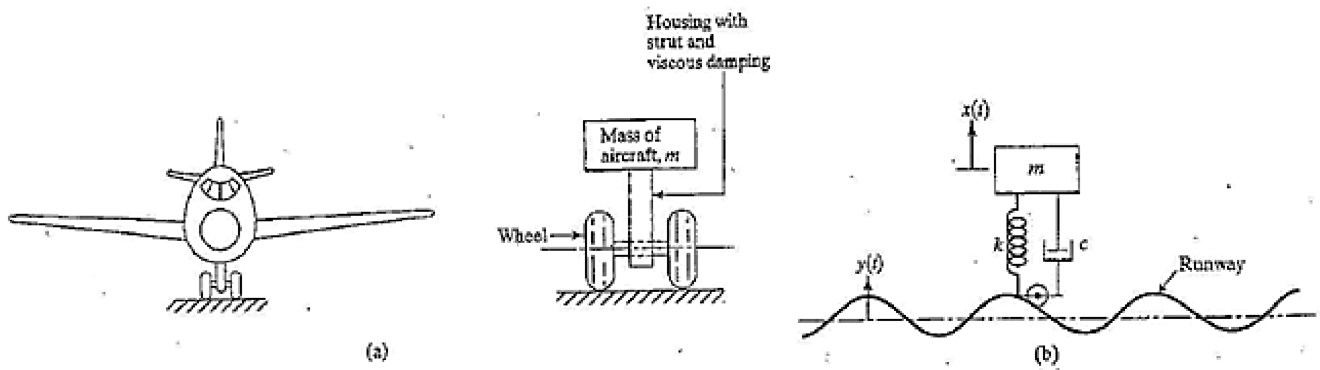
a) Find the equivalent mass of the system.
b) Find the equivalent stiffness of the system.
c) Use Raleigh's method to find the natural frequency of the system



Question 2

The landing gear of an airplane can be idealized as the spring-mass-damper system shown in the figure. Your modelling assumptions indicate that m = 2000 kg, c = 8000 Ns/m and k = 8000 N/m.

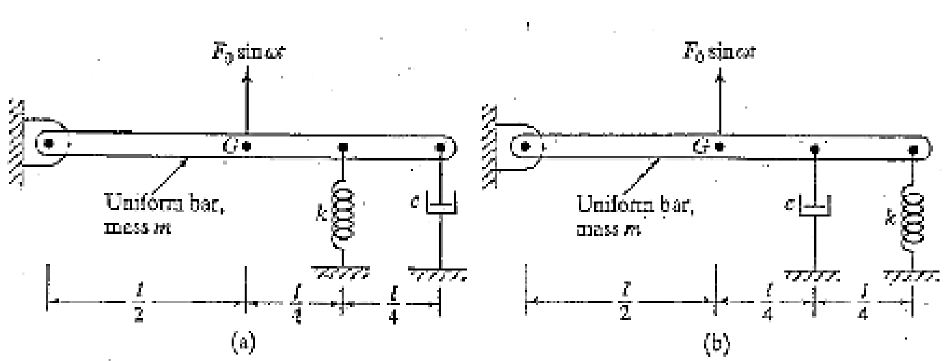
a) Find the natural frequency and damping ratio of the landing gear of the aircraft.
b) Is the landing gear underdamped, overdamped or critically damped?
c) Write the general mathematical form for the *free* vibration response of the landing gear.
d) *Sketch* how you expect your answer to part (c) to look like.
e) Subsequent testing indicates that the stiffness, k, of the landing gear is much larger than your initial 8000 N/m estimate. Does this change your answer to parts (b), (c) and (d)? If so, sketch how you would expect the free vibration response to look with a much stiffer landing gear.



Question 3

A slender uniform bar has mass m, length l and moment of inertia, J_0, about the point where it is rotating. It may be supported in one of two ways as indicated in the figure. In the following, use the angular displacement (assumed small) of the bar as your generalized coordinate.

a) Derive the equation of motion for configuration (a).
b) Derive the equation of motion for configuration (b).
c) Find the magnitude of the steady state response for configuration (a) under a harmonic force, $F_0 \sin(\omega t)$ applied to the middle of the bar as shown. Express your answer in terms of system variables $\omega, F_o, c, k, l, J_o$.
d) Repeat part (c) for configuration (b).
e) Assuming an *extremely* small stiffness $(k \approx 0)$, which configuration results in a **reduced** steady-state response of the bar under a harmonic force applied to its middle? Explain your reasoning.



Potentially Useful formulae

Quadratic formula: roots of $ax^2 + bx + c$ are given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$\sin(u) = \frac{e^{iu} - e^{-iu}}{2i} = \operatorname{Im}\left\{e^{iu}\right\} \qquad \cos(u) = \frac{e^{iu} + e^{-iu}}{2} = \operatorname{Re}\left\{e^{iu}\right\}$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$A_1 \sin \omega t + A_2 \cos \omega t = \sqrt{A_1^2 + A_2^2}\,\sin\left(\omega t + \phi\right) \quad \text{where } \phi = \arctan\left(\frac{A_2}{A_1}\right)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \text{ for all } x$$