2.6

(a) $p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}}{550\times10^{-9}}$

$= 1.205\times10^{-27}$ kg-m/s

$\upsilon = \frac{p}{m} = \frac{1.2045\times10^{-27}}{9.11\times10^{-31}} = 1.32\times10^{3}$ m/s

or $\upsilon = 1.32\times10^{5}$ cm/s

(b) $p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}}{440\times10^{-9}}$

$= 1.506\times10^{-27}$ kg-m/s

$\upsilon = \frac{p}{m} = \frac{1.5057\times10^{-27}}{9.11\times10^{-31}} = 1.65\times10^{3}$ m/s

or $\upsilon = 1.65\times10^{5}$ cm/s

(c) Yes

2.9

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h}\cdot 2mc = \frac{2mc^2}{100}$$

$$= \frac{2(9.11\times10^{-31})(3\times10^{8})^2}{100}$$

$$= 1.64\times10^{-15}\text{ J} = 10.25\text{ keV}$$

2.10

(a) $p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}}{85\times10^{-10}}$

$= 7.794\times10^{-26}$ kg-m/s

$\upsilon = \frac{p}{m} = \frac{7.794\times10^{-26}}{9.11\times10^{-31}} = 8.56\times10^{4}$ m/s

or $\upsilon = 8.56\times10^{6}$ cm/s

$E = \frac{1}{2}m\upsilon^2 = \frac{1}{2}(9.11\times10^{-31})(8.56\times10^{4})^2$

$= 3.33\times10^{-21}$ J

or $E = \frac{3.334\times10^{-21}}{1.6\times10^{-19}} = 2.08\times10^{-2}$ eV

(b) $E = \frac{1}{2}(9.11\times10^{-31})(8\times10^{3})^2$

$= 2.915\times10^{-23}$ J

or $E = \frac{2.915\times10^{-23}}{1.6\times10^{-19}} = 1.82\times10^{-4}$ eV

$p = m\upsilon = (9.11\times10^{-31})(8\times10^{3})$

$= 7.288\times10^{-27}$ kg-m/s

$\lambda = \frac{h}{p} = \frac{6.625\times10^{-35}}{7.288\times10^{-27}} - 9.09\times10^{-8}$ m

or $\lambda = 909\ \overset{o}{A}$

2.12

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-6}}$$

$$= 1.054\times10^{-28}\text{ kg-m/s}$$

2.23

(a) $\Psi(x,t) = Ae^{-j(kx+\omega t)}$

(b) $E = (0.025)(1.6\times10^{-19}) = \frac{1}{2}m\upsilon^2$

$= \frac{1}{2}(9.11\times10^{-31})\upsilon^2$

so $|\upsilon| = 9.37\times10^{4}$ m/s $= 9.37\times10^{6}$ cm/s

For electron traveling in $-x$ direction,

$\upsilon = -9.37\times10^{6}$ cm/s

$p = m\upsilon = (9.11\times10^{-31})(-9.37\times10^{4})$

$= -8.537\times10^{-26}$ kg-m/s

$\lambda = \frac{h}{|p|} = \frac{6.625\times10^{-34}}{8.537\times10^{-26}} = 7.76\times10^{-9}$ m

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.76\times10^{-9}} = 8.097\times10^{8}\text{ m}^{-1}$

$\omega = k\cdot|\upsilon| = (8.097\times10^{8})(9.37\times10^{4})$

or $\omega = 7.586\times10^{13}$ rad/s

2.25

$$E_n = \frac{\hbar^2 n^2\pi^2}{2ma^2} = \frac{n^2(1.054\times10^{-34})^2\pi^2}{2(9.11\times10^{-31})(75\times10^{-10})^2}$$

$$E_n = n^2(1.0698\times10^{-21})\text{ J}$$

or

$$E_n = \frac{n^2(1.0698\times10^{-21})}{1.6\times10^{-19}}$$

or $E_n = n^2(6.686\times10^{-3})\,\text{eV}$

Then

$$E_1 = 6.69\times10^{-3}\,\text{eV}$$
$$E_2 = 2.67\times10^{-2}\,\text{eV}$$
$$E_3 = 6.02\times10^{-2}\,\text{eV}$$

2.26

(a) $$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2(1.054\times10^{-34})^2\pi^2}{2(9.11\times10^{-31})(10\times10^{-10})^2}$$

$$= n^2(6.018\times10^{-20})\,\text{J}$$

or $$E_n = \frac{n^2(6.018\times10^{-20})}{1.6\times10^{-19}} = n^2(0.3761)\,\text{eV}$$

Then

$$E_1 = 0.376\,\text{eV}$$
$$E_2 = 1.504\,\text{eV}$$
$$E_3 = 3.385\,\text{eV}$$

(b) $$\lambda = \frac{hc}{\Delta E}$$

$$\Delta E = (3.385 - 1.504)(1.6\times10^{-19})$$
$$= 3.01\times10^{-19}\,\text{J}$$

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{8})}{3.01\times10^{-19}}$$
$$= 6.604\times10^{-7}\,\text{m}$$

or $\lambda = 660.4\,\text{nm}$

2.29

Schrodinger's time-independent wave equation

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

We know that

$$\psi(x) = 0 \text{ for } x \geq \frac{a}{2} \text{ and } x \leq \frac{-a}{2}$$

We have

$$V(x) = 0 \text{ for } \frac{-a}{2} < x < \frac{+a}{2}$$

so in this region

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

The solution is of the form

$$\psi(x) = A\cos kx + B\sin kx$$

where

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

Boundary conditions:

$$\psi(x) = 0 \text{ at } x = \frac{-a}{2},\ x = \frac{+a}{2}$$

First mode solution:

$$\psi_1(x) = A_1\cos k_1 x$$

where

$$k_1 = \frac{\pi}{a} \Rightarrow E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$

Second mode solution:

$$\psi_2(x) = B_2\sin k_2 x$$

where

$$k_2 = \frac{2\pi}{a} \Rightarrow E_2 = \frac{4\pi^2\hbar^2}{2ma^2}$$

Third mode solution:

$$\psi_3(x) = A_3\cos k_3 x$$

where

$$k_3 = \frac{3\pi}{a} \Rightarrow E_3 = \frac{9\pi^2\hbar^2}{2ma^2}$$

Fourth mode solution:

$$\psi_4(x) = B_4\sin k_4 x$$

where

$$k_4 = \frac{4\pi}{a} \Rightarrow E_4 = \frac{16\pi^2\hbar^2}{2ma^2}$$

2.33

(a) For region II, $x > 0$

$$\frac{\partial^2\psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_O)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2\exp(jk_2x) + B_2\exp(-jk_2x)$$

where

$$k_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_O)}$$

Term with B_2 represents incident wave and term with A_2 represents reflected wave.

Region I, $x < 0$

$$\frac{\partial^2\psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

General form of the solution is

$$\psi_1(x) = A_1\exp(jk_1x) + B_1\exp(-jk_1x)$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave and the term involving A_1 represents reflected wave: but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\psi_1(x) = B_1 \exp(-jk_1 x)$$
$$\psi_2(x) = A_2 \exp(jk_2 x) + B_2 \exp(-jk_2 x)$$

(b)

Boundary conditions:

(1) $\psi_1(x=0) = \psi_2(x=0)$

(2) $\left.\frac{\partial \psi_1}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_2}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$
$$k_2 A_2 - k_2 B_2 = -k_1 B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{k_2 - k_1}{k_2 + k_1}\right) \cdot B_2$$

$$B_1 = \left(\frac{2k_2}{k_2 + k_1}\right) \cdot B_2$$

The reflection coefficient is

$$R = \frac{A_2 A_2^*}{B_2 B_2^*} = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)^2$$

The transmission coefficient is

$$T = 1 - R \Rightarrow T = \frac{4k_1 k_2}{(k_1 + k_2)^2}$$

Question 3

a) $En = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \quad n = 1,2,3 \; for\ the\ 3\ lowest\ energy\ levels$

b) $E_1 = 1/2\ m\ v_1^2$ (m is free electron rest mass)

c) The wave function is

$\psi(x) = \sqrt{\frac{2}{a}} \sin(\frac{n\pi x}{a})$ Where a=3Å

For the position x, the expectation values defined as:

$< x > = \int_{-\infty}^{\infty} \psi^*(x,t)\, x\, \psi(x,t)\, dx$ The average value of x

The meaning of this value is the average value of position for a large number of particles.

we know that $P_{operator} = \frac{\hbar}{i}\frac{\partial}{\partial x} \rightarrow$ *the expectation value of momnetum is*

$< p >= \frac{\int \psi * \hat{p}\, \psi dx}{\int \psi \psi * dx}$ Where $\hat{p} = \frac{\hbar}{j}\frac{\partial}{\partial x}$

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\sin\left(\frac{n\pi x}{a}\right) = \frac{\hbar n\pi}{ia}\cos\left(\frac{n\pi x}{a}\right)$$

$$< p > = \int_{-\infty}^{\infty} \psi^*(x)\ \frac{\hbar}{i}\ \frac{\partial}{\partial x}\psi(x)\ dx = \frac{2}{a}\left(-i\frac{n\pi\hbar}{a}\right)\int_0^a \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{n\pi x}{a}\right) dx = 0$$

$$< E >= \frac{< p^2 >}{2m}$$

$$< E > = \int_{-\infty}^{\infty} \psi^*(x)\ \frac{-\hbar^2}{2m}\ \frac{\partial^2}{\partial x^2}\psi(x)\ dx = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$