ECON 303 C : MIDTERM 1, Winter 2017

Instructions: The test is closed book/notes. Calculators are allowed. There are 100 points.

1. In the Cobb-Douglas production function $Y = K^{\frac{1}{3}}L^{\frac{1}{3}}H^{\frac{1}{3}}$ where K is capital, L is labor and H stands for human capital, if the skilled worker earns the marginal products of labor plus the marginal product of human capital and the unskilled worker earns the marginal product of labor, show that the wage paid to the skilled worker is always greater than the wage paid to the unskilled worker. (10 points).

$w_u = MP_L = \frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}}H^{\frac{1}{3}}$ and $w_S = \frac{1}{3}K^{\frac{1}{3}}L^{\frac{1}{3}}H^{-\frac{2}{3}} + \frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}}H^{\frac{2}{3}}$

$\Rightarrow \frac{w_S}{w_u} = 1 + \frac{L}{H} > 1 \Rightarrow w_S > w_L$

w_s: the real wage of skilled worker

w_u: the real wage of unskilled worker

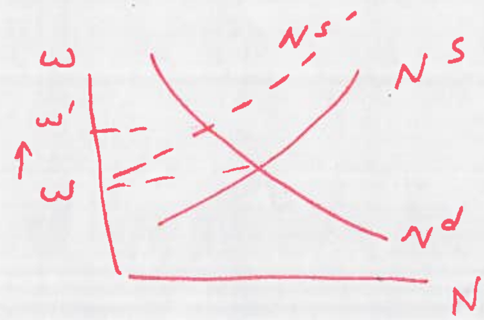
2. If total factor productivity rises in the one-period model, explain what happens to the real wage in equilibrium, and why (Discuss your answer in terms of substitution and income effects). (15 points)

Answer: The increase in TFP shifts the production possibilities frontier (PPF) out, and makes it steeper. There is an income effect, that increases consumption and leisure, and the substitution effect causes leisure to decrease and consumption to increase. So, consumption must rise, but leisure may rise or fall. The substitution effect is a movement along an indifference curve from the initial equilibrium point, up the indifference curve — the indifference curve becomes steeper. The income effect results in a movement down the new PPF — the PPF gets steeper. Thus, the indifference curve that is tangent to the PPF at the new equilibrium point must be steeper than the indifference curve tangent to the initial PPF at the initial equilibrium point. Therefore the real wage must increase.

or because TFP ↑ labor demand for sure will increase, but labor supply will either decrease or increase by less than the increase in labor demand Therefore w ↑

3. The Black Death: In the middle of the fourteenth century, an epidemic known as the Black Death killed about a third of Europe's population, about 34 million people. While this was an enormous tragedy, the macroeconomic consequences might surprise you: over the next century, wages are estimated to have been **higher** than before the Black Death. Use the production model assumed in class to explain why wages might have been higher. Use graph and if necessary math to justify your answer. (15 points).

Wages were higher after the black death because of diminishing returns. Our production models exhibits diminishing returns to labor. Because the amount of labor is reduced, the marginal product of labor – and hence the wage – increases. The reason is that capital stays the same. Each remaining worker is able to work with more machines, so his productivity rises



4. Suppose household's preferences are described by the utility function

$$U(C, l) = \sqrt{C} + b\sqrt{l}$$

where C stands for consumption of market goods, l stands for leisure, and b is a constant parameter. The household's time constraint is $l + N^s = 1$. Output is produced using the production function: $Y = \phi ln(N^d)$, where $\phi > 0$ is a constant parameter. In answering to the first three parts assume, for simplicity, that there is no government in this economy and $\pi = 0$. (60 points).

(a) Write down the Lagrangian for this problem and compute the representative hosehold's first-order conditions. (10 points).

$$L = \sqrt{c} + b\sqrt{l} + \lambda \cdot \left[\omega - \omega l - C\right]$$

$$\frac{\partial L}{\partial c} = \frac{1}{2\sqrt{c}} - \lambda = 0, \quad (1) \qquad \frac{\partial L}{\partial l} = \frac{b}{2\sqrt{l}} - \lambda\omega = 0, \quad (2) \qquad \frac{\partial L}{\partial \lambda} = \omega - \omega l - C = 0 \quad (3)$$

(b) Assuming the market (real) wage is w, derive the labor supply and labor demand curves. Discuss whether the income effect can ever dominate the substitution effect or not? (10 points).

To derive the labor supply we must solve the above F.O.C.s for l^*.

from (1) & (2) $\Rightarrow$ $c^* = l\frac{\omega^2}{b^2}$ (4) substitute into (3) $\Rightarrow$ $\omega - \omega l^* - l\frac{\omega^2}{b^2}$

Sorry $l\frac{\omega^2}{b^2} + \omega l = \omega$ $\Rightarrow l^* = \frac{\omega}{\omega + \frac{\omega^2}{b^2}}$ $\Rightarrow$ $\boxed{N^s = 1 - l^* = \frac{\omega b^2}{\omega b^2 + \omega^2} = \frac{\omega}{b^2 + \omega}}$

$\Rightarrow l\frac{\omega}{b^2} + l = 1 \Rightarrow l\left(\frac{\omega}{b^2} + 1\right) = 1 \Rightarrow l = \frac{1}{\frac{\omega}{b^2} + 1} \Rightarrow$

$l = \frac{b^2}{\omega + b^2} \Rightarrow 1 - l = \frac{\omega}{\omega + b^2}$

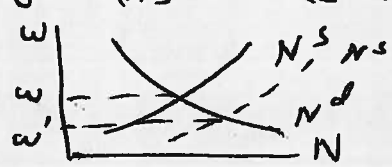
since $\frac{\partial N^s}{\partial \omega} > 0$ or $\frac{\partial l}{\partial \omega} < 0 \Rightarrow$ The substitution effect dominates the income effect. For labor demand from the firm's problem we know $MP_N = \omega \Rightarrow \frac{\phi}{N^d} = \omega \Rightarrow \boxed{N^d = \frac{\phi}{\omega}}$

(c) Compute the competitive equilibrium wage rate and employment for this economy. How do b and ϕ affect the equilibrium wage and output level? Illustrate and explain your answers with a labor supply/labor demand graph. (10 points).

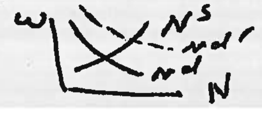
The eq. w^* is where $N^d = N^s \Rightarrow \frac{\omega}{b^2 + \omega} = \frac{\phi}{\omega} \Rightarrow$ or $\cancel{\frac{\omega^2}{b^2+\omega} = \phi}$ $\cancel{\omega = \frac{\phi b^2}{b^2 - \phi}}$

or $\frac{\omega^2}{b^2 + \omega} = \phi$

when $b\uparrow$ $\omega\downarrow$ or when $b\uparrow$ labor supply shifts to the right, but labor demand doesn't change $\Rightarrow \omega\downarrow$



when $\phi\uparrow$ $N^d\uparrow \Rightarrow \omega\uparrow$



when $\phi\uparrow$ $Y\uparrow$ from $Y = \phi\ln(N^d)$

~~As long as they unders~~

when $b \uparrow$ $N^s \uparrow$ therefore $Y \uparrow$

(d) Now suppose that there is a government in the economy and its purchases are equal to G, where it finances them with imposing lump-sum taxes equal to T. Does in the presence of the government labor supply depend on G? Why or why not? (Hint: Your answer should refer to income and substitution effects). If G increases, how does output and employment respond? Explain. (10 points).

The problem that we solve now is $\max \sqrt{c} + b\sqrt{\ell}$ s.t

$c + w\ell = w - G$ (5)

and using $N^s = 1 - \ell^*$

if we plugg now (4) into (5) $\Rightarrow$ $N^s = \frac{w^2 + b^2 G}{wb^2 + w^2}$

It's clear that now when $G \uparrow$ $N^s \uparrow$, in fact when the gov. spending increases it's like a pure negative income effect ~~because~~ and it decreases C and so persuades people to work more

(e). Define the competitive equilibrium of this economy. Be careful to specify the problems that must be solved individually and the variables that each problem take as given. (4 points)

The Comp. equilibrium is a set of endogenous variables C, N^d, N^s, w, Y, T that given G, z and K the followings are satisfied.

1- Given T, w and π, the rep. consumer maximizes her utility subject to her BC by changing C and N^s

2- The rep. firm chances N^d to maximize its profit, given Y and w.

3. The market for labor clears, that is, $N^d = N^s$,

4. The gov. budget constraint is satisfied. $G = T$

(f) Write down the planner's problem. (5 points)

The planner's problem is to maximize $\sqrt{c} + b\sqrt{\ell}$

subject to the resource constraint that is $C + G = \phi \ln N$

~~for~~ $\max \sqrt{c} + b\sqrt{\ell}$

$$C + G = \phi \ln(1-\ell)$$

(g) What are the first-order conditions of the planner's problem? (4 points)

$$L = \sqrt{c} + b\sqrt{\ell} + \lambda \cdot \left[\phi \ln(1-\ell) - C - G\right]$$

① $\frac{\partial L}{\partial c} = \frac{1}{2\sqrt{c}} - \lambda = 0$, ② $\frac{\partial L}{\partial \ell} = \frac{b}{2\sqrt{\ell}} - \frac{\lambda \cancel{\phi}}{1-\ell} = 0$

$$\frac{\partial L}{\partial \lambda} = \phi \ln(1-\ell) - C - G = 0$$

(h) Find the solution to the planner's problem. What does this tell you about the economy? (7 points)

from ② $\Rightarrow \frac{b}{2\sqrt{\ell}} = \frac{\lambda}{(1-\ell)} \Rightarrow \lambda = \frac{b(1-\ell)}{2\sqrt{\ell}}$, substitute into

① $\Rightarrow \frac{1}{2\sqrt{c}} = \frac{b(1-\ell)}{2\sqrt{\ell}}$ This is exactly what we get from the firm and household's maximization problems. This means the competitive market is efficient and