Fluid Statics

- CVG2116 Fluid Mechanics

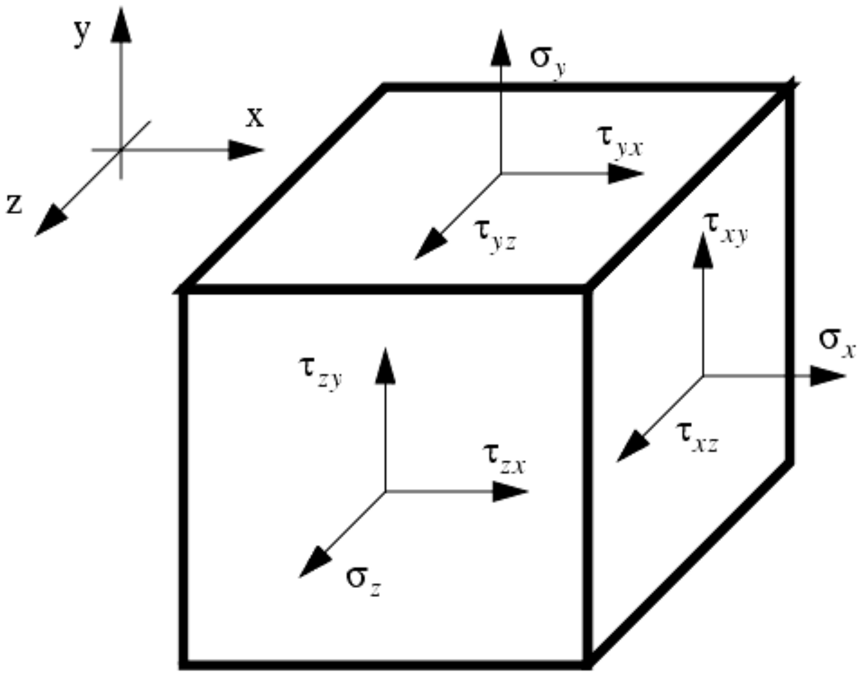
Fluid Statics

1. Basic equation of fluid statics
2. Pressure variation with depth
    a. Case 1: constant density fluids (most liquids)
    b. Case 2: Ideal gases
3. Pressure forces on surface
    a. Horizontal plane
    b. Tanks and pressure vessels
4. Pressure measurement
5. Buoyancy

1. Basic Equation of Fluid Statics

- Pressure is the normal force per unit area for a differential volume of fluid

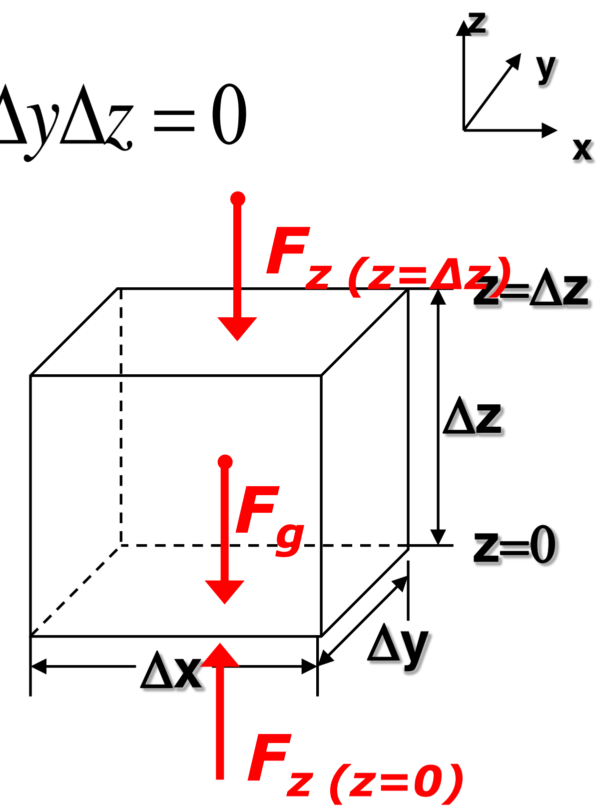
- Fluid at rest: Sum of forces on any part of fluid in any direction is zero.



1. Basic Equation of Fluid Statics

- Sum of forces in each direction:

$$(P_{z=0})\Delta x\Delta y-(P_{z=\Delta z})\Delta x\Delta y-\rho g\Delta x\Delta y\Delta z=0$$



divide by $-\Delta x\Delta y\Delta z$, rearrange:

$$\frac{P_{z=\Delta z}-P_{z=0}}{\Delta z}=-\rho g$$

$$\lim_{\Delta z\to 0}\frac{\Delta P}{\Delta z}=\frac{dP}{dz}=-\rho g$$

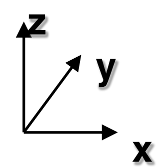
The barometric equation or hydrostatic equation.

2. Pressure variation with depth

- <u>Case 1: Constant density fluids (most liquids) :</u>

$$\frac{dP}{dz} = -\rho g \quad \Rightarrow \quad \int_{P_1}^{P_2} dP = -\int_{z_1}^{z_2} \rho g dz$$

$$P_2 - P_1 = -\rho g (z_2 - z_1)$$

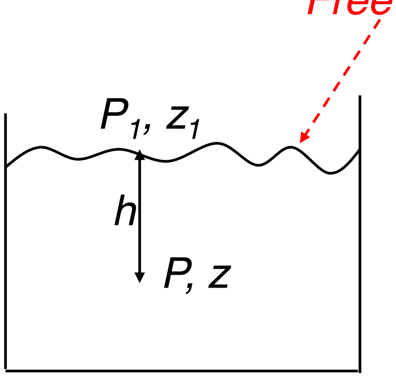


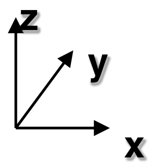
2. a) Pressure variation with depth

- In hydraulics problems and problems involving a free surface: (gauge pressure, constant density):

$$P_{1,gauge} = 0$$

$$h = z_{free\ surface} - z$$





$$P - P_1 = -\rho g (z - z_1) \quad \Rightarrow \quad P = \rho g h$$

2. a) Pressure variation with depth

Example:

The deepest point in the oceans of the world is believed to be in the Marianas Trench, southeast of Japan; there the depth is about 36,201 feet. Assume density of the ocean water is about 1028 kg/m^3.

What is the pressure at that point?

2. b) Pressure variation with depth

- Case 2: Ideal gases :
  - Density of gases changes with pressure.

$$\rho = \frac{PM}{RT}$$

  - So, back to the hydrostatic equation and substitute the ideal gas law:

$$\frac{dP}{dz} = -\rho g$$

$$\frac{dP}{dz} = -\frac{PM}{RT} g$$

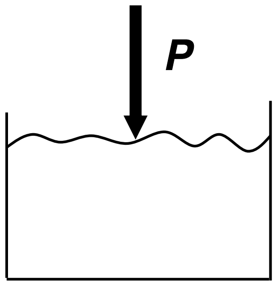
3. b) Pressure variation with depth

- After separation and integration we get for the *isothermal, perfect gas* case:

$$P_2 = P_1 \exp^{\left(-\frac{gM\Delta z}{RT}\right)}$$

3. Pressure forces on surfaces

$$dF = PdA \longrightarrow F = \int PdA$$



a) Horizontal plane surfaces exposed to static fluids:

- pressure is constant over the entire surface:

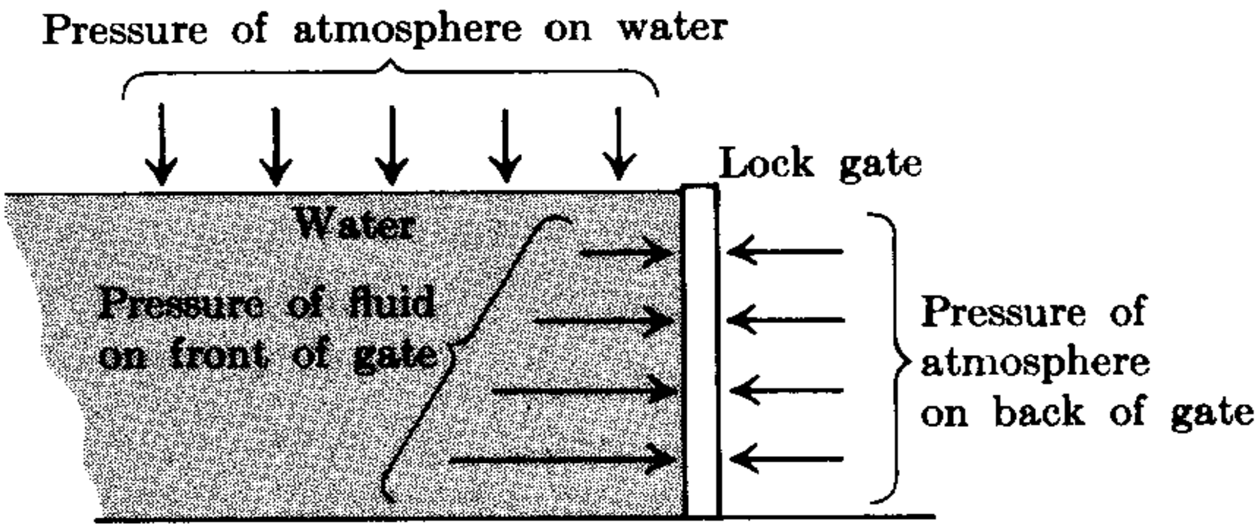
$$F = PA$$

3. Pressure forces on surfaces

b) Vertical plane surfaces exposed to static fluids:

- pressure is not constant, so use integral expression.

$$F = \int P dA$$



3. Pressure forces on surfaces

c) *Curved surfaces or inclined plane*:

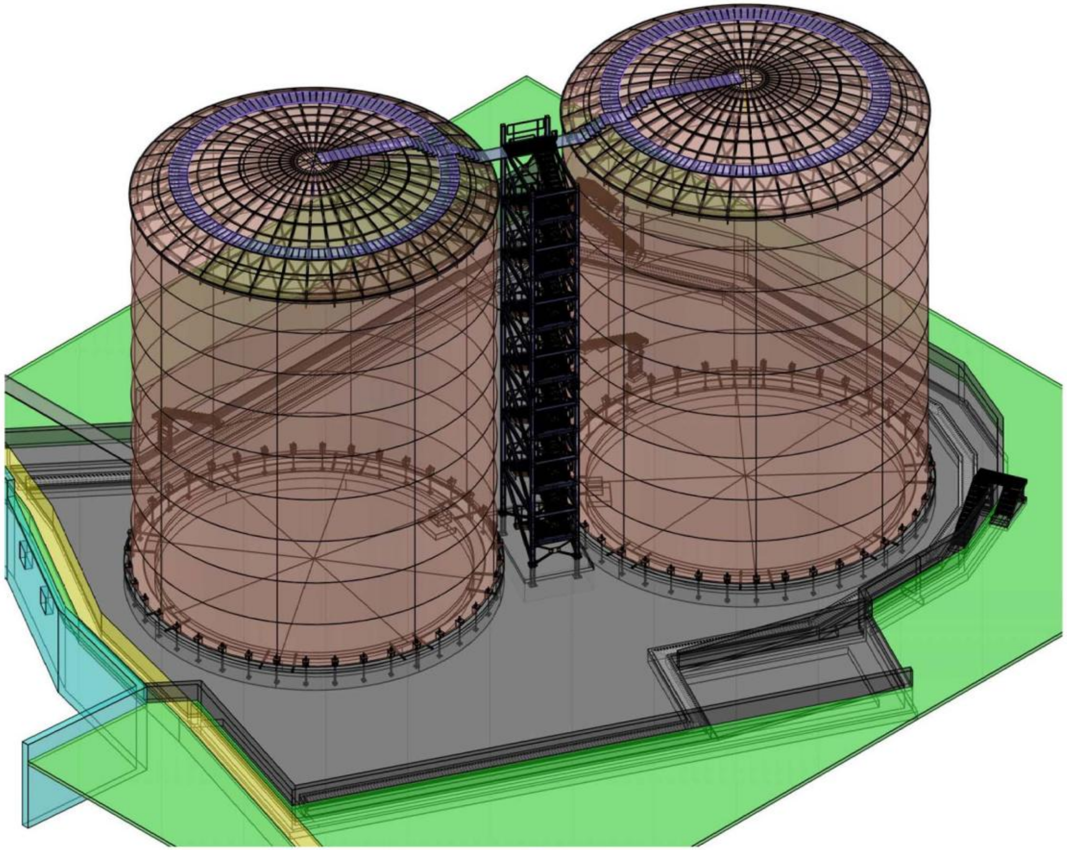
– It is usually of more practical interest to know the x- and y-components of the pressure force on the surface:

$$dF_x = \left(x\text{-component of } dF\right) = \sin\theta \cdot dF$$
$$= \sin\theta \cdot P\, dA$$

$$dF_y = \left(y\text{-component of } dF\right) = -\cos\theta \cdot dF$$
$$= -\cos\theta \cdot P\, dA$$

3. Pressure forces on surfaces

Design of water tanks



3. Pressure forces on surfaces

- It is usually of more practical interest to know the x- and y-components of the pressure force on the surface:
  - Cylindrical tanks

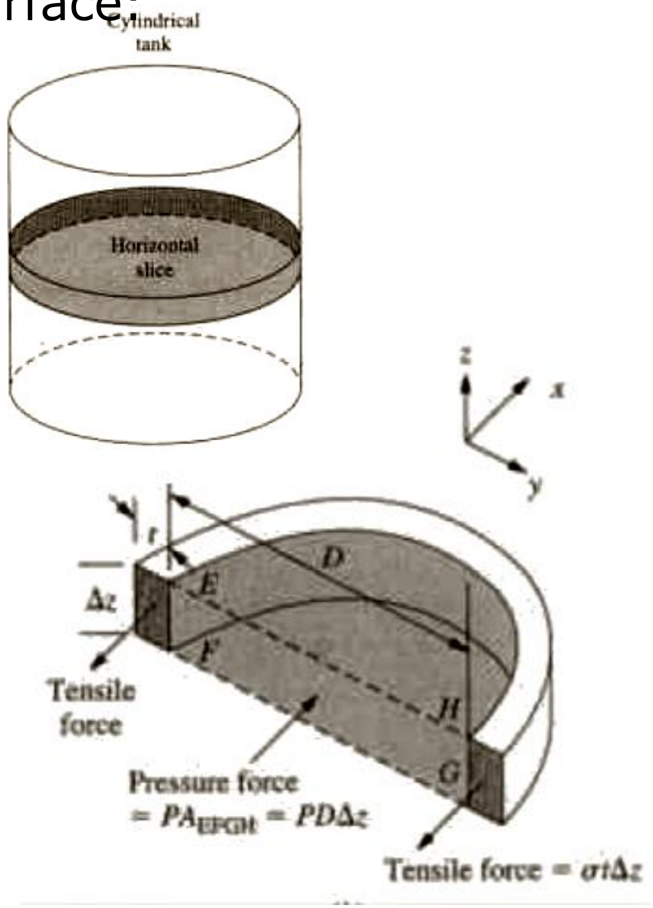
$$P \cdot D \cdot \Delta Z = 2 \cdot \sigma_{\text{tensile}} \cdot \Delta Z \cdot t$$

$$t = \frac{P\,D}{2\sigma_{\text{tensile}}}$$

  - Spherical tanks

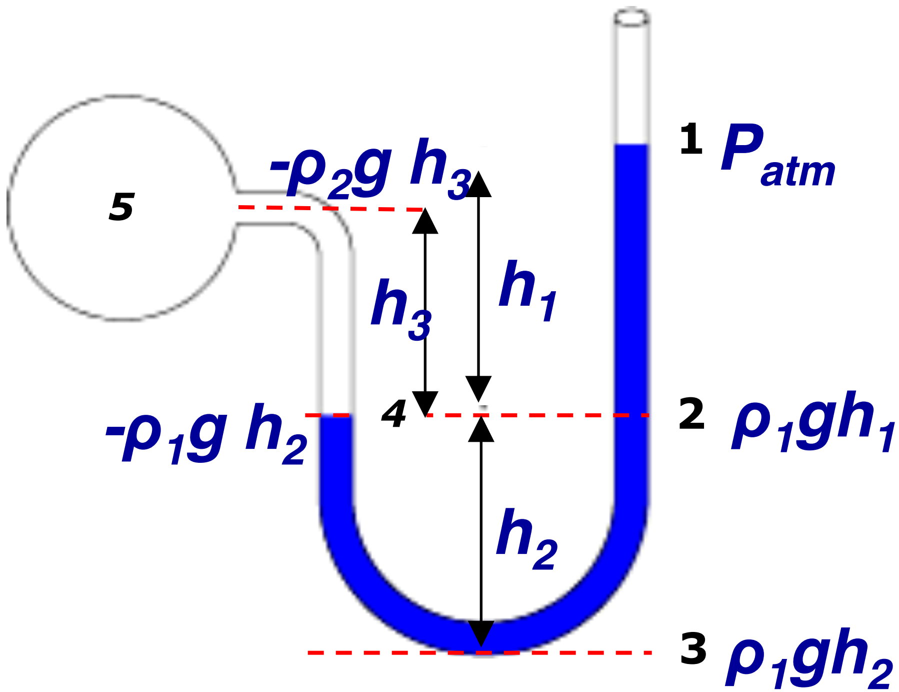
$$P \cdot \frac{\pi}{4} D^2 = \sigma_{\text{tensile}} \cdot \pi \cdot D \cdot t$$

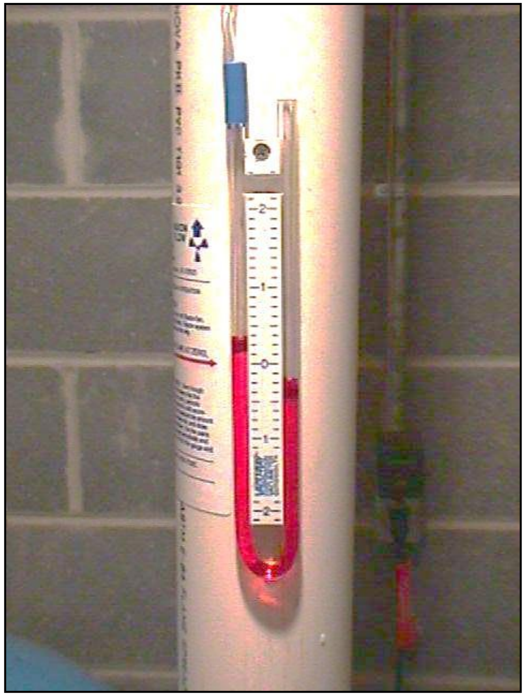
$$t = \frac{P\,D}{4\sigma_{\text{tensile}}}$$



4. Pressure Measurement

- A manometer can be constructed to read gauge pressure, differential pressure and absolute pressure.





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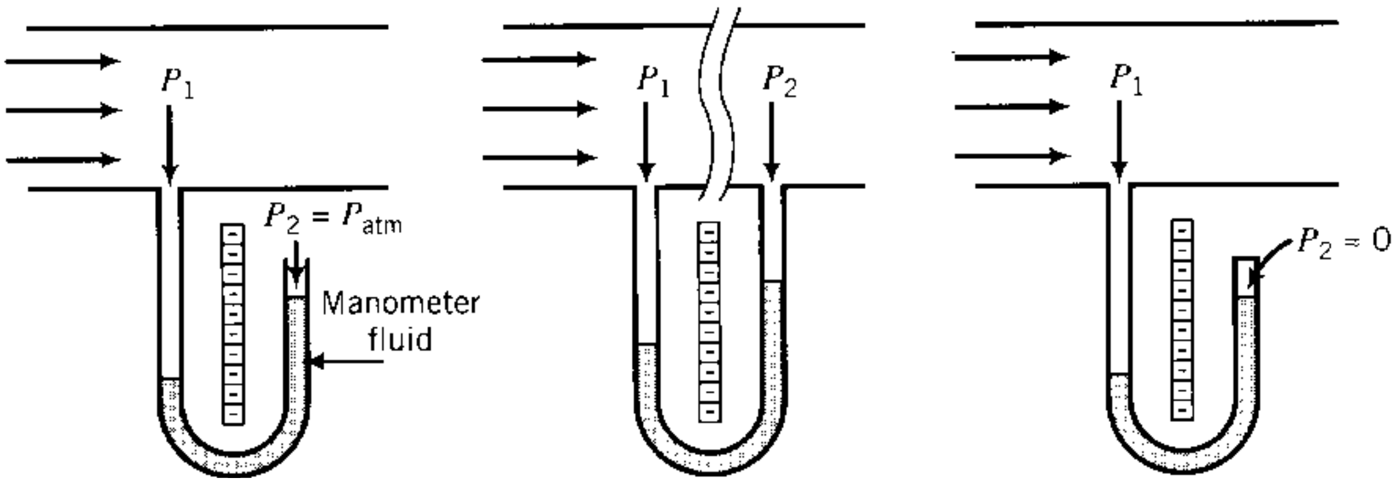


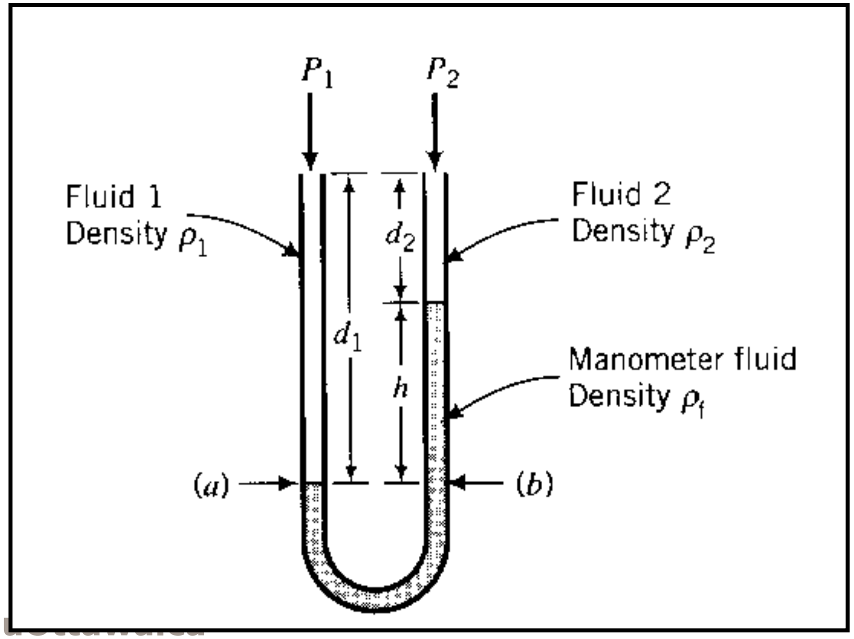
Gauge pressure

Differential pressure

Absolute pressure

4. Pressure measurement

- To measure a pressure difference, it is necessary to correct for the pressure exerted by the other fluid. It is enough to follow the variation of the hydrostatic pressure from one point to another.



$$P_1 + \rho_1 g d_1 - \rho_f g h - \rho_2 g d_2 - P_2 = 0$$

$$P_1 - P_2 = \rho_f g h + \rho_2 g d_2 - \rho_1 g d_1$$

- **Differential manometer with fluid 1 and 2 being the same.**

$$P_1 - P_2 = (\rho_f - \rho)\, gh$$

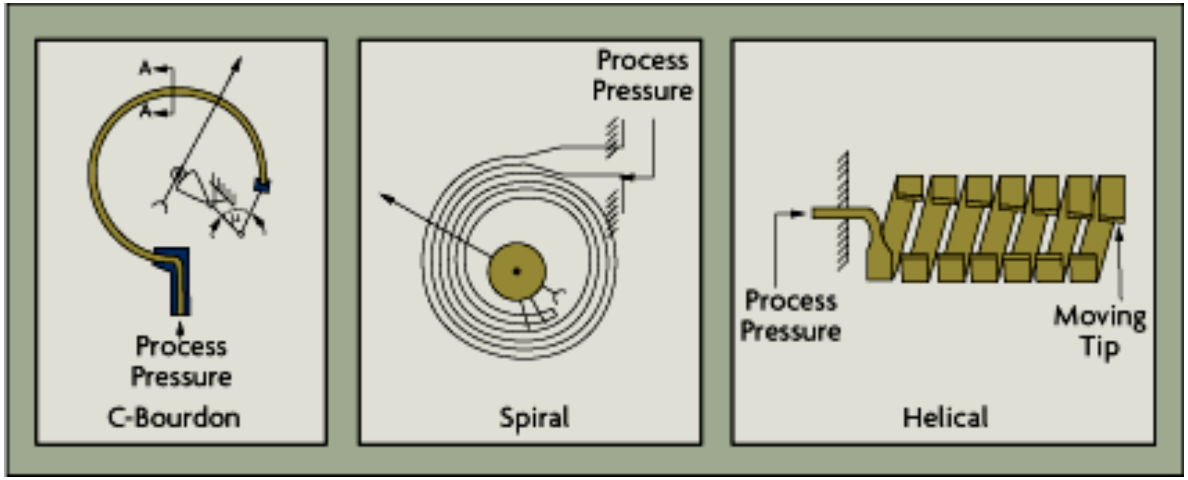
4. Pressure measurement

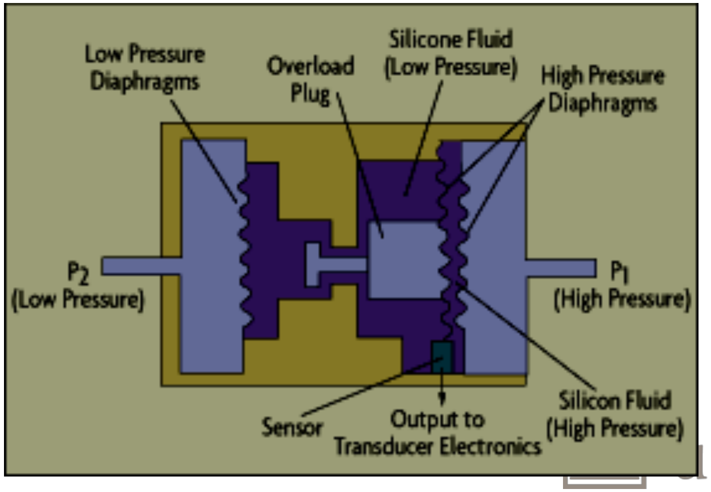
- To increase precision when reading low pressures, one can use a fluid of lower density and a tilted or curved pressure gauge.

4. Pressure measurement

- Contribution of the section of the manometer full of gas can generally be neglected.
- Calculations should be in gauge pressure.
- Usually refer to the pressures as "inches of water" or "mm of mercury".
- No calibration required.
- Use this calculation method to avoid making mistakes.
- Manometers are still very common in industry.
- One problem is their slow response time to rapidly changing pressures.
- Other types:

4. Pressure measurement





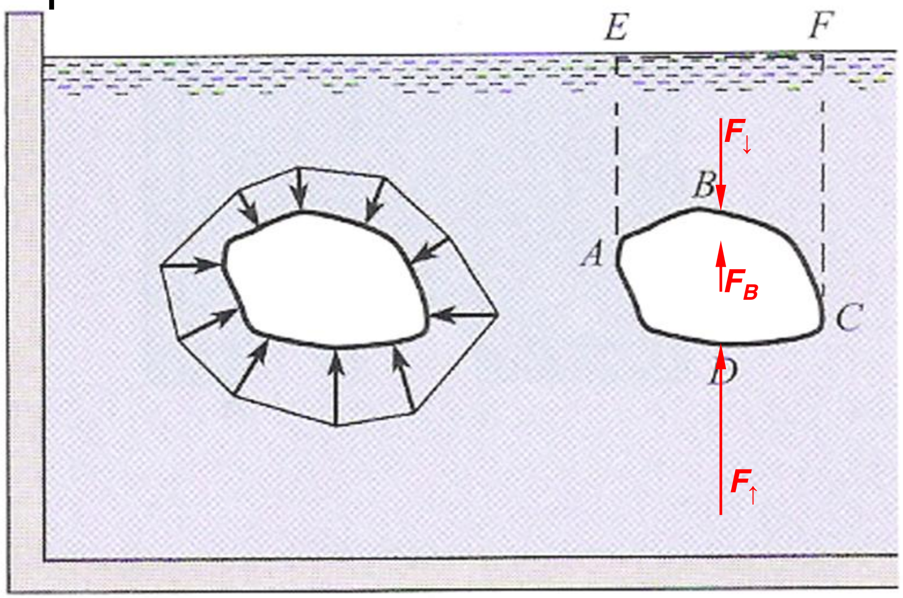
5. Buoyancy

Archimedes Principle: for an object partially or completely submerged in a fluid, there is a net upward force (buoyant force) equal to the weight of displaced fluid.

$$F_{\uparrow}=\rho g(V_b+V_a)$$

$$F_{\downarrow}=\rho g V_a$$

$$F_B=F_{\uparrow}-F_{\downarrow}=\rho g V_b$$

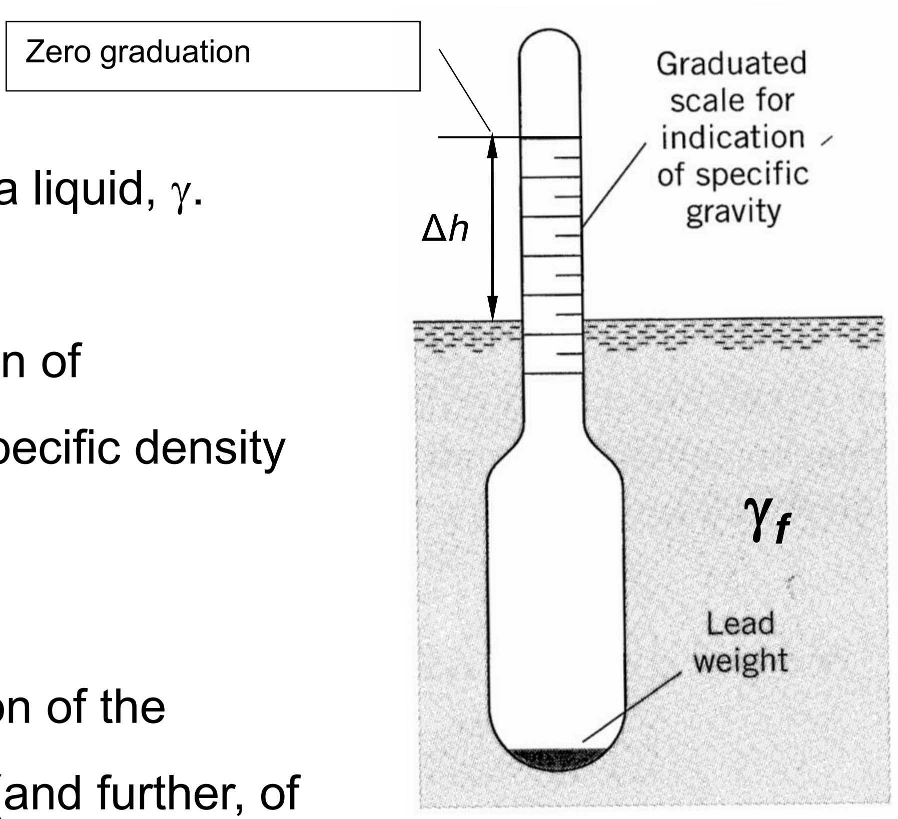


5. Buoyancy

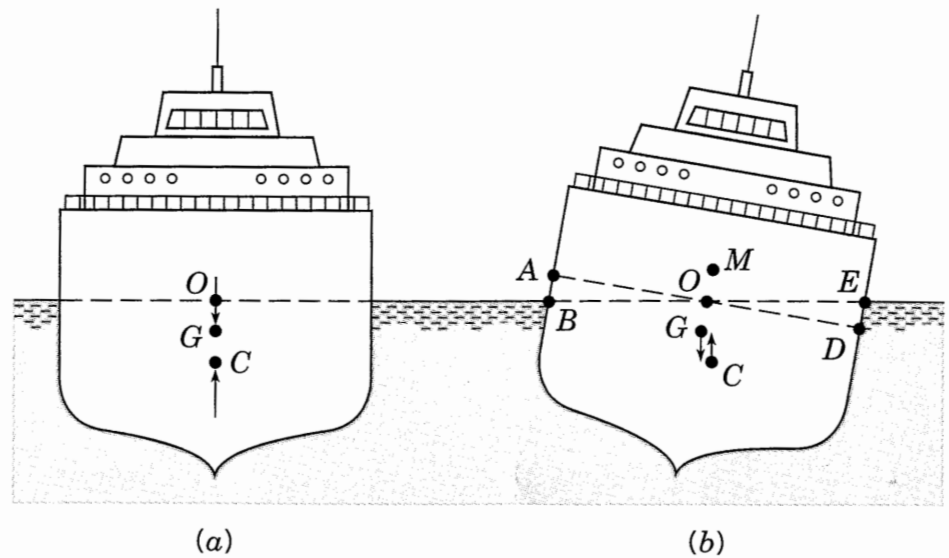
Hydrometer: measure the specific weight of a liquid, γ.

1. $\Delta h = V_o/A\,(S-1)/S$ for the calibration of liquids whose specific density is known

2. $S = (V_o - \Delta h A)/V_o$ for the calculation of the specific gravity (and further, of specific weight)



5. Buoyancy



The stability of an immersed body depends on the relative position of the center of gravity (G) and the center of buoyancy (C) (centroid of the volume displaced by the fluid).

(a) Center of buoyancy above the center of gravity $\Rightarrow$ stability

(b) Center of buoyancy coincide with the center of gravity $\Rightarrow$ neutral

(c) Center of buoyancy below the center of gravity $\Rightarrow$ instability