Electrostatic force and Electric field (Chapter 23)

1. Electric charge

Matter is formed of elementary particles
$\Rightarrow$ part of nuclei, atoms, molecules, cristals, ..

The main interaction you've seen so far is due to: Mass
$\Rightarrow$ Gravitational forces caused by the mass of two particles

The mass $m > 0$ (in some cases, $m = 0$ neutrinos)

However some interactions cannot be explained by mass
$\Rightarrow$ interactions at distance

$\Rightarrow$ Elementary particles are associated with an electric charge q

	example
$q > 0$	proton $+e$
$q = 0$	neutron
$q < 0$	electron $-e$

q is quantized
$\Rightarrow$ Smallest elementary charge

electron $-e = -1.60218 \times 10^{-19}$ C $\rightarrow$ Coulomb

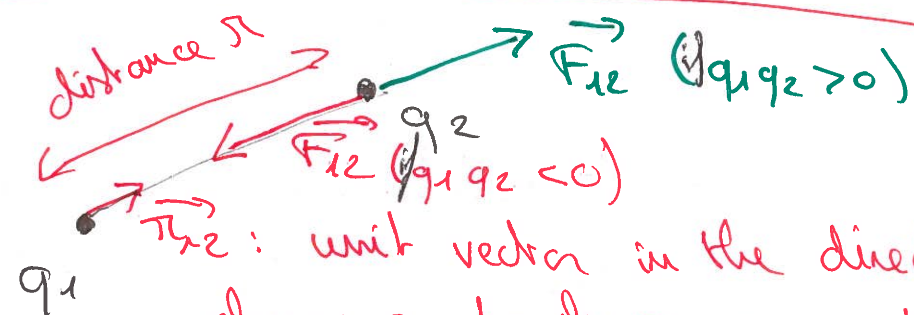
2 charges of opposite sign attract each other

2 charges of same sign repel each other

Definition : Insulator : material where electrons are bound to atoms and cannot freely move

Conductor : material composed of free electrons that can move easily in the material (eg: metals)

2. Electrostatic force and Coulomb's law



$\vec{r}_{12}$: unit vector in the direction from charge q_1 to charge q_2 : $\|\vec{r}_{12}\| = 1$

Force exerted by a charge q_1 on q_2

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \vec{r}_{12}$$

→ independant of q_2 ⇒ Force exerted where we place the charge q_2

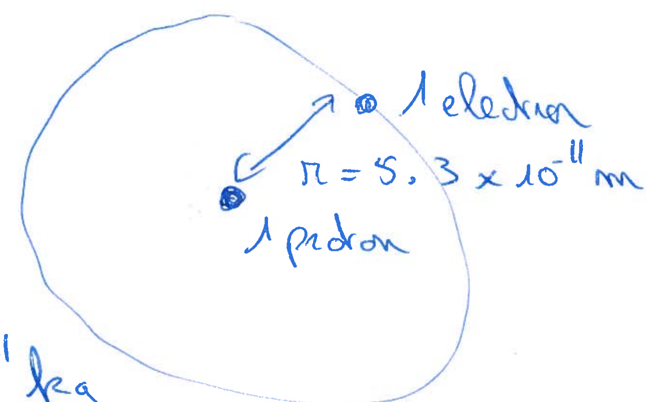
if $q_1 q_2 < 0 \Rightarrow$ attraction

if $q_1 q_2 > 0 \Rightarrow$ repulsion

$$k_e = \frac{1}{4\pi \varepsilon_0} = 8.9876 \times 10^9 \ N.m^2/C^2$$

electrical permittivity in free space

ex: The hydrogen atom



Electron (e) $m = 9.1094 \times 10^{-31}$ kg

$q = -e = -1,60218 \times 10^{-19}$ C

Proton (p) $m = 1.67262 \times 10^{-27}$ kg

$q = +e = +1.60218 \times 10^{-19}$ C

Electrostatic force : $\|\vec{F_e}\| = k_e \dfrac{|e||-e|}{r^2} = 8,2 \times 10^{-8}$ N

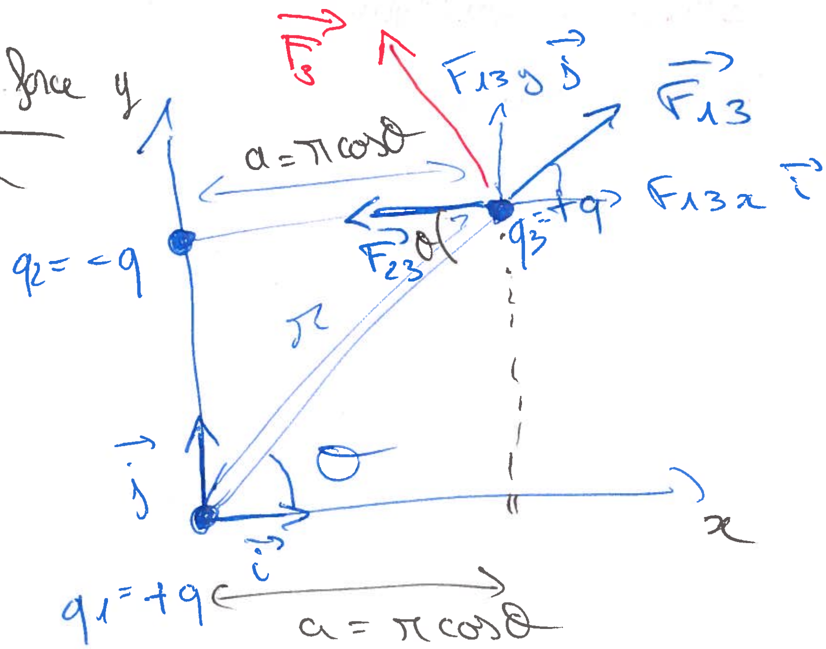
Gravitational force : $\|\vec{F_g}\| = G \dfrac{m_e m_p}{r^2} = 3.6 \times 10^{-47}$ N

$G = 6.674 \times 10^{-11}$ N.m^2 / kg^2

$$\Rightarrow \quad \|\vec{F_e}\| \gg \|\vec{F_g}\|$$

In general we <u>neglect</u> gravitational forces in presence of electrostatic forces.

<u>Example of force calculation</u>



<u>Forces applied on q_3</u>

$$\vec{F_3} = \vec{F}_{13} + \vec{F}_{23}$$

$\vec{F}_{13}$: force applied by q_1 on q_3

$\vec{F}_{23}$: force applied by q_2 on q_3

$$\vec{F}_{13} = F_{13x}\,\vec{i} + F_{13y}\,\vec{j}$$

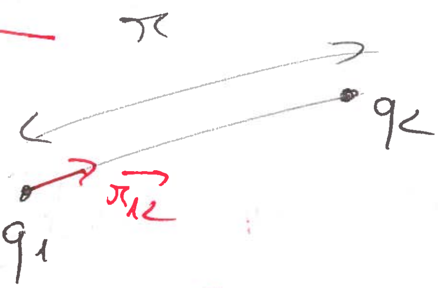
$$= k_e \frac{q^2}{r^2}\cos\theta\,\vec{i} + k_e \frac{q^2}{r^2}\sin\theta\,\vec{j}$$

$$\vec{F}_{23} = -\frac{k_e\, q^2}{a^2}\,\vec{i} + 0\,\vec{j}$$

$$= -\frac{k_e\, q^2}{(r\cos\theta)^2}\,\vec{i} + 0\,\vec{j}$$

So: $$\vec{F}_3 = \frac{k_e\, q^2}{r^2}\left(\cos\theta - \frac{1}{(\cos\theta)^2}\right)\vec{i} + \frac{k_e\, q^2}{r^2}\sin\theta\,\vec{j}$$

3. Electric field

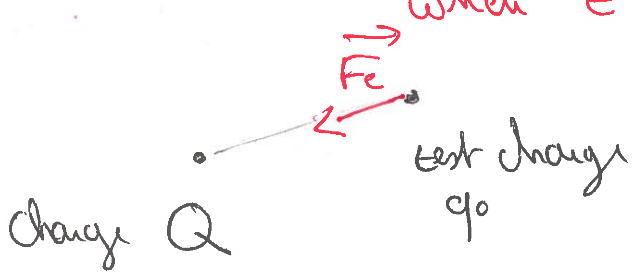


$$\vec{F}_{12} = k_e\,\frac{q_1 q_2}{r^2}\,\vec{r}_{12}$$

$\Downarrow$

This is the electric field $\vec{E}$ generated by a charge q_1 in position of q_2

$\Rightarrow$ it results in an electrostatic force when $\vec{E}$ is applied on a test charge q_2



By definition

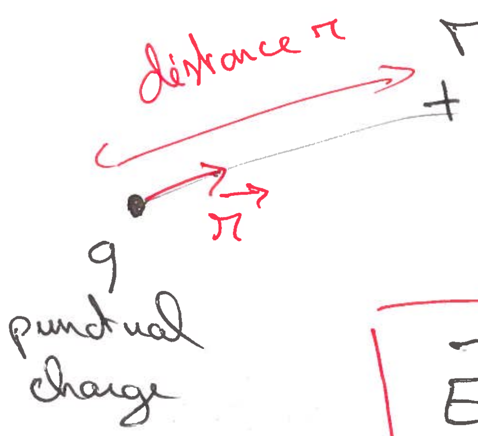
$$\vec{E} = \frac{\vec{F_e}}{q_0} \quad (\text{unit } N/C)$$

When placing a test charge q_0 in a point in space where an electric field $\vec{E}$ exists, the resulting force $\vec{F_e} = q_0 \vec{E}$ will be exerted on q_0

Analogy : gravitational field vs gravitational force

$$\underbrace{\vec{F_g}}_{\text{force}} = \underbrace{m}_{\text{mass}} \underbrace{\vec{g}}_{\text{gravitational field}}$$

Electric field generated by a punctual charge q



What is the electric field $\vec{E}$ in r

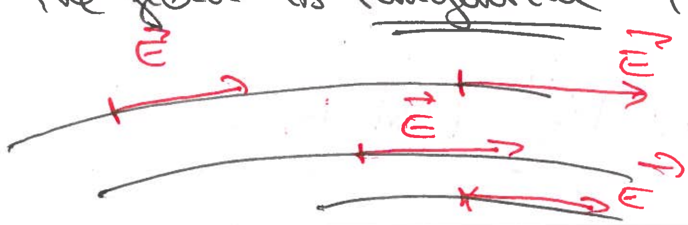
$\vec{r}$: unit vector: $\|\vec{r}\| = 1$

$$\boxed{\vec{E}(r) = k_e \frac{q}{r^2} \vec{r}}$$

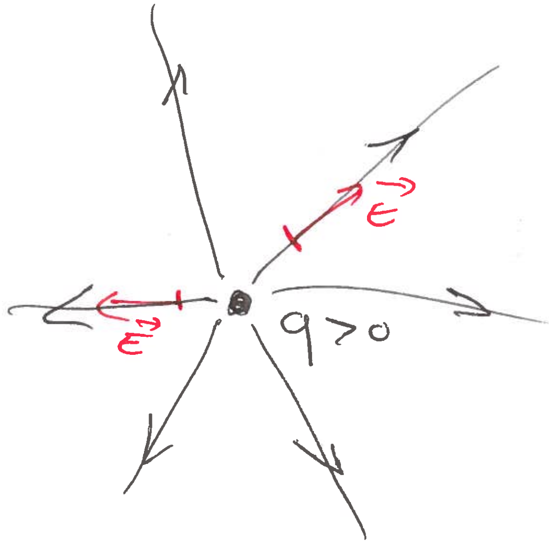
This is a vector field defined in any point in 3D space

Definition of a field line

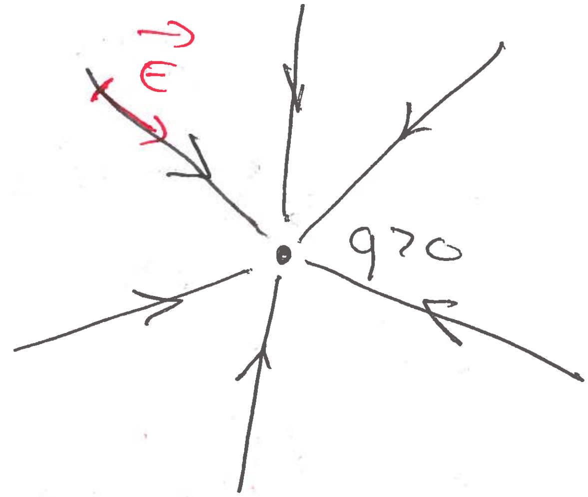
The field is tangential to the line in any point along the line



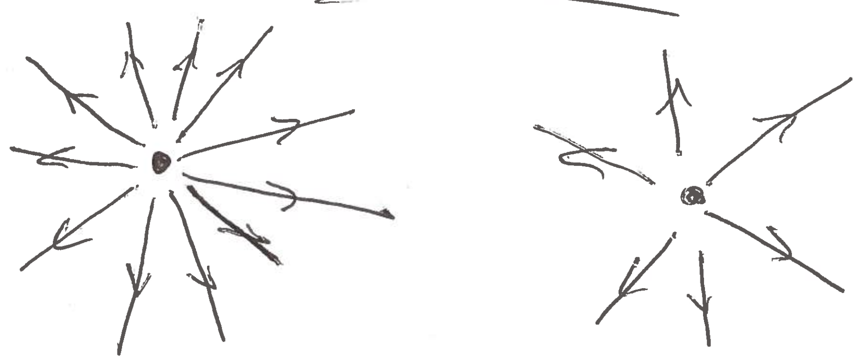
Positive charge $q > 0$



Negative charge $q < 0$



If $Q > q > 0$

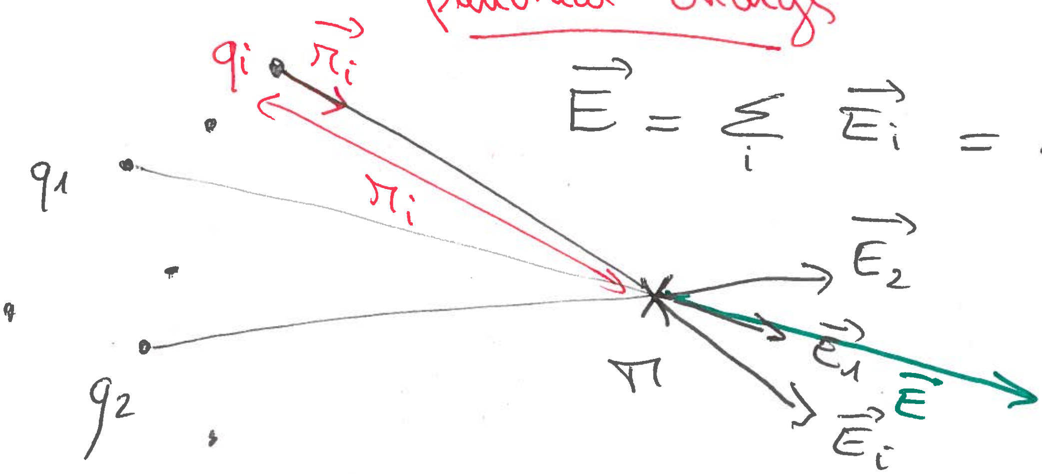


$Q \quad > \quad q$

The "density" of line fields is in relation with the field strength

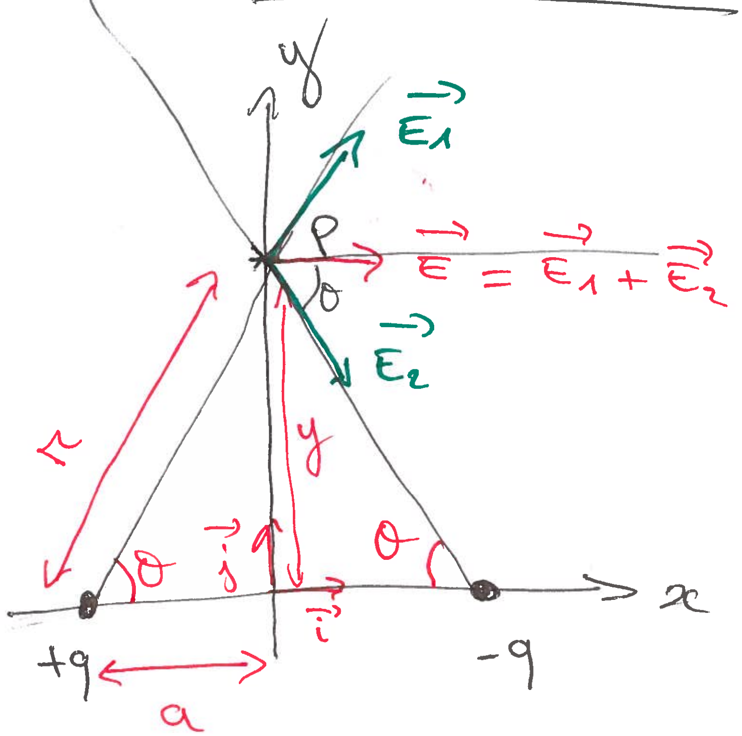
4. Electric field generated by a distribution of discrete punctual charges

$$\vec{E} = \sum_i \vec{E}_i = \sum_i k_e \frac{q_i}{r_i^2} \vec{r}_i$$



$\Rightarrow$ Superposition Principle

Example The electrostatic dipole



What is the electric field $\vec{E}$ generated by the dipole in P ?

$$\vec{E}_1 = E_{1x}\,\vec{i} + E_{1y}\,\vec{j}$$

$$\vec{E}_1 = k_e \frac{q}{r^2} \cos\theta\,\vec{i} + k_e \frac{q}{r^2} \sin\theta\,\vec{j}$$

$$\vec{E}_2 = E_{2x}\,\vec{i} + E_{2y}\,\vec{j}$$

$$= k_e \frac{q}{r^2} \cos\theta\,\vec{i} - k_e \frac{q}{r^2} \sin\theta\,\vec{j}$$

cancellation

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2\,\frac{k_e q}{r^2} \cos\theta\,\vec{i} + 0\,\vec{j}$$

Note that $r^2 = a^2 + y^2$

and $\cos\theta = \dfrac{a}{r} = \dfrac{a}{(a^2+y^2)^{1/2}}$

$$\Rightarrow E_x = \frac{2 k_e q}{(a^2+y^2)} \frac{a}{(a^2+y^2)^{1/2}}$$

$$\boxed{E_x = k_e \frac{2aq}{(a^2+y^2)^{3/2}}} \qquad \boxed{\vec{E} = E_x \vec{i}}$$

Electrostatic dipole $\quad y \gg a$

$$\Rightarrow \boxed{\vec{E} = k_e \frac{2aq}{y^3} \vec{i}}$$

$\|\vec{E}\|$ is deacreasing as $1/y^3$, y being the distance from the dipole to the point M on the midline

5. Electric field of a continuous charge distribution

dV_1 elementary volume of charge dq_1

$$d\vec{E}_1 = k_e \frac{dq_1}{r_1^2}\vec{r}_1$$

$$\vec{E} = \int_V d\vec{E} = \int_V k_e \frac{dq}{r^2}\vec{r}$$

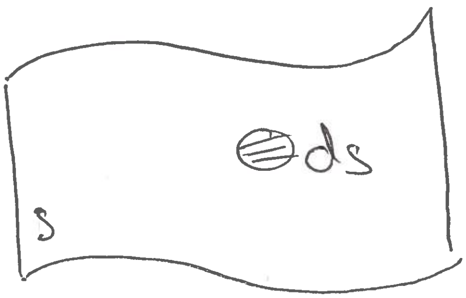
Volume charge density : $\rho \equiv Q/V$ (uniform charge distribution

$dq = \rho \, dV$

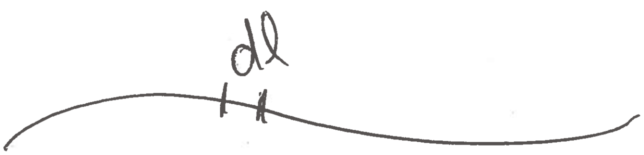
$$\Rightarrow \vec{E} = \int_V k_e \frac{\rho}{r^2}\vec{r}\, dV$$

Surface charge density

$$\sigma \equiv Q/S \Rightarrow \| dq = \sigma \, dS$$

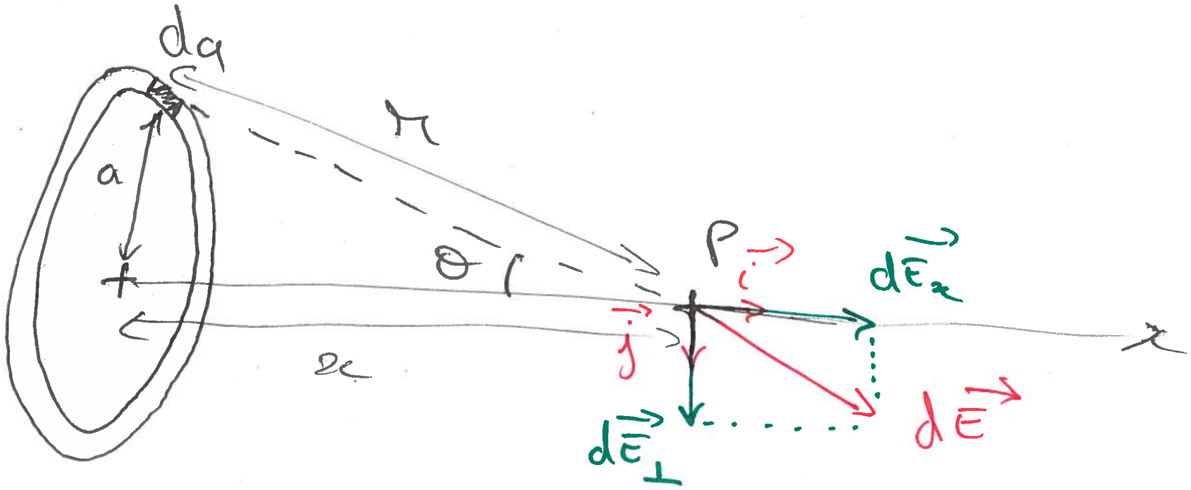


Linear charge density : $\lambda \equiv Q/\ell$



$$\Rightarrow dq = \lambda \, d\ell \, \|$$

Example : Electric field generated by a uniform ring



Transverse component of the electric field

$$\vec{E}_\perp = \int_{\text{along the ring}} d\vec{E}_\perp = \vec{0} \quad \text{by } \underline{\underline{\text{symmetry}}}$$

$$d\vec{E}_x = k_e \frac{dq}{r^2} \cos\theta \, \vec{i} = k_e \frac{dq}{a^2 + x^2} \cos\theta$$

$r^2 = a^2 + x^2$ and $\cos\theta = \dfrac{x}{r} = \dfrac{x}{(a^2+x^2)^{1/2}}$

$$\Rightarrow d\vec{E}_x = k_e \frac{x}{(a^2+x^2)^{3/2}} \, dq \, \vec{i}$$

$$\vec{E} = \int_{\text{along the ring}} d\vec{E}_x = \frac{k_e \, x}{(a^2+x^2)^{3/2}} \underbrace{\int_{\text{along the ring}} dq}_{\text{total charge } Q}$$

$\underline{\vec{E} \text{ near the center of the ring when } x \ll a}$

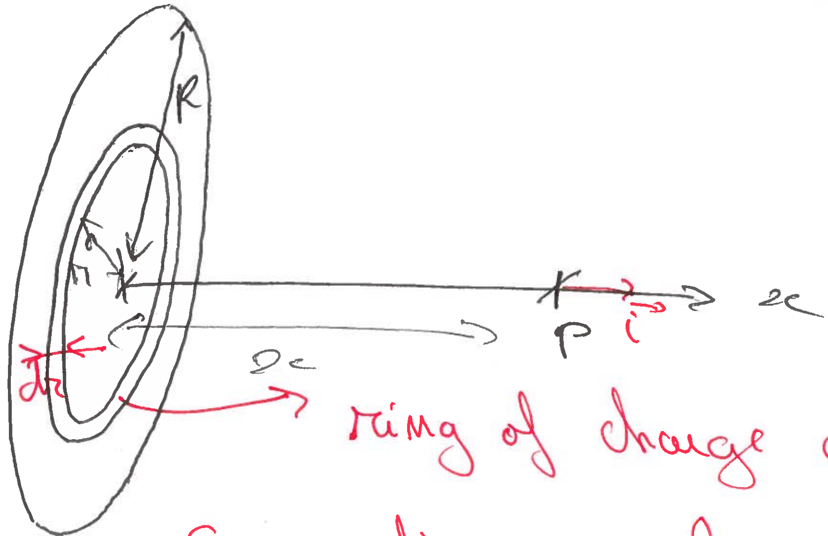
$$\vec{E} = k_e \frac{x}{a^3} Q \, \vec{i}$$

If we place a negative charge $-q$, near the center of the ring

$$\Rightarrow \text{Force } \vec{F}_x = -q\vec{E} = -\frac{k_e q Q}{a^3} x \, \vec{i}$$

of Hookes' law
Particle in simple harmonic motion model

Ex 2 : Electric field generated by a uniform charged disk of radius R



Superposition principle

To compute $\vec{E}$ generated by the disk, we integrate (sum) all fields generated by the rings from 0 to R.

The elementary ring of charge dq is generating the electric field

$$d\vec{E}_x = \frac{k_e\, x}{(r^2+x^2)^{3/2}}\, dq\ \vec{i}$$

<u>When</u> : $dq = \sigma\, dS = \sigma\, \underbrace{2\pi r\, dr}_{\text{surface of the ring}}$

$$\Rightarrow \vec{E} = \int_0^R d\vec{E}_x = \int_0^R k_e\, x\, \pi\, \sigma\, \frac{2r\, dr}{(r^2+x^2)^{3/2}}\ \vec{i}$$

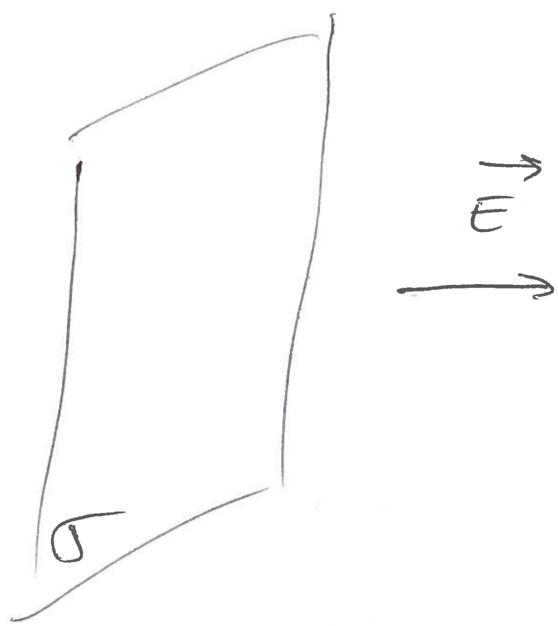
Note that $2r\,dr = d(r^2)$

Then: $\vec{E} = k_e\, x\, \pi\, \sigma \displaystyle\int_0^R (r^2+x^2)^{-3/2}\, d(r^2)\ \vec{i}$

$$\vec{E} = k_e\, x\, \pi\, \sigma \left[\frac{(r^2+x^2)^{-1/2}}{-1/2}\right]_0^R \vec{i}$$

$$\vec{E} = 2\pi k_e \sigma \left[1 - \frac{x}{(R^2+x^2)^{1/2}}\right] \vec{i}$$

Note than when $R \to +\infty$: Infinite uniformly charged plane



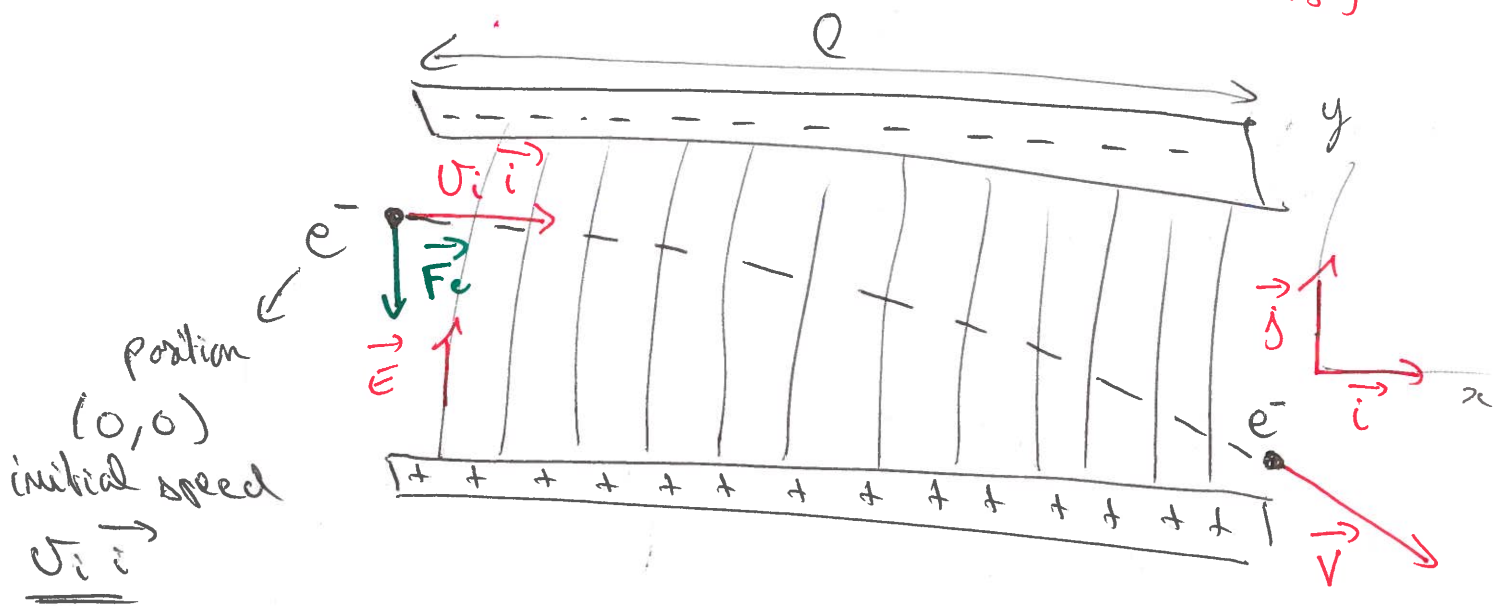
$$\vec{E} = 2\pi k_e \sigma\, \vec{i} = \frac{\sigma}{2\varepsilon_0}\vec{i}$$

$\Rightarrow$ an infinite uniformly charged plane is creating an uniform field $\vec{E}$

(impossible to realize in practice, only finite planes)

6. Motion of a charged particle in a uniform electric field $\vec{E}$

ex: An electron is projected horizontally into a uniform electric field produced by two charged plates (as in a cathodic tube in old TVs)



- Initial speed of the electron : $v_i = 3.00 \times 10^6$ m/s
- Uniform electric field : $\|\vec{E}\| = 200$ N/C
- Length of the plates : $l = 0.100$ m

Electrostatic force applied on the electron

$$\vec{F_e} = -e\vec{E} = 0\vec{i} - eE\vec{j}$$

The net force causes the particle to accelerate

$$\boxed{\vec{F_e} = \underset{\text{mass}}{m}\ \underset{\text{acceleration}}{\vec{a}}}$$

Therefore

$$\vec{a} = -\frac{eE}{m_e}\vec{j} \qquad a_y = -\frac{eE}{m_e} = -3.51 \times 10^{13}\ m/s^2$$

m_e ⇓ mass of the electron

Model with null acceleration, therefore constant speed along the x axis ($\vec{i}$)

$$\Rightarrow x_f = x_i + v_i t$$

[i: init (input)
f: final (output)

The electron will leave the plates at time

$$t = \frac{x_f - x_i}{v_i} = \frac{l - 0}{v_i} = 3.33 \times 10^{-8}\ s$$

Model of a particle with constant acceleration along the y axis ($\vec{j}$)

$$\Rightarrow y_f = \underset{0}{\underset{\parallel}{y_i}} + \underset{0}{\underset{\parallel}{v_{y_i}}}\, t + \frac{1}{2} a_y t^2 \Rightarrow \underbrace{y_f = -\frac{1}{2}\frac{eE}{m_e}t^2}_{\text{parabolic trajectory}}$$

Position of the electron when leaving the plate

$$\Rightarrow y_f = -1.95\ cm$$