CARLETON UNIVERSITY

DEFERRED FINAL EXAMINATION
MATH 1004 H
Winter 2016

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004 H.
Course Instructor(s): Mr. Gary Bazdell.

AUTHORIZED MEMORANDA
Non-Programmable Calculator.

This exam may be released to the Library and may be taken away by the student. **In addition to the examination paper students will require a SECONDARY EXAMINATION BOOKLET, and a SCANTRON SHEET.**

1. Please verify that you are in possession of a Scantron FORM.
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., H), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination consists of two sheets of legal size paper.** It is out of a total of 100 and consists of 25 multiple choice questions each worth 4 marks **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.,* you get 0 marks for that question).

Return only the Scantron form not the examination nor your work.

1 Find the parabola with equation $y = ax^2 + bx + c$ that has slope 4 at $x = 1$, slope -8 at $x = -1$, and passes through the point $(2, 15)$.
(a) $y = 3x^2 - 2x + 23$ **(b)** $y = 3x^2 - 2x + 7$ **(c)** $y = 2x^2 + 2x - 7$ **(d)** None of these

2 Let $f(x) = \dfrac{\sin 2x}{2}$, for $x \neq 0$, and $f(x) = L$, for $x = 0$. What value of L will make f continuous at $x = 0$?
(a) $L = 1$ **(b)** $L = 0$ **(c)** $L = -1$ **(d)** $L = 2$.

3 Evaluate $L = \lim\limits_{x \to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$.
(a) $L = 1$ **(b)** $L = \frac{1}{2}$ **(c)** $L = 2$ **(d)** $L = 0$.

4 Let $f(x) = x \sin(\pi/x)$. Evaluate $L = \lim\limits_{x \to \infty} f(x)$.
(a) $L = \pi$ **(b)** $L = -\pi$ **(c)** $L = -1/\pi$ **(d)** This limit does not exist.

5 Two functions f, g are defined by $f(x) = \sin^2(x) - \cos x$ and $g(x) = \sin 2x$.
What is the value of their composition $f(g(0))$?
(a) 0 **(b)** -1 **(c)** 5 **(d)** $5\sqrt{2}$

6 Find the derivative of the function $y = x^{\sqrt{x}}$ at $x = 1$.
(a) 1 **(b)** 2 **(c)** $\dfrac{1}{2}$ **(d)** 0

7. Find the derivative of the function $\sin(x+y) = y^2 \cos x$.

(a) $\dfrac{\cos(x+y)}{2y\cos x - y^2 \sin x}$ **(b)** $\dfrac{2y\cos x - \cos(x+y)}{y^2 \sin x + \cos(x+y)}$ **(c)** $\dfrac{\cos(x+y)}{2y\cos x}$ **(d)** $\dfrac{y^2 \sin x + \cos(x+y)}{2y\cos x - \cos(x+y)}$

8. Find the derivative of the function $y = \ln(e^{-x} + xe^{-x})$.

(a) $-\dfrac{x}{1+x}$ **(b)** $\dfrac{1}{1+x}$ **(c)** $\dfrac{2+x}{1+x}$ **(d)** $\dfrac{1+x}{-1}$

9. Find the absolute maximum and minimum values of the function $y = x^3 - 3x^2 + 1$ on the interval $[-1/2, 4]$.

(a) Maximum at $(4, 17)$ **(b)** Minimum at $(-\frac{1}{2}, \frac{1}{8})$, maximum at $(4, 17)$ **(c)** Minimum at $\left(-\frac{1}{2}, \frac{1}{8}\right)$
(d) Minimum at $(2, -3)$, maximum at $(4, 17)$

10. Which of the following statements is true?

(a) $f(x) = x^3 + 7$ is concave up for all x, and has no points of inflection.
(b) $f(x) = x^2 - 1$ is concave down for $x < 0$, concave up for $x > 0$, and has a point of inflection at $(0, -1)$.
(c) $f(x) = -3e^x$ is concave up for all x, and has no points of inflection.
(d) $f(x) = x + 2\sin x$ between $[0, 2\pi]$ has a local maximum at $(2\pi/3, (2\pi/3) + \sqrt{3})$

11. Evaluate $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} dx$.

(a) $\frac{2}{3}\cos^3(\sqrt{x}) - 2\cos(\sqrt{x}) + C$ **(b)** $\cos^3(\sqrt{x}) - 2\sin(\sqrt{x}) + C$ **(c)** $-\dfrac{1}{2}\cos^3(\sqrt{x}) + C$
(d) $\frac{2}{3}\cos^3(\sqrt{x}) + 2\sin(\sqrt{x}) + C$

12. Evaluate the definite integral $\displaystyle\int_0^{\pi} \sin^2(x)\cos^4(x) dx$.

(a) 0 **(b)** $\dfrac{\pi}{4}$ **(c)** $\dfrac{\pi}{16}$ **(d)** $\dfrac{\pi}{8}$

13 Evaluate the definite integral $\displaystyle\int_0^{\pi} \cos^6(x) dx$.

(a) 0 **(b)** $-\dfrac{2\pi}{32}$ **(c)** -1 **(d)** $\dfrac{5\pi}{16}$

14 Evaluate the integral

$$\int e^{-x}\cos(2x)\ dx$$

(a) $5e^x \sin(2x) - 5e^x \cos(2x) + C$ **(b)** $2e^{-x}\sin(2x) - e^{-x}\cos(2x) + CC$
(c) $\dfrac{2}{5}e^{-x}\sin(2x) - \dfrac{1}{5}e^{-x}\cos(2x) + C$ **(d)** $\dfrac{1}{5}e^x \sin(2x) - \dfrac{1}{5}e^x \cos(2x) + C$

15 Evaluate the integral

$$I = \int \cos x \ln(\sin x)\ dx$$

(a) $\sin x(\ln\sin x - 1) + C$ **(b)** $\sin x(\ln\sin x + 1) + C$
(c) $\cos x(\ln\sin x - 1) + C$ **(d)** $\cos x(\ln\sin x + 1) + C$

16 Evaluate the integral

$$\int \frac{xdx}{(x+1)(x-2)}\ dx$$

using partial fractions.

(a) $\frac{2}{3}\ln|x+1| - \frac{4}{3}\ln|x-2| + C$ **(b)** $\frac{1}{3}\ln|x+1| - \frac{2}{3}\ln|x-2| + C$ **(c)** $\frac{1}{3}\ln|x+1| + \frac{2}{3}\ln|x-2| + C$
(d) $\dfrac{1}{2}\ln|x+1| + \dfrac{1}{3}\ln|x-2| + C$

17 Evaluate the integral

$$\int \frac{x^3+x}{x-1}\,dx$$

using partial fractions.

(a) $\frac{x^3}{3}+\frac{x^2}{2}+C$ **(b)** $\frac{x^3}{3}+\frac{x^2}{2}+2x+2ln|x-1|+C$ **(c)** $\frac{x^2}{2}+2ln|x-1|+C$ **(d)** $\frac{x^3+x^2}{|x-1|}+C$

18 Find an equation of the tangent line to the curve $y=\ln(xe^{x^2})$ at the given point $(1,1)$.

(a) $y=2x+3$ **(b)** $y=12x-24$ **(c)** $y=3x-2$ **(d)** $y=x+1$

19 Find $\int \frac{dx}{\sqrt{x^2-a^2}}$, where $a>0$.

(a) $\ln|x|-\ln a+C$ **(b)** $\ln|\sqrt{x^2-a^2}|$ **(c)** $\ln|x+\sqrt{x^2-a^2}|-\ln a+C$ **(d)** None of these

20 Evaluate $\int_0^1 \ln x\,dx$.

(a) 2 **(b)** -1 **(c)** 0 **(d)** 1/2

21 Evaluate $\int \frac{x^3}{\sqrt{1+x^2}}\,dx$.

(a) $\sqrt{3}+\frac{x}{3}+C$ **(b)** $\sqrt{1+x^2}+c$ **(c)** $\frac{1}{3}(x^2-2)+c$ **(d)** $\frac{1}{3}(x^2-2)\sqrt{1+x^2}+c$

22 Let $f(x)=x|x|$. Calculate $L=\lim_{h\to 0}\frac{f(-5+h)-f(-5)}{h}$.

(a) $L=10$ **(b)** $L=1$ **(c)** $L=5$ **(d)** This limit does not exist.

23 Find an expression for the volume V of the solid of revolution obtained by rotating the region bounded by the graph of $y=ln(x)$, $y=1$, $y=2$ and $x=0$ about the $y-$axis.

(a) $\int_1^2 2\pi y(e^y)^2\,dy$ **(b)** $\int_1^2 \pi(e^y)^2\,dy$ **(c)** $\int_1^2 2\pi(e^y)\,dy$ **(d)** $\int_1^2 \pi y(e^y)\,dy$

24 Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y=2x^2-x^3$ and $y=0$

(a) $\frac{16}{5}\pi$ **(b)** π **(c)** $-\pi$ **(d)** $\frac{8}{5}\pi$ does not exist

25 Evaluate the improper integral $\int_0^3 \frac{dx}{x-1}$.

(a) $-\infty$ **(b)** -1 **(c)** $\frac{\pi}{4}$ **(d)** 1/2

[Total: 100 marks]

END OF THE EXAMINATION.