Electromagnetic waves

Propagation of waves as sound waves, water waves ⇒ but in free space

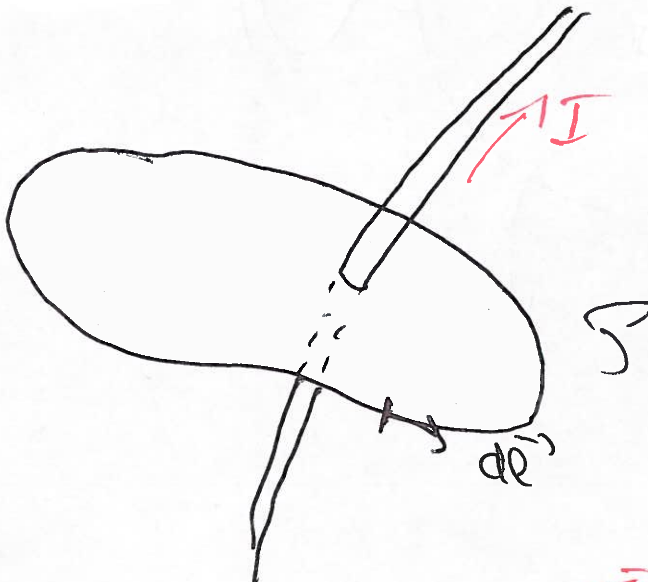
⇓

no particle involved

(light)

(carries energy)

Limitations and general form of Ampère's law



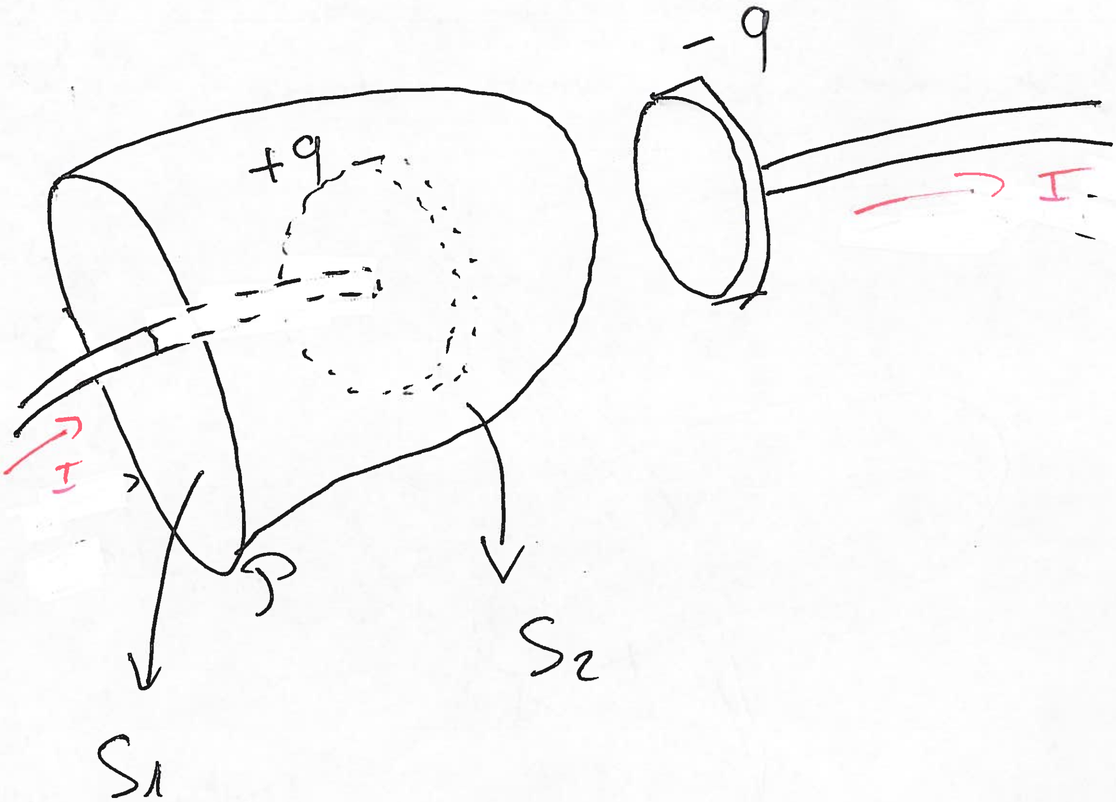
$$\oint_{\mathcal{S}} \vec{B}\cdot d\vec{l} = \mu_0 \, I$$

Conduction current $I = dQ/dt$

⇓

Σ of all currents crossing any surface bounded by $\mathcal{S}$

Problem ⇒ charging a capacitor



* <u>For S_1</u>: $\oint_S \vec{B} \cdot \vec{dl} = \mu_0 I$

* <u>For S_2</u> $\oint_S \vec{B} \cdot \vec{dl} = 0$

No charge crossing between the plates of the capacitor

⇒ <u>contradictory</u>

$\Rightarrow$ Maxwell introduced the notion of

Displacement current $I_d \equiv \varepsilon_0 \dfrac{d\phi_E}{dt}$

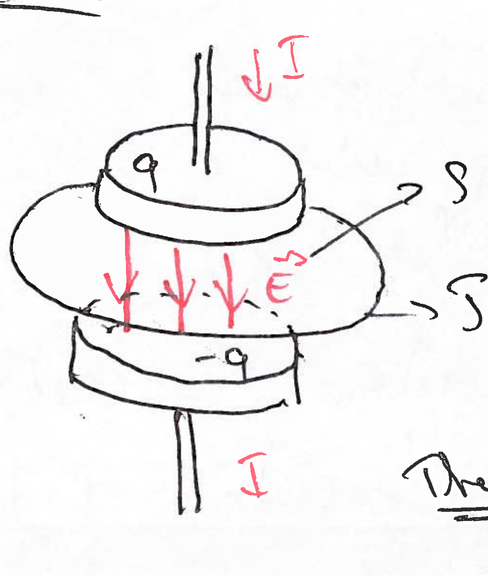
$$\phi_E = \int_S \vec{E} \cdot d\vec{S}$$: flux of electric field $\vec{E}$

Generalization **Ampère - Maxwell law**

$$\oint_{\mathcal{P}} \vec{B} \cdot d\vec{l} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

$$\phi_E = \int_S \vec{E} \cdot d\vec{S}$$

Example: Capacitor



$\Rightarrow$ $\vec{E}$ is uniform between the plates : $E = q/\varepsilon_0 S$

$$\phi_E = E S = \frac{q}{\varepsilon_0}$$

surface of the plate of the capacitor

Therefore $I_d = \varepsilon_0 \dfrac{d\phi_E}{dt} = \dfrac{dq}{dt} = I$

conduction current

Maxwell's equation (in free space)

$$\oint_S \vec{E} \cdot d\vec{S} = q/\varepsilon_0$$: Gauss law $\iff \operatorname{div}\vec{E} = q/\varepsilon_0$

(close surface)

Electric field and charge distribut.

(flux of $\vec{E}$ through a close surface)

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$: Gauss law in Magnetism $\iff \operatorname{div}\vec{B} = 0$

close surface

Net magnetic flux through a close surface is 0

$\Rightarrow$ no magnetic monopole

$$\oint_\ell \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \iff \overrightarrow{\operatorname{curl} E} = -\frac{\partial \vec{B}}{\partial t}$$

close path

Faraday law of induction

Change in Φ_B creates an electric field $\Rightarrow$ emf

$$\oint_\ell \vec{B} \cdot d\vec{\ell} = \mu_0 I + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt} \iff \overrightarrow{\operatorname{curl} B} = \mu_0 I + \varepsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t}$$

close path

change in $\Phi_E \Rightarrow$ creates a magnetic field $\vec{B}$

Presence of $\vec{E}$ and $\vec{B}$

$\Rightarrow$ action on a particle of charge q and speed $\vec{v}$

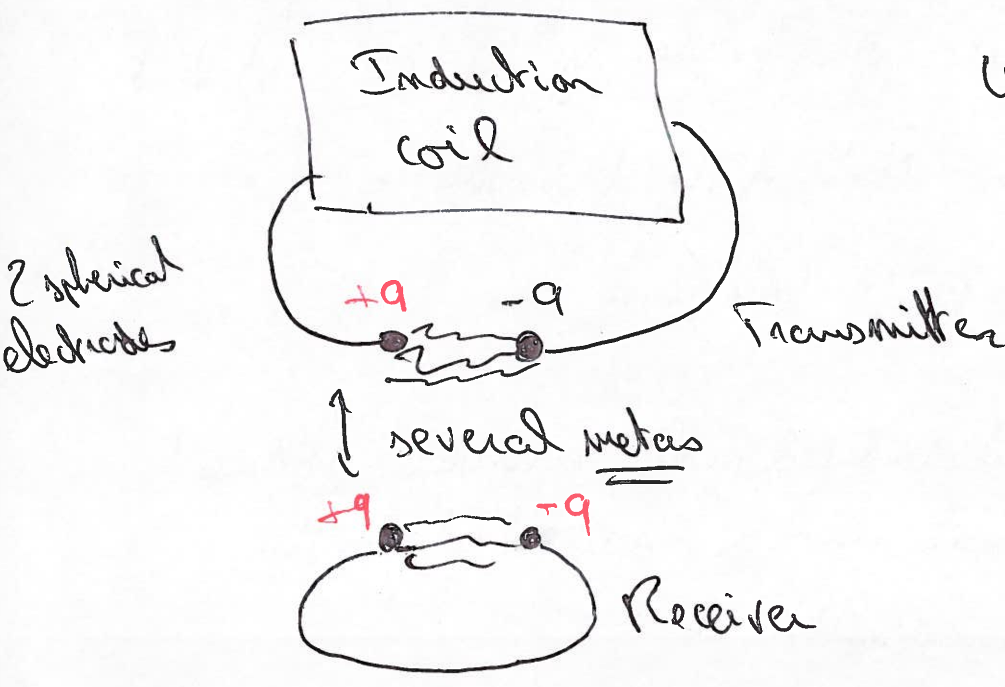
Lorentz force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

$\Rightarrow$ Electro magnetic wave equation

$\Rightarrow$ even when $q = 0$ and $I = 0$ (free space)

$\Rightarrow$ electromagnetic wave travelling at the speed of light

Hertz experiment



When $\vec{E}$ >> surpasses the dielectric strength of air $\Rightarrow$ spark

$\Downarrow$

ionization $\Rightarrow$ more electron $\Rightarrow$ more sparks $\Rightarrow$ ...

$\Rightarrow$ oscillatory behavior $\Rightarrow$ similar to a LC circuit

$$\omega = 1/\sqrt{LC} \Rightarrow \text{high frequency}$$

$$f \simeq 100\,MHz$$

$\Rightarrow$ Electromagnetic waves are radiated at this frequency as a result of the oscillation of free charges in the transmitter circuits

$\Rightarrow$ Hertz demonstrated at sparks were induced in the receiver circuits

when the receiver frequency is adjusted to match that of the transmitter

Application : Radio, telephone, ...

Oscillatory current induced in the receiver produced by electromagnetic waves radiated by the transmitter

⇒ Properties of interference, diffraction, reflection, refraction and polarization.

⇒ similar to light waves

Hertz demonstrated that the propagation speed of these electromagnetic waves was

$c = 3 \cdot 10^{8}$ m/s

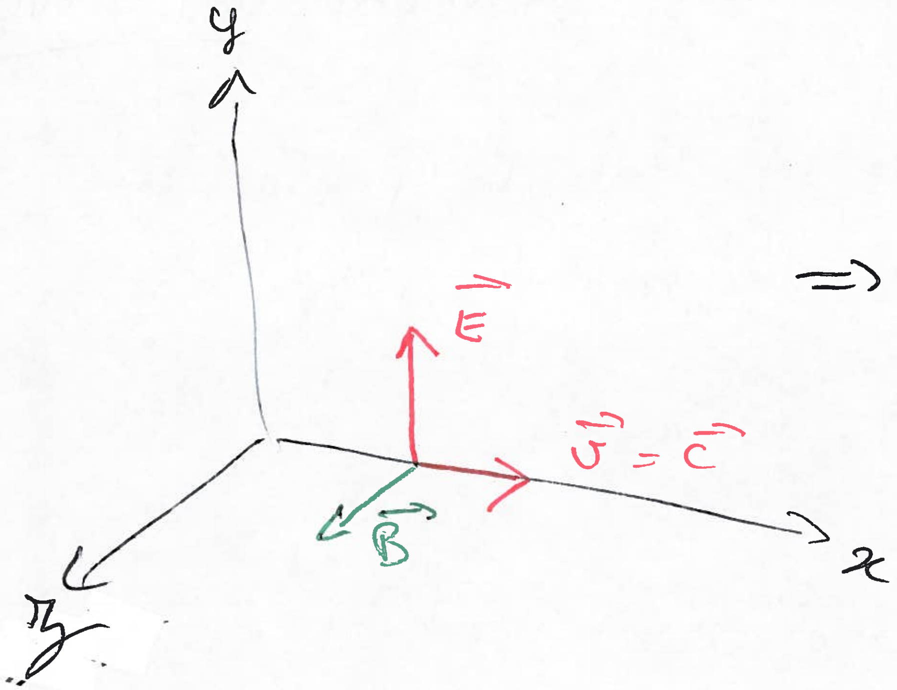
speed of visible light

⇒ Light is an electromagnetic wave

⇒ derived from Maxwell equation

Plane electromagnetic Waves

Solving Maxwell equations in a particular case



⇒ Linearly polarized wave

Let's assume that $\vec{B}$ and $\vec{E}$ depend on x and t only

* Wave radiated from any point of the yz plane
  ⇒ propagating in x direction, in phase

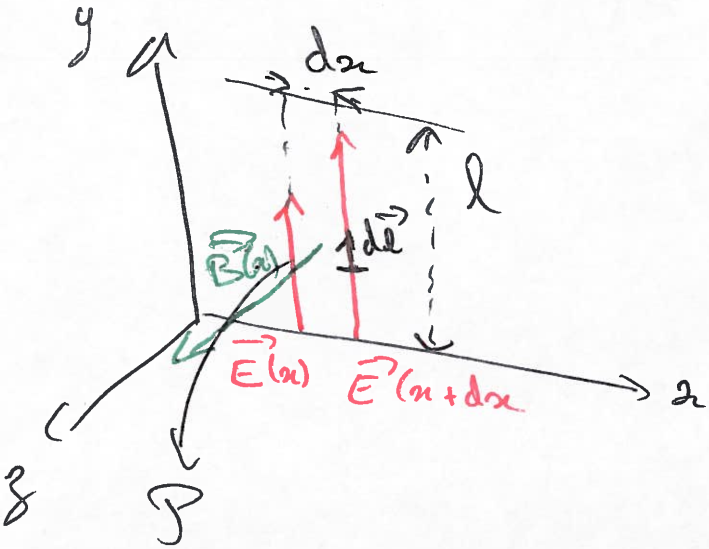
⇒

Ray = line along which the wave travels
⇒ all rays are //

Collection of waves = plane wave

Surface connecting points of equal phase = wave front
⇒ plane wave

When point source of radiation ⇒ <u>spherical wave</u>



$$\oint_S \vec{E}\cdot\vec{dl} = E(x+dx)\,l - E(x)\,l$$

<u>With</u> : $E(x+dx) \simeq E(x) + \underbrace{\left.\frac{dE}{dx}\right|_{t=cte}}_{\displaystyle \frac{\partial E}{\partial x}} dx$

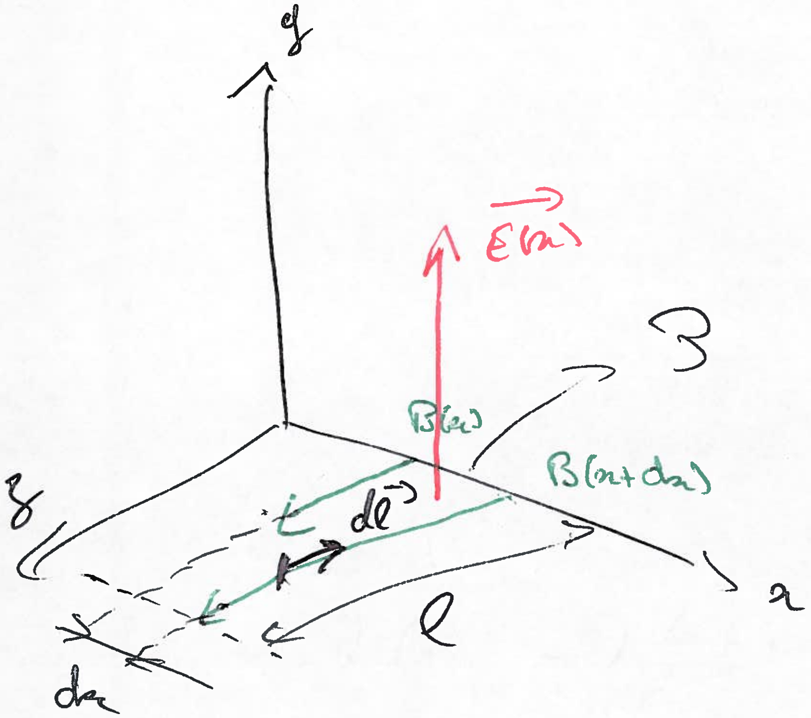
(partial derivative along x, keeping all other variable constant, including t)

$$\oint_S \vec{E}\cdot\vec{dl} = \frac{\partial E}{\partial x}\,dx = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = B\,l\,dx$$

$$\frac{d\Phi_B}{dt} = l\,dx \left.\frac{dB}{dt}\right|_{x\,cte} = l\,dx\,\frac{\partial B}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$



$$\oint_S \vec{B}\cdot d\vec{l} = B(x)\,l - B(x+dx)\,l$$

$$= -l\,\frac{\partial B}{\partial x}\,dx$$

$$\Phi_E = \int_S \vec{E}\cdot d\vec{S} = E\,l\,dx$$

$$\frac{d\Phi_E}{dt} = l\,dx\,\frac{\partial E}{\partial t}$$

$$\oint_S \vec{B}\cdot d\vec{l} = \cancel{\mu_0 I} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\Leftrightarrow \quad -\ell\left(\frac{\partial B}{\partial x}\right)dx = \mu_0 \varepsilon_0 \, \ell \, dx\left(\frac{\partial E}{\partial t}\right)$$

$$\Rightarrow \quad \frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

We have

$$\frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right)$$

$$= -\frac{\partial}{\partial t}\left(-\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\boxed{\begin{aligned} &\Rightarrow \quad \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \\ &\text{Similarly} \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \end{aligned}}$$

Wave equation with speed $v = c = 1/\sqrt{\varepsilon_0 \mu_0}$

$$c = \frac{1}{\sqrt{(4\pi \cdot 10^{-7}\ T.m/A)(8.854 \times 10^{-12}\ C^2/N.m^2)}}$$

$$c = 2.99792 \times 10^8\ m/s$$

Speed of light in empty space

Solution

$$E = E_{max} \cos(kx - \omega t)$$

$$B = B_{max} \cos(kx - \omega t)$$

k = wave number $= 2\pi / \lambda$

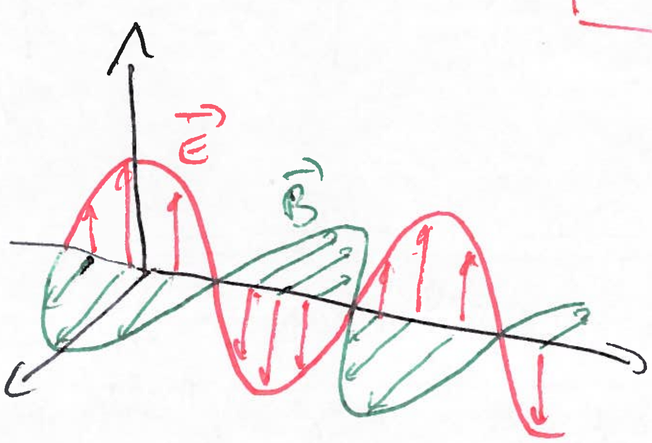
λ → wavelength in m

ω = angular frequency $= 2\pi f$

f → wave frequency in Hz

$$\frac{\omega}{k} = c \iff \frac{2\pi f}{2\pi/\lambda} = c$$

$$\iff \boxed{\lambda f = c}$$



We have

$$\frac{\partial E}{\partial x} = -k\, E_{max} \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega\, B_{max} \sin(kx - \omega t)$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \quad k\, E_{max} = \omega\, B_{max}$$

$$\boxed{\frac{E_{max}}{B_{max}} = \frac{E}{B} = \frac{\omega}{k} = c}$$

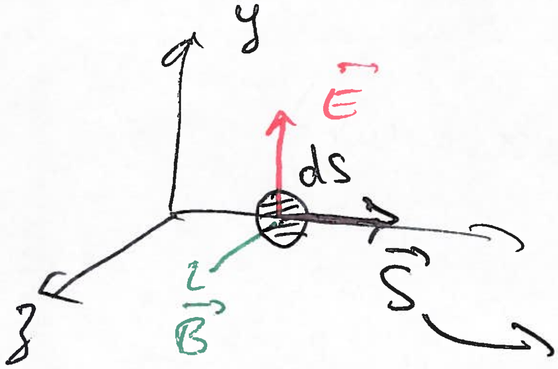
Energy carried by electromagnetic waves

T_{ER} = amount of energy transferred by electromagnetic wave

Rate of transfer of energy Poynting vector $\vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B}$

↳ rate of energy passing throug a surface ⊥

to the direction of wave propagati-



in $J/s.m^2 = W/m^2$

<u>Plane electromagnetic wave</u> $\Rightarrow S = \|\vec{S}\| = \dfrac{EB}{\mu_0}$

$B = E/c$

$$\Rightarrow \quad S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0} = \frac{EB}{\mu_0}$$

S is instantaneous ⇒ we prefer refering to

average power, ie average over 1 cycle

or more (coeff 1/2 = avg of $\cos^2$)

$$I = S_{avg} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{c B_{max}^2}{2\mu_0}$$

Note Instantaneous energy density

for $\vec{E}$ $\Rightarrow$ $u_E = \frac{1}{2} \varepsilon_0 E^2$

for $\vec{B}$ $\Rightarrow$ $u_B = \frac{1}{2} \frac{B^2}{\mu_0}$

using $E/B = c$ and $c = 1/\sqrt{\mu_0 \varepsilon_0}$

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0 \varepsilon_0}{2\mu_0} E^2 = \frac{1}{2} \varepsilon_0 E^2 = u_E$$

Total instantaneous energy

$$u = u_E + u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$u_{avg} = \varepsilon_0 (E^2)_{avg} = \varepsilon_0 \frac{1}{2} E_{max}^2 = \frac{B_{max}^2}{2\mu_0}$$

$$\Rightarrow \boxed{I = S_{avg} = c\, u_{avg}}$$