Concordia University
ENCS
Engineering & Computer Science

# ENGR 213

## Applied Ordinary Differential Equations

## Second Mid Term Exam

## March 12th 2017

**Student Name:** ____________________________________________

**Student ID Number:** ______________________________

**ENGR 213 Section:** ____________________

**Course Given BY:** ______________________________________.

| | |
|---|---|
| Q1 | |
| Q2 | |
| Q3 | |
| Q4 | |
| Q5 | |
| Total | |

- **Exam is closed book close notes.**
- **No use of any electronic devices.**
- **Use only the approved calculator.**
- **Write your answers in the provided space.**

**Problem#1**

Write the number $(3+6i)$ + (4-i) (3+5i) + $\frac{1}{2-i}$ in the form $a+bi$.

**Solution:**

$$(17 + 17i)$$

$$(3 + 6i) + (12 + 5 - 3i + 20i) + \frac{1}{2-i} \quad \frac{2+i}{2+i}$$

$$(20 + 23i) + \frac{2+i}{4+1} = (20 + 23i) + \frac{2}{5} + \frac{i}{5}$$

$$\frac{102}{5} + \frac{116}{5} i$$

**Problem#2**

Verify that $y_1 = e^{5x}$ and $y_2 = e^{-7x}$ are form a fundamental set of solutions of the differential equation $y''+2y'-35y=0$ on the interval $(-\infty,\infty)$. Write the form of the general solution.

**Solution:**

$y_1 = e^{5x}$ $\qquad$ $y_2 = e^{-7x}$

$y_1' = 5e^{5x}$ $\qquad$ $y_2' = -7e^{-7x}$

$y_1'' = 25e^{5x}$ $\qquad$ $y_2'' = 49\,e^{-7x}$

For $y_1$ $\qquad$ $y'' + 2y' - 35y$

$25e^{5x} + 10e^{5x} - 35e^{5x} = 0$ $\qquad$ o.k

For $y_2$ = $49e^{x} - 14e^{-7y} - 35e^{-7x} = 0$ $\qquad$ o.k

$y = c_1 e^{5x} + c_2 e^{-7x}$

**Problem#3**

**Solution:**

(a) Write the characteristic equation and the general solution of the differential equation:

$y^{(6)} - 13y^{(5)} + 70y^{(4)} - 198y^{(3)} + 308y'' - 248y' + 80y = 0.$

$y^{(k)}$ is the k-th derivative of y. For Your convenience, the roots of the characteristic equation are: 1, 2, 2, 2, 3 + i, 3 − i. You do not have to check this.

(b) Find the general solution of the differential equation: $x^2 y'' - 7xy' + 16y = 0$, given one solution $y_1 = x^4$.

**Solution:**

(a) characteristic Eq:

$$m^6 - 13m^5 + 70m^4 - 198m^3 + 308m^2 - 248m + 80$$

general solution:

$$y = C_1 e^x + C_2 e^{2x} + C_3 x e^{2x} + C_4 x^2 e^{2x} + C_5 e^{3x} \cos x$$

$$+ C_6 e^{3x} \sin x$$

(b) $x^2 y'' - 7xy' + 16y = 0$ given $y_1 = x^4$

$y_2 = u x^4$ $\quad y_2' = u' x^4 + 4u x^3$

$y_2'' = u'' x^4 + 8u' x^3 + 12u x^2$

$$u'' x^6 + 8u' x^5 + 12u x^4 - 7(u' x^5 + 4u x^4)$$

$$+ 16u x^4 = 0$$

$$u'' x^6 + u' x^5 = 0$$

Subtitute: $w = u'$ $\quad w' x^6 + w x^5 = 0$

$$w' + \frac{1}{x} w = 0$$

$$\frac{dw}{dx} = -\frac{1}{x} w$$

$$\int \frac{dw}{w} = \int -\frac{1}{x} dx$$

$$\ln w = -\ln x \Rightarrow \ln w = \ln \frac{1}{x}$$

$$w = \frac{1}{x} \Rightarrow u = \int w = \int \frac{1}{x} dx = \ln x$$

$$y_2 = x^4 \ln x$$

general solution $y = c_1 x^4 + c_2 x^4 \ln x$

$$\boxed{y = x^2 e^x}$$

**Problem#4**

Using the method of undetermined coefficients, solve the initial value problem

*y''+y'-2y= (6x+2) e^x; y(0)=0; y'(0)=0.*

**Solution:**

$m^2 + m - 2 = 0 \Rightarrow (m+2)(m-1) = 0$

$m = -2$ or $m = 1$

$y_c = c_1 e^x + c_2 e^{-2x}$

$y_p = (Ax^2 + Bx) e^x$

$y_p' = (2Ax + B) e^x + (Ax^2 + Bx) e^x$

$y_p'' = (2A) e^x + (2Ax + B) e^x + (2Ax + B) e^x + (Ax^2 + Bx) e^x$

$= 2A e^x + (4Ax + 2B) e^x + (Ax^2 + Bx) e^x$

$x^2$: $-2A + A + A = 0$

$x$: $-2B + 2A + B + 4A + B = 0 \Rightarrow 6A = 6 \Rightarrow A = 1$

$c$: $B + 2A + 2B = 2 \Rightarrow B = \frac{2 - 2A}{3} = 0$

$y_p = x^2 e^x$

Solution $\Rightarrow y = c_1 e^x + c_2 e^{-2x} + x^2 e^x$

$y(0) = 0 \Rightarrow 0 = c_1 + c_2 \Rightarrow c_1 = -c_2$

$y'(0) = 0 \quad 0 = c_1 e^x - 2c_2 e^{-2x} + (2x e^x + x^2 e$

$0 = c_1 - 2c_2 \Rightarrow c_1 = 2c_2$

**Problem#5**

Solve the following differential Equation $y'' + (3/x)\,y' + (5/x^2)\,y=0$

**Solution:**

$$y'' + \frac{3}{x} y' + \frac{5}{x^2} y = 0$$

$$x^2 y'' + 3xy' + 5y = 0 \qquad \text{Cauchy Eular Eq}$$

$$y = x^m$$

$$m(m-1) + 3(m) + 5 = 0$$

$$m^2 - m + 3m + 5 = 0$$

$$m^2 + 2m + 5 = 0$$

$$m = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2}$$

$$= -1 \pm 2i$$

$$y = \cancel{c_1 x^{-1} \cos 2\ln x + c}$$

$$= c_1 x^{-1} \cos(2\ln x) + c_2 x^{-1} \sin(2\ln x)$$