Solution for Midterm Exam.

#1:

a) (x_1, p_1): (7500, 2.28)

(x_2, p_2): (7900, 2.37)

$$m = \frac{2.37 - 2.28}{7900 - 7500} = 0.000225$$

$$\Rightarrow p - 2.28 = (.000225)(x - 7500)$$

$$\Rightarrow p = 0.000225x + 0.5925 \quad \text{(Price supply)}$$

b) (x_1, p_1): (7900, 2.28)

(x_2, p_2): (7800, 2.37)

$$m = \frac{2.37 - 2.28}{7800 - 7900} = -0.0009$$

$$p - 2.28 = (-0.0009)(x - 7900)$$

$$\Rightarrow p = -0.0009x + 9.39 \quad \text{(Price - demand)}$$

c) $.000225x + 0.5925 = -0.0009x + 9.39$

$$\Rightarrow x = \frac{8.7975}{0.001125} = 7820$$

$\therefore p = 2.352$

$(x, p) = (7820, 2.352)$

#2

A) $64^{x^2} = 256^{x}$

$\Rightarrow 4^{3x^2} = 4^{4x} \Leftrightarrow 3x^2 - 4x = 0$

$\Rightarrow x = 0, \frac{4}{3}$

B) $3^{\log_2 x} = 3^5 \Leftrightarrow \log_2 x = 5$

$\Rightarrow x = 2^5 = 32$

C) $e^{-x^2-1} = e^{3x+1} \Rightarrow -x^2 - 1 = 3x + 1$

$\Rightarrow -x^2 - 3x - 2 = 0$

$\Rightarrow x^2 + 3x + 2 = 0$

$\Rightarrow (x+2)(x+1) = 0$

$\Rightarrow x = -1, -2$

D) $\log_{10}(x+1) - \log_{10}(10x-3) = 1$

$\Rightarrow \frac{x+1}{10x-3} = 10$

$\Rightarrow 100x - 30 = x + 1$

$\Rightarrow 99x = 31$

$\Rightarrow x = \frac{31}{99}$

#3

A) $a_4 = -5, \quad a_{15} = 23$

$$a_n = a_4 + (n-1)d$$

$$\Rightarrow a_{15} = (-5) + (15-1)\cdot d$$

$$\Rightarrow 23 = -5 + 14d \iff d = 2$$

$$a_{73} = (-5) + 72(2) = 139$$

B) $a_1 = 3, \quad a_{10} = 30$

$$a_n = a_1 r^{n-1}$$

$$\Rightarrow a_{10} = 30 = 3 \cdot r^{10-1}$$

$$\Rightarrow 10 = r^9 \iff r = 10^{\frac{1}{9}}$$

$$\therefore a_{40} = 3 \cdot \left(10^{\frac{1}{9}}\right)^{40-1} = 3 \cdot (10)^{\frac{39}{9}} = 64633.041$$

#4

$$A = P(1+rt) = 5500\left(1 + .08\left(\frac{90}{365}\right)\right) = 5608.493$$

5610

Third Party pays $5560 and will receive 5608.493 $ after 60 days.

$$P = 5560, \quad t = \frac{60}{365}$$

$$r = \frac{A-P}{Pt} = \frac{5608.493 - 5560}{5560\left(\frac{60}{365}\right)} = \frac{48.493}{913.97}$$

$$= 0.0531 = 5.31\%$$

5.40%

#5:

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i} = 500 \frac{\left(1 + \frac{.08}{4}\right)^4 - 1}{\left(\frac{.08}{4}\right)}$$

$$= 2060.80\ \$$$

Total deposit in one year $= 4(500) = 2000\ \$$

Interest earned in first year $= FV - 2000 = 60.80\ \$$

At the end of second year:

$$FV = 500 \frac{(1+.02)^8 - 1}{.02} = 4291.48\ \$$$

Total deposit + interest in the second year

$$= 4291.48 - 2060.80 = 2230.68\ \$$$

∴ Interest earned in second year $= 2230.68 - 2000$

$$= 230.68\ \$$$

At the end of 3rd year:

$$FV = 500 \frac{(1.02)^{12} - 1}{.02} = 6706.04\ \$$$

Total deposit + Interest $= (6706.99 - 4291.48)\ \$$

$$= 2414.56\ \$$$

∴ Interest earned in 3rd year $= (2414.56 - 2000)$

$$= 414.56\ \$$$

#6:

A) $PV = 240{,}000\ \$$

$i = \frac{0.0575}{12}$, $t = 20$ years.

$$PMT = \frac{(240000)\cdot\left(\frac{.0575}{12}\right)}{1-\left(1+\frac{.0575}{12}\right)^{-240}} = 1685.00\ \$$$

Total amount paid in 240 payments:

$$(1685.00)(240) = 404400.00\ \$$$

Total Interest paid $= (404400 - 240000)\ \$$

$= 164{,}400\ \$$

B) Unpaid balance after 8 years:

$$= (1685)\cdot\frac{1-\left(1+\frac{.0575}{12}\right)^{-144}}{\left(\frac{.0575}{12}\right)}$$

$= 174{,}980.97\ \$$