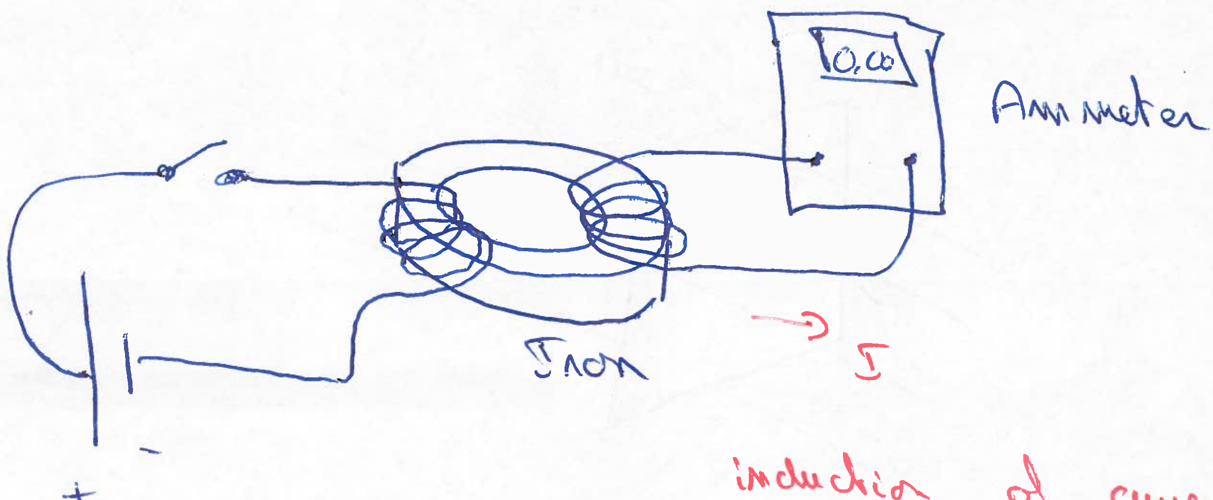
Faraday's law

Chapter 31

Faraday law of induction.



induction of current

* when switchin on the the battery
* ——— off the battery

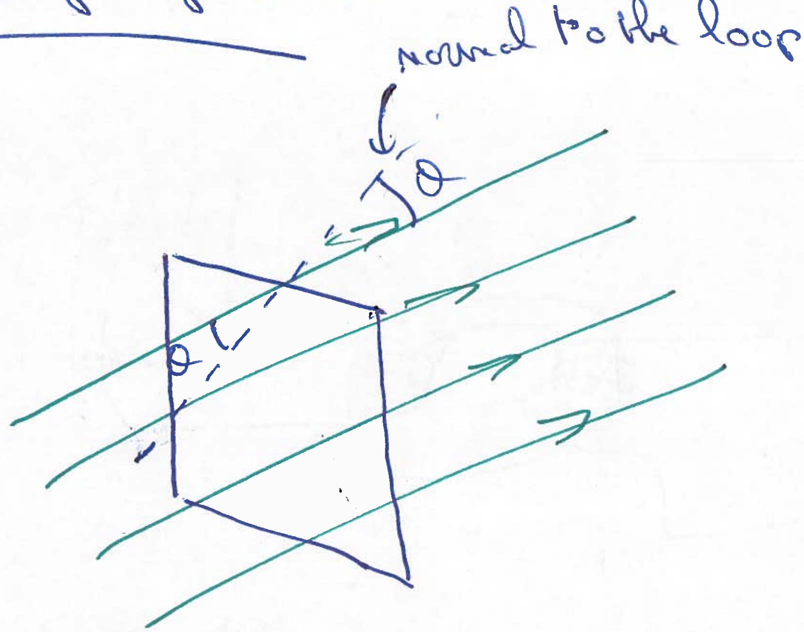
$\Rightarrow$ transient

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \text{with } \Phi_B = \int \vec{B} \cdot d\vec{S}$$

if a coil consist of N loops , emf in serie

$$\varepsilon = -\underline{N}\ \frac{d\phi_B}{dt}$$

in a uniform field $\vec{B}$



$$\Rightarrow \quad \varepsilon = -\frac{d}{dt}\left(B\,S\,\cos\theta\right)$$

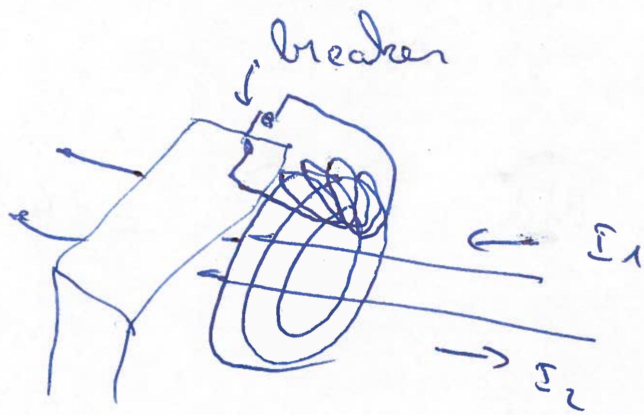
Induction of a current :

→ by varying B

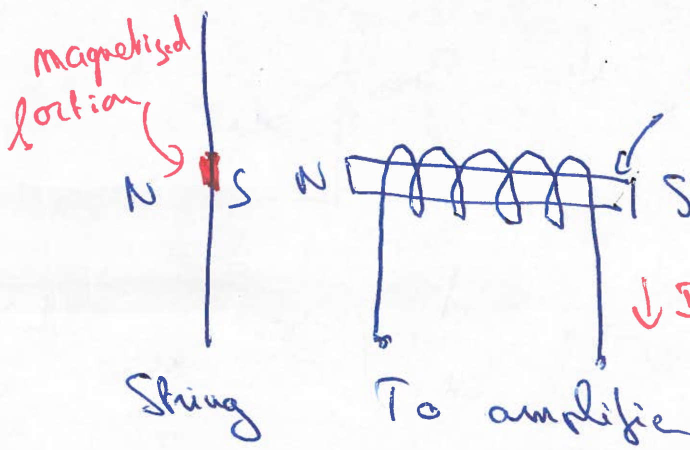
→ by varying S

→ by varying θ

ex: Ground fault circuit interrupter



Guitar pickup coil



↓ I created by vibration of the string.

Inducing an emf in a coil

coil of 200 loop, each loop is a square ($d = 18$ cm)

in a uniform field $\vec{B}$ $\perp$ to the plane of the coil

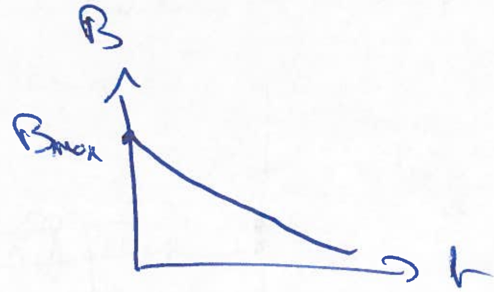
$B \nearrow$ linearly from 0 to 0,5 T in 0,80 s

$\Rightarrow$ induced emf ??

$$|\mathcal{E}| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BS)}{\Delta t}$$

$$|\mathcal{E}| = Nd^2\,\frac{B_f - B_i}{\Delta t} = 4.0\ \text{V}$$

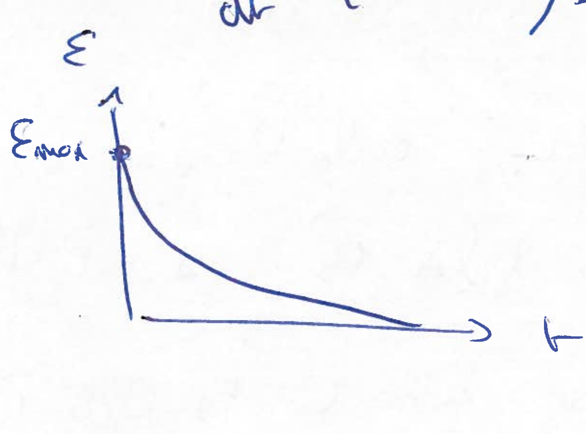
* A loop with a surface S, $\vec{B} \perp$ to the plane of the loop



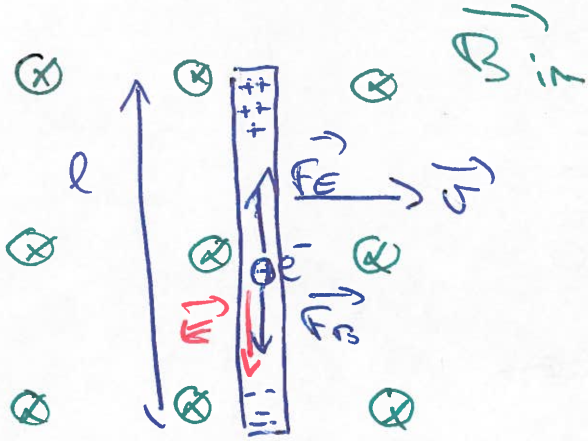
at $t=0$: $B = B_{max}\, e^{-at}$

$$\Rightarrow \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(S B_{max}\, e^{-at}\right)$$

$$= -S B_{max}\frac{d}{dt}\left(e^{-at}\right) = \underbrace{a S B_{max}}_{\mathcal{E}_{max}}\, e^{-at}$$



Motional emf



Straight conductor of length l, moving at a speed $\vec{v}$ because of an external agent

$\Rightarrow$ on each free electron:

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

$\Rightarrow$ generation of an electric field $\vec{E}$, $\vec{F_E} = q\vec{E}$

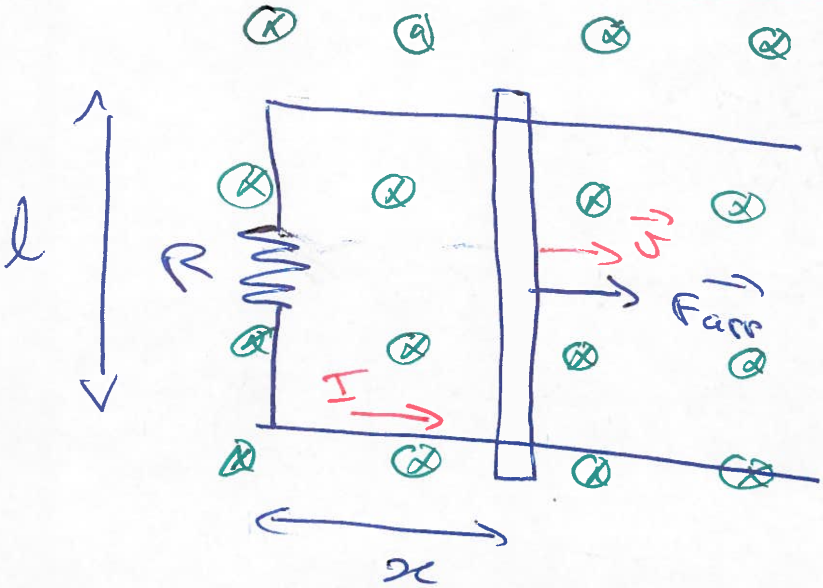
Equilibrium $\Rightarrow \vec{F_B} = -\vec{F_E}$

$$\Rightarrow qvB = qE \Rightarrow \boxed{E = vB}$$

$\vec{E}$ is uniform $\Rightarrow \Delta V = El$

$$\Rightarrow \boxed{\Delta V = El = Blv}$$

Moving a conductor in a closed conducting path



$$\underline{\underline{\phi_B = B l x}}$$

$$\Rightarrow \quad \mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt}$$

$$\Rightarrow \boxed{\mathcal{E} = -Blv}$$ Motional emf

$$\Rightarrow I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$$

Energy consideration

A work is applied on the bar

Conservation of energy $W = \Delta E_{int}$

$\Rightarrow$ input energy $\Rightarrow$ internal energy in the resistor

When moving the bar $\Rightarrow$ force $\vec{F_B}$ is experienced

$\Rightarrow$ magnitude $\underline{\underline{I \ell B}}$

$\vec{v} = \text{cte}$, ie $\Rightarrow$ $\underline{\underline{\vec{a} = 0}}$ $\Rightarrow$ $\Sigma \vec{F} = \vec{0}$

$\Rightarrow$ $F_{app} = F_B = I \ell B$

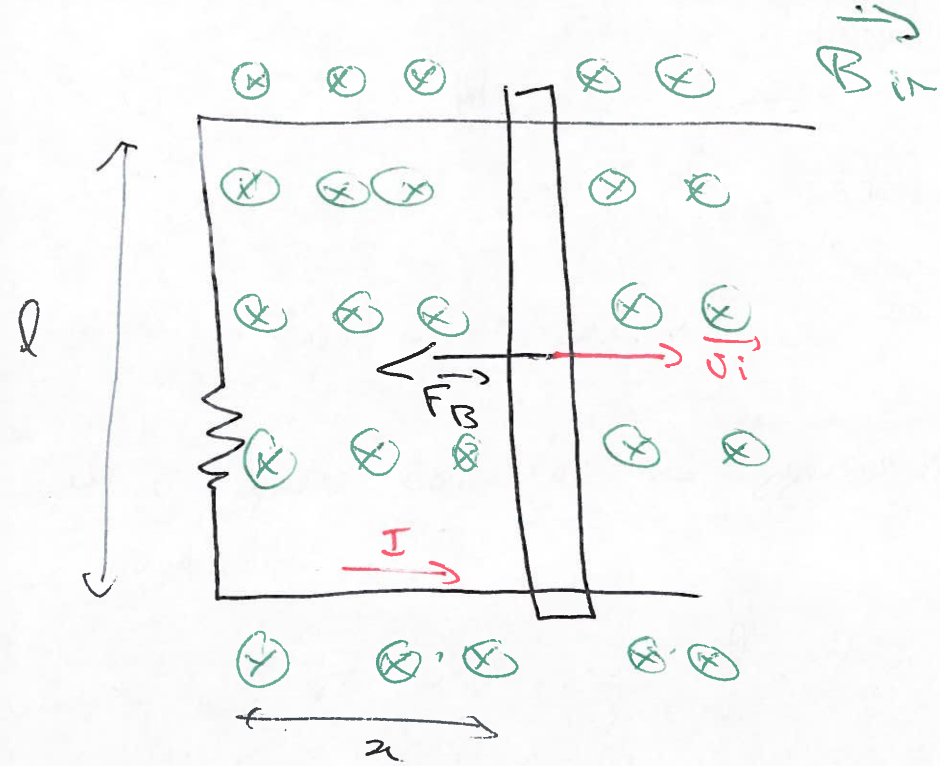
$\Rightarrow$ Power delivered by the applied force

$$P = F_{app}\, v = (I \ell B)\, v = \frac{B^2 \ell^2 v^2}{R} = \frac{\varepsilon^2}{R}$$

$(W = F_{app}\, x)$ $\left(I = \dfrac{B \ell v}{R} \right)$

$P = \dfrac{dW}{dt}$

$$\boxed{P = \frac{\varepsilon^2}{R}}$$



Initial speed at $t=0$ $\vec{v_i}$

$$\vec{F_B} = -I l B \vec{i}$$

$$\sum \vec{F} = m\vec{a} \Rightarrow F_x = ma$$

$$\Rightarrow -IlB = m\frac{dv}{dt}$$

$$I = Blv/R$$

$$\Rightarrow m\, dv/dt = -\frac{B^2l^2}{R} v \Leftrightarrow \frac{dv}{v} = -\frac{B^2l^2}{mR} dt$$

$$\int_{v_i}^{v} \frac{dv}{v} = \int_0^t \left(-\frac{B^2l^2}{mR}\right) dt$$

$$\ln\left(v/v_i\right) = -\frac{B^2 l^2}{mR}\, t$$

$$\Rightarrow \boxed{v = v_i\, e^{-t/\tau} \quad \text{with } \tau = \frac{mR}{B^2 l^2}}$$

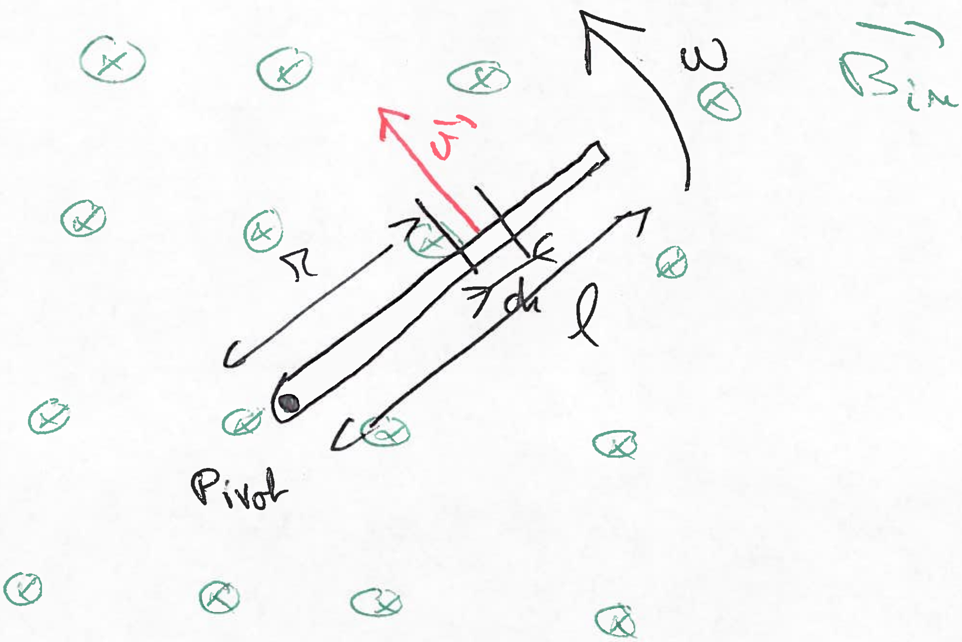
<u>Or</u> : $P_{resistor} = -P_{bar}$

$$I^2 R = -\frac{d}{dt}\underbrace{\left(\frac{1}{2} m v^2\right)}_{OK}$$

$$\Rightarrow \frac{B^2 l^2 v^2}{R} = -m v \frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{v} = -\left(\frac{B^2 l^2}{mR}\right) dt$$

Motional emf induced in a rotating bar



$$|d\mathcal{E}| = B\,v\,dr \Big\} \text{ small elements in serie}$$

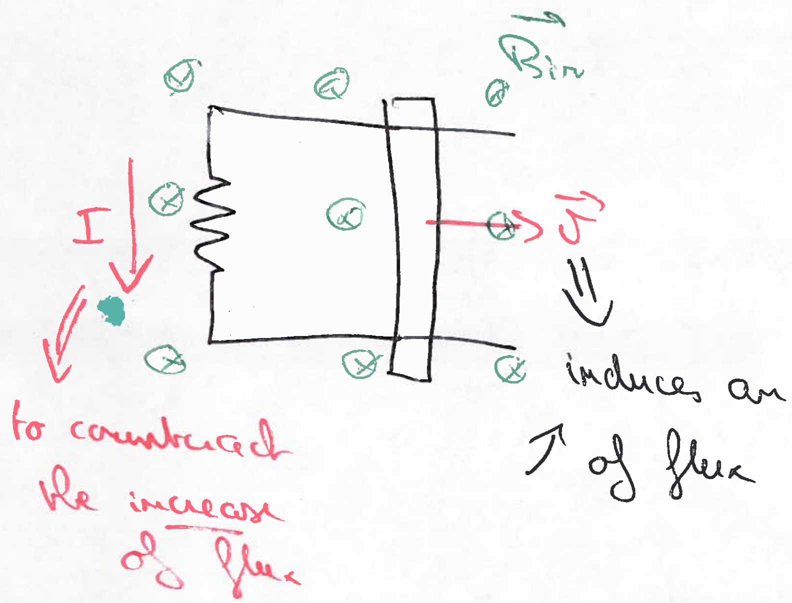
$$\Rightarrow |\mathcal{E}| = \int B\,v\,dr \quad \text{with} \quad v = r\omega$$

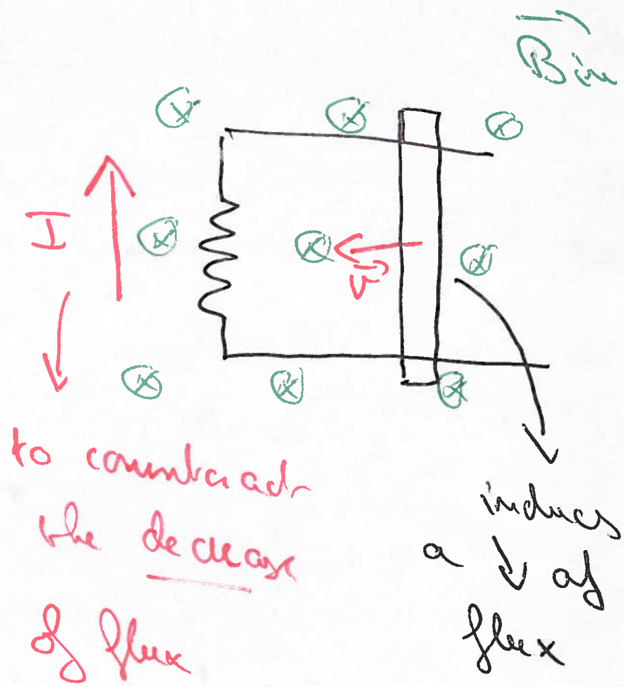
$$\Rightarrow |\mathcal{E}| = B\int v\,dr = B\omega\int_0^l r\,dr$$

$$|\mathcal{E}| = \frac{1}{2}B\omega l^2$$

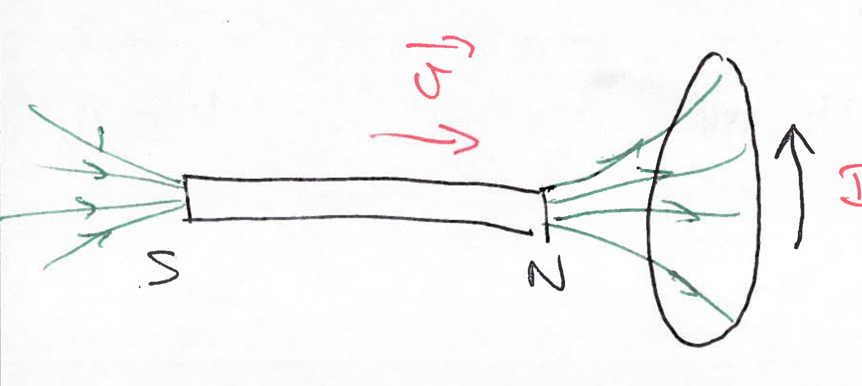
Lentz 's Law

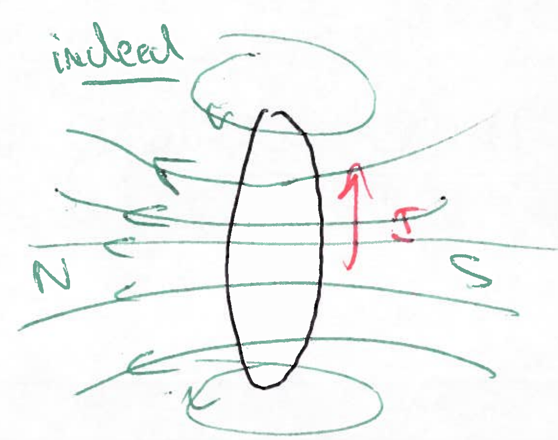
The induced current in a loop is in the direction that creates a magnetic field that opposes the change of magnetic flux through the area enclosed by the loop.

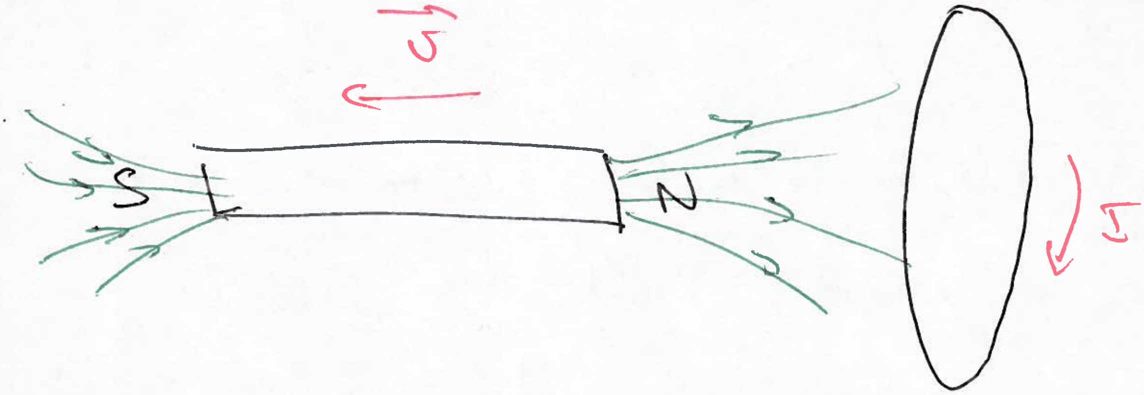


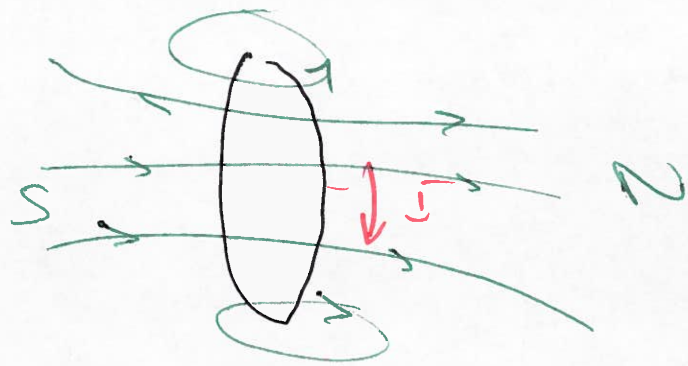


$$\mathcal{E} = - d\Phi_B/dt$$

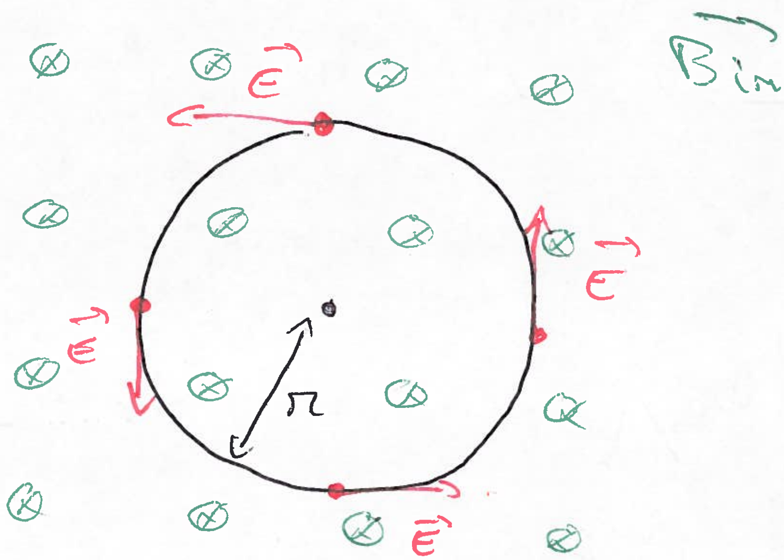








Induced emf and electric fields



If $\vec{B}$ is varying with time $\Rightarrow$ $\mathcal{E} = -d\Phi_B/dt$

Note : In electrostatic, $\vec{E}$ exists even if there is not test charge $\Rightarrow$ Moving charges $\vec{E}$

$\Rightarrow$ change in magnetic flux $\Rightarrow$ induced $\vec{E}$

$\vec{E}$ induced is non conservative.

$\vec{E}$ electrostatic (created by static charges) is conservative

Work done by the electric field in moving a charge q around the loop $= q\mathcal{E}$

Force $q\vec{E}$ $\Rightarrow$ work around the loop $qE(2\pi r)$

We should have: $q\mathcal{E} = qE(2\pi r)$

$$\Rightarrow E = \mathcal{E}/2\pi r$$

$\mathcal{E} = -d\Phi_B/dt \Rightarrow$ $\boxed{E = -\frac{\pi}{2}\, dB/dt}$

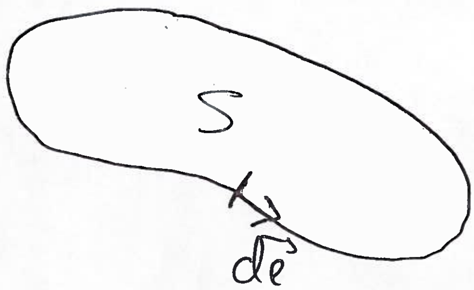
$\Phi_B = BS = B\pi r^2$

$\Rightarrow$ time varying magnetic field.

$\Rightarrow$ induced electric field

$$\mathcal{E} = \oint \vec{E} \cdot \vec{dl}$$

For any close path Γ



$$\oint_{\Gamma} \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$$

Note when $\vec{B}$ is varying $\Rightarrow$ $\vec{E}$ induced is

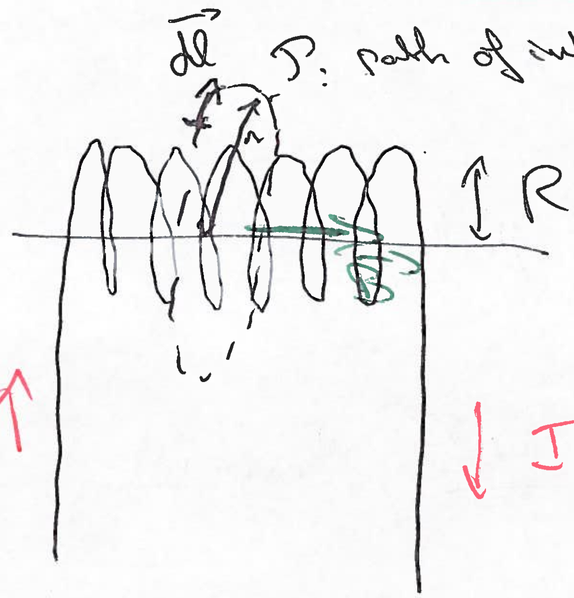
Not conservative.

Note 2: Electrostatic field: $\Delta V = -\int \vec{E} \cdot \vec{dl}$

$$\Delta V = -\oint \vec{E} \cdot \vec{dl} = 0$$

Conservative fielld

Electric field induced by a change in magnetic field in a solenoid



$\vec{B}$ uniform inside the solenoid

$B = \mu_0 n I$

$I = I_{max} \cos \omega t$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(B \pi R^2\right)$$

$B = \mu_0 n I$, $I = I_{max} \cos \omega t$

For $r > R$

$$\Rightarrow -\frac{d\Phi_B}{dt} = -\pi R^2 \mu_0 n I_{max} \frac{d}{dt}(\cos \omega t) = \pi R^2 \mu_0 n I_{max}\, \omega \sin \omega t$$

$$\oint_{\mathcal{P}} \vec{E}\cdot\vec{dl} = E\oint dl = E\,(2\pi r)$$

$\vec{E}$: constant along $\mathcal{P}$

$$E\,(2\pi r) = \pi R^2 \mu_0\, n\, I_{max}\, \omega \sin \omega t$$

$$\boxed{E = \frac{\mu_0\, n\, I_{max}\, \omega R^2}{2 r} \sin \omega t \quad \text{for } r > R}$$

$E \searrow$ as $1/r$

<u>For $r < R$</u>

$$-\frac{d\phi_B}{dt} = -\frac{d}{dt}\left(B \pi \underline{\underline{r^2}}\right)$$

$$E\,(2\pi r) = \pi r^2 \mu_0\, n\, I_{max}\, \omega \sin \omega t$$

$$\boxed{E = \frac{\mu_0\, n\, I_{max}\, \omega}{2}\, r \sin \omega t \quad \text{for } r < R}$$

Generators and Motors

Generator : energy by work → electrical transmission

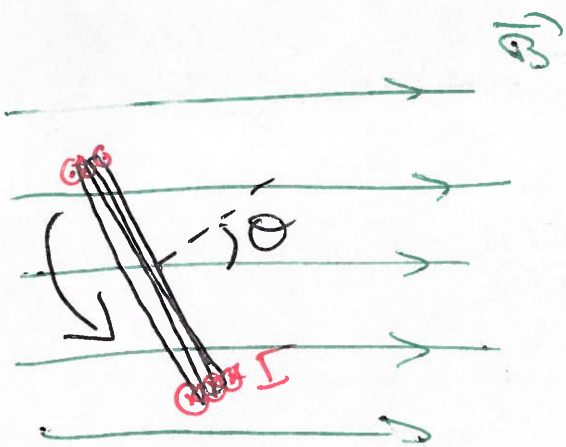
Motor : current ⇒ electrical transmission ⇒ energy transferred out by work.

Generator of AC current

$$\phi_B = B\,S\cos\theta = B\,S\cos\underline{\omega t}$$

N loop

$$\varepsilon = -N\frac{d\phi_B}{dt} = NBS\,\omega\,\underline{\underline{\sin\omega t}}$$

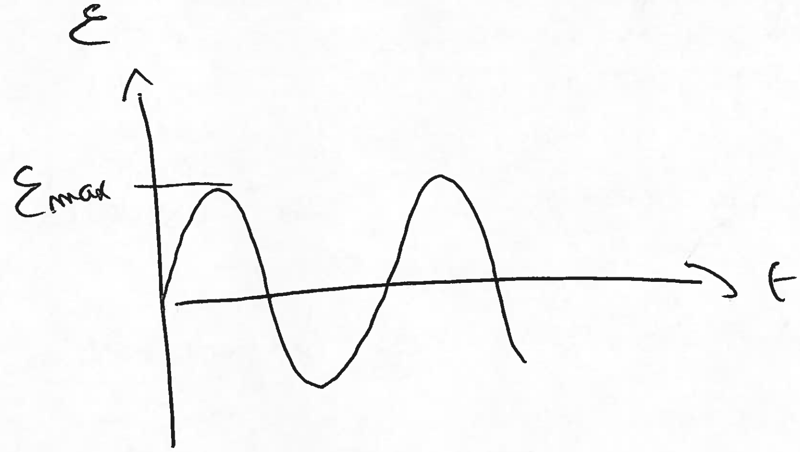


$$\varepsilon_{max} = NBS\omega$$

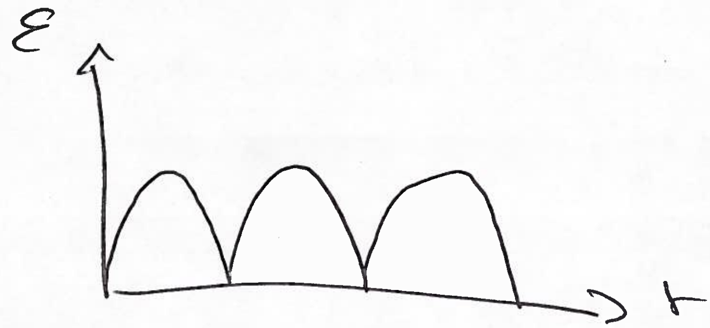
$$\omega = 2\pi f$$

$f = 60\,Hz$ in USA, Canada

$f = 50\,Hz$ in Europe



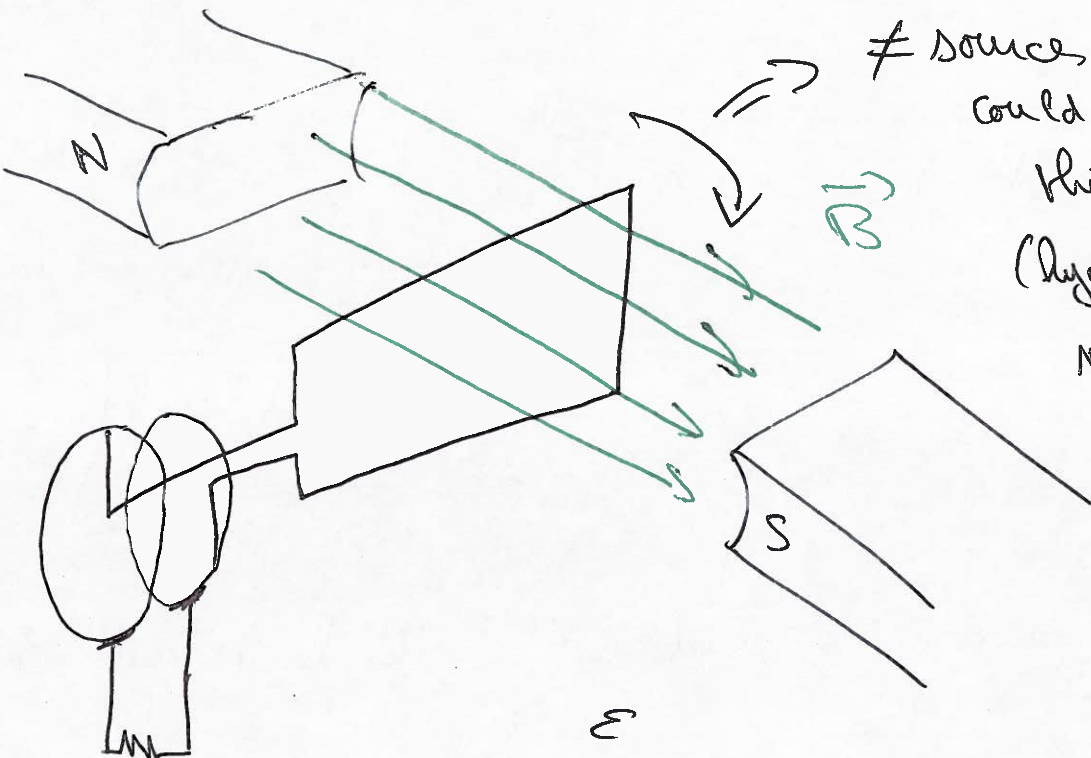
DC current generator : use of a commutator



Various coils out of phase to produce a steady $\mathcal{E}$.

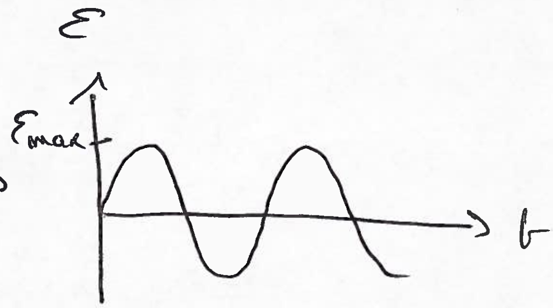
Induced current in a motor.

AC current generator

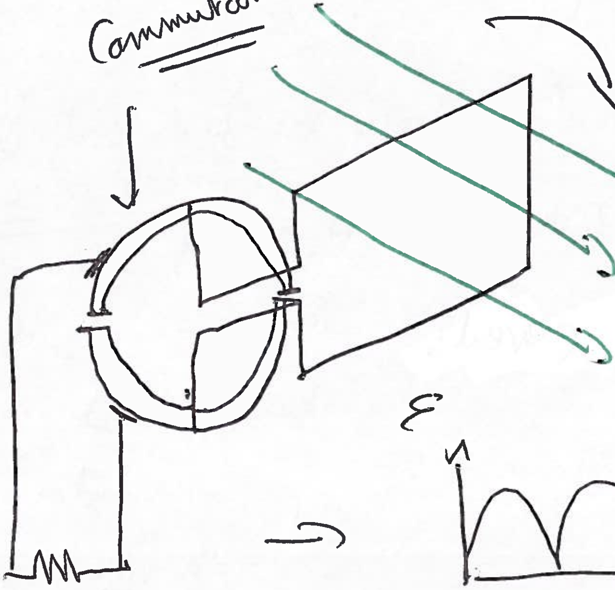


$\Rightarrow$ ≠ source of energy could produce this rotat. (hydro, steam, nuclear, ...)

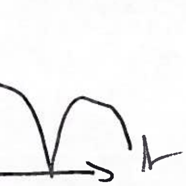
light bulb, ... $\Rightarrow$



DC current generator



$\rightarrow$



various coils out of phase to produce a steady $\mathcal{E}$

Motor : electrical energy ⇒ transfered to work (rotation)

Current supplied to a coil (from a battery) ⇒ torque on the current carrying coil in $\vec{B}$

⇒ **Rotation**

Transmission of mechanical work to external device.

However ⇒ coil rotating in a magnetic field $\vec{B}$

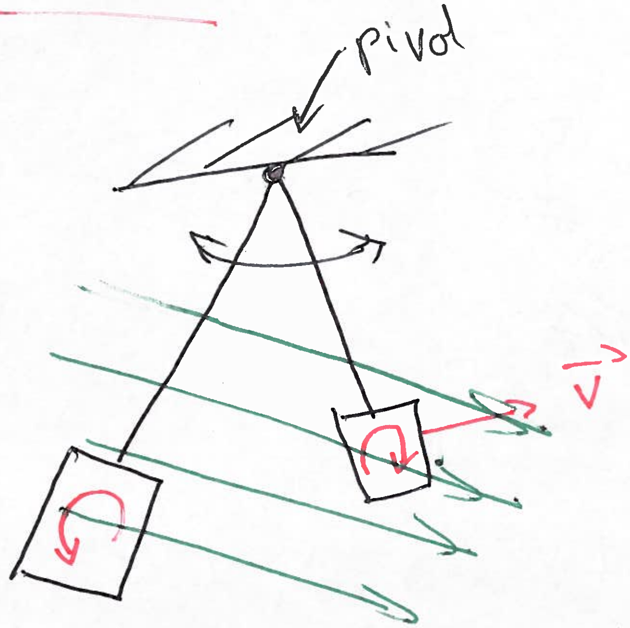
⇒ emf induced

From Lentz's law ⇒ **back** emf to reduce the current in the coil

If mechanical load ↑ ⇒ motor slows down ⇒ back emf ↓
⇒ more current in the coil ⇒ more consumption.

Heavy load ⇒ motor jam ⇒ lack of back emf
can lead to dangerous high current in the motor
⇒ heat

Eddy currents

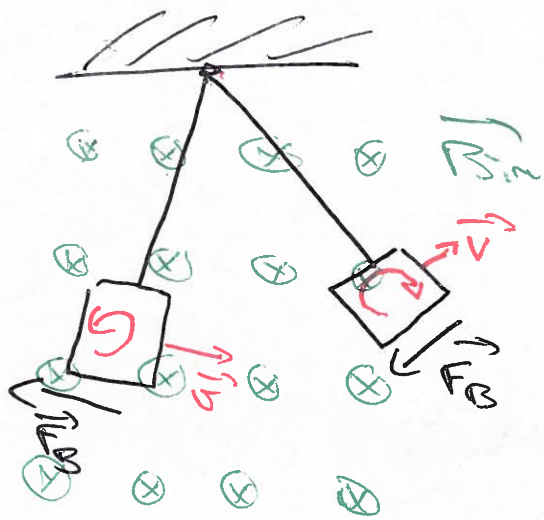


⇒ Movement of piece of metal in a magnetic field ⇒ induced circulating currents

⇒ eddy current

Lentz law ⇒ magnetic field generated by the currents in the plates ⇒ poles that repel by the poles of the magnet ⇒

‖ Repulsive force that oppose to the motion of the plate



- Applicati **Brakes** (train, truck) (brake without contact)
- Cuts in the plates to prevent the creation of eddy currents (to eliminate them since they are undesirable in most case