Solutions
Assignment #5
Physics 205I

Q1 A silver wire 1 mm in diameter carries a charge of 90 C in one hour and 15 min. Silver contains 5.8×10^{28} free electrons per m^3. (a) What is the current in the wire? (b) What is the drift velocity of electrons in the wire? (0.02 A, 2.76×10^{-6} m/s)

Solution: Radius of the wire, $r = 0.5\,mm = 0.5\times10^{-3}\,m$

Charge Carried, $Q = 90C$

Time spent in Carrying Charge, $t = 1\,hr.\,15\,m = 4.5\times10^{3}\,S$.

Number of electrons per meter Cube, $n = 5.8\times10^{28}/m^3$

(a) Current, $I = \dfrac{\text{Total Charge}}{\text{Time}} = \dfrac{90C}{4500\,S} = 2\times10^{-2}\,A$

(b) Area of Cross-section of the wire, $A = \pi r^2 = \pi(0.5\times10^{-3})^2\,m^2$

Current, $I = A n e v$

$$\therefore\ v = \frac{I}{Ane} = \frac{(2\times10^{-2})}{\pi(0.5\times10^{-3})^2(5.8\times10^{28})(1.6\times10^{-19})}$$

$$= 2.74\times10^{-6}\,m/s$$

Q2 A vacuum diode can be approximated to a plane cathode, and a plane anode, parallel to each other, and 5mm apart. The area of both cathode and anode is 2 cm^2. In the region between cathode and anode, the current is solely carried by electrons. If the electron current is 50 mA, and the electrons strike the anode surface with a velocity of 1.2×10^7 m/s, find the number of electrons per cubic millimeter in the space just outside the surface of anode. ($1.3\times10^5/(mm)^3$)

Solution: Area of anode, $A = 2\,cm^2 = 2\times10^{-4}\,m^2$

Electron Current, $I = 50\,mA = 50\times10^{-3}\,A$

velocity of electrons at the anode, $v = 1.2\times10^{7}\,m/s$

$$I = nevA$$

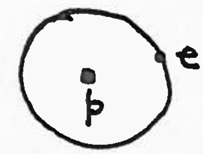
$$50\times10^{-3} = n(1.6\times10^{-19})(1.2\times10^{7})(2\times10^{-4})$$

$$\therefore\ n = \frac{50\times10^{-3}}{(1.6\times10^{-19})(1.2\times10^{7})(2\times10^{-4})} = 13.02\times10^{13}/m^3$$

$$= 1.3\times10^{5}/(mm^3)$$

Q3 In the Bohr model of the hydrogen atom the electron makes about 6x10^{15} rev s^{-1} around the nucleus. What is the average current at a point on the orbit of the electron? (9.6x10^{-4} A)

Solution: No of times per second, the electron goes around the nucleus, $n = 6\times10^{15}\,\frac{rev}{sec}$

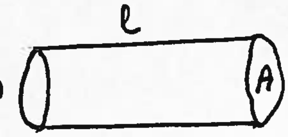


Therefore, the electron will pass any point on the orbit, 6×10^{15} times per second.

$\therefore$ Average Current, $I = ne = (6\times10^{15})(1.6\times10^{-19}) = 9.6\times10^{-4}\,A$

Q4 A wire 100 m long and 2mm in diameter has a resistivity of 4.8x10^{-8} Ωm. (a) What is its resistance? (b) A second wire of the same material has the same weight as the 100 m length, but twice its diameter. What is its resistance?(1.53 Ω, 0.096 Ω)

Solution: Length of the wire, $l = 100m$



Radius of the wire, $r = \frac{2}{2}mm = 10^{-3}\,m$

Resistivity, $\rho = 4.8\times10^{-8}\,\Omega m$

(a) Resistance, $R = \frac{\rho l}{A} = \frac{(4.8\times10^{-8})(100)}{\pi(10^{-3})^2} = 1.53\,\Omega$ — (1)

(b) weight of the wire, $w = lAdg$ → density, → grav. acc.

Since the weights of two wires are equal

$$w = w'$$

or $lAdg = l'A'd'g$ (2) $\quad d = d'$ (same material)

Since the diameter of l' is twice that of l

$$2r = r'$$

$$\therefore 4A = A'$$

From (2) $lAdg = l'(4A)dg$

or $l' = \frac{l}{4} = \frac{100}{4} = 25m.$

$\therefore$ Resistance, R' of 25m long resistance, $R' = \frac{4.8\times10^{-8}\times25}{\pi(2\times10^{-3})^2}$

$= 0.095\,\Omega$

Q5 The resistance of a coil of copper wire is 200Ω at 20° C. What is its resistance at 50°C? (223.6 Ω)

Solution:

Resistance of wire at 20°, $R_{20} = 200\,\Omega$

Temperature Coefficient of Cu, $\alpha_{cu} = 0.00393$ (from table)

$\therefore$ Resistance at 50°C, $R_{50} = R_{20}\left[1 + \alpha_{cu}(50-20)\right]$

$= 200\left[1 + 0.00393 \times 30\right]$

$= 223.58\,\Omega$

Q6 A certain resistor has a resistance of 150.4 Ω at 20°C and a resistance of 162.4 at 40°C. What is the temperature coefficient of resistivity? (3.99×10^{-3}/c)

Solution: Resistance at any temperature T is given by

$$R_T = R_0\left[1 + \alpha(T - T_0)\right] \quad (1)$$

Resistance at 20°C, $R_{20} = 150.4\,\Omega$

Resistance at 40°C, $R_{40} = 162.4\,\Omega$

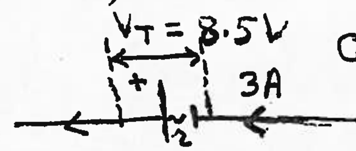
From (1), $162.4 = 150.4\left[1 + (40-20)\alpha\right] = 150.4\left[1 + 20\alpha\right]$

$\therefore 162.4 - 150.4 = 150.4 \times 20 \times \alpha$

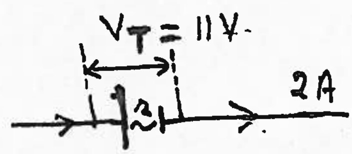
$\therefore \alpha = 12/(150.4 \times 20) = 3.99 \times 10^{-3}\ C^{-1}$

Q.7 The potential difference across the terminals of a battery is 8.5 V when there is a current of 3A in the battery from the negative to the positive terminal. When the current is 2A in the reverse direction, the potential difference becomes 11V. (a) What is the internal resistance of the battery? (b) What is the emf of the battery? (emf=10V,r=0.5Ω)

Solution: If $\mathcal{E}$ is the emf of the Cell, and r is the internal resistance of the battery



Case I



Case II

In Case I, $V_T = 8.5 = \mathcal{E} - 3r \quad (1)$

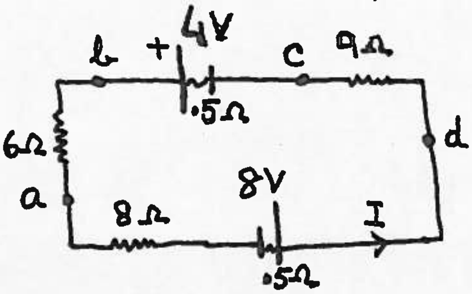
In Case II, $V_T = 11.0 = \mathcal{E} + 2r \quad (2)$

From (1) and (2) $\mathcal{E} = 10\,V$, $r = 0.5\,\Omega$

Q8 (a) What is the potential difference V_{ad} in the circuit shown in the figure?

(b) What is the terminal voltage of the 4V-battery?

Solution: If I is the Current in the circuit shown,



$I(0.5+9+0.5+6+8) = 8+4$

$\therefore \; 24I = 12 \quad \therefore I = 0.5A$

(a) Potential difference between a and d, V_{ad} is

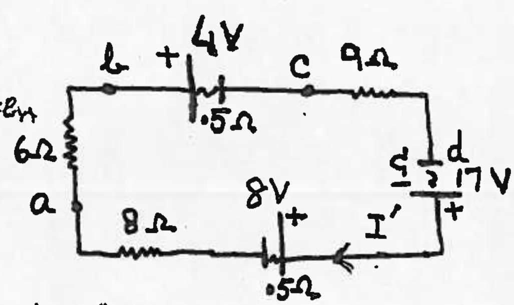
$$V_{ad} = 8V - 0.5I - 8I$$
$$= 8 - 0.5\times0.5 - 8\times0.5 = 3.75V$$

(b) Terminal voltage of 4V battery, V_T is

$$V_T = \mathcal{E} - Ir = 4 - 0.5\times0.5 = 3.75V.$$

© A battery of emf 17V and internal resistance 1Ω is inserted in the circuit at d, its positive terminal being connected to the positive terminal of the 8V-battery. What is now the difference of potential between the terminals of the 4V-battery? (3.75V, 3.75V, 4.10V)

When 17V- battery is connected at d, the current I' is now, given by, $I'(0.5+9+0.5+6+8+1) = 8-4+17$

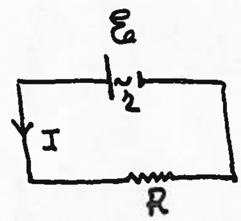


$25I' = 5$ or $I' = 0.2A$

Potential Difference between the terminals of 4V- battery, $V_T = 4 + 0.5\times0.2 = 4.1V$.

Q9 The internal resistance of a dry cell increases gradually with age, even though the cell is not used. The emf, however, remains fairly constant at about 1.5V dry cell may be tested for age at the time of purchase by connecting an ammeter directly across the terminals of the cell and reading the current. The resistance of the ammeter is so small that the cell is practically short-circuited. (a) The short-circuit current of a fresh No.6 dry cell is about 30A. Approximately, what is the internal resistance? (b) What is the internal resistance if the short-circuit current is only 10A? (c) The short-circuit current of a 6-v0lt storage battery may be as great as 1000A. What is the internal resistance? (0.05, 0.15, 0. 006)

Solution:



(a) The terminal voltage of a cell of emf, $\mathcal{E}$ is given by,

$$V_T = \mathcal{E} - Ir = IR$$

When the cell is short-circuited, $R = 0$, and $\mathcal{E} = Ir$

where $r \rightarrow$ internal resistance of the cell.

$$\therefore \quad 1.5 = 30 \times r \quad \text{or } r = \frac{1.5}{30} = 0.05\,\Omega.$$

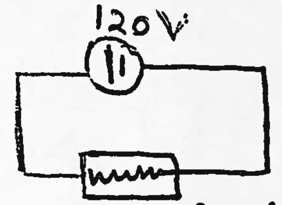
(b) When the short-circuit current is 10 A,

$$1.5 = 10 \times r \quad \text{or} \quad r = \frac{1.5}{10} = 0.15\,\Omega$$

(c) For a 6V-cell, short-circuit current being 1000A,

$$6 = 1000 \times r \quad \text{or} \quad r = \frac{6}{1000} = 0.006\,\Omega$$

Q.10 A 660W electric heater is designed to operate from 120V-line. (a) What is its resistance? (b) What current does it draw? (c) What is the rate of dissipation of energy, in calories per second? (d) If the line voltage drops to 110V, what power does the heater take, in watts? (21.8Ω, 5.5A, 157.1 cal/s, 555W)



Solution:

(a) $\frac{V^2}{R} = P \quad , \quad \therefore \frac{(120)^2}{R} = 660$

$$\therefore R = \frac{(120)^2}{660} = 21.8\,\Omega$$

$\therefore$ Resistance of the heater $= 21.8\,\Omega$

(b) Also, $P = I^2 R \quad \therefore I^2(21.8) = 660$

$$\therefore I = \sqrt{\frac{660}{21.8}} = 5.5\,A$$

(c) Rate of dissipation, $P = 660\ J/s = \frac{660}{4.2}\ \frac{Cal}{s} = 157.1\ \frac{Cal}{s}$

(d)

$$P = \frac{V^2}{R}$$

$V = 110\,V \qquad \therefore P = \frac{(110)^2}{21.8} = 555\,W.$

Q11 (a) Calculate the equivalent resistance of the circuit of the figure between x and y. (b) What is the potential difference between x and a if the current in the 8Ω resistor is 0.5A? (8Ω, 12V)

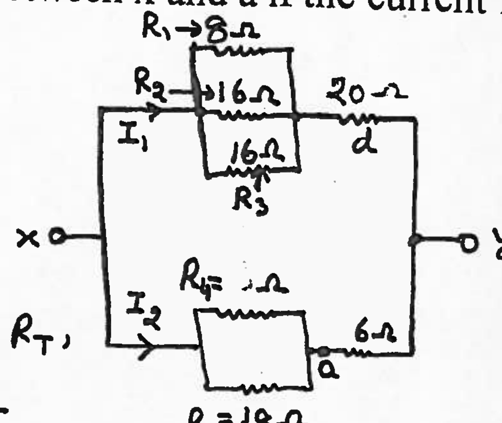


Fig. 1

Solution:

(a) resistances $R_1 = 8\Omega$, $R_2 = 16\Omega$ and $R_3 = 16\Omega$ are in parallel and the equivalent resistance is given by R_T,

$$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{2+1+1}{16} = \frac{4}{16}$$

$$\therefore R_T = 4\Omega$$

Similarly, $R_5 = 18\Omega$ and $R_4 = 9\Omega$ are in parallel and the equivalent resistance R_T' is given by

$$\frac{1}{R_T'} = \frac{1}{9} + \frac{1}{18} = \frac{2+1}{18} = \frac{3}{18} = \frac{1}{6}$$

$$\therefore R_T' = 6\Omega$$

The equivalent circuit is now given in Fig. 2

$R_T = 4$ and 20Ω resistances are in series in the branch ef of the circuit. The equivalent resistance, R_T'' is

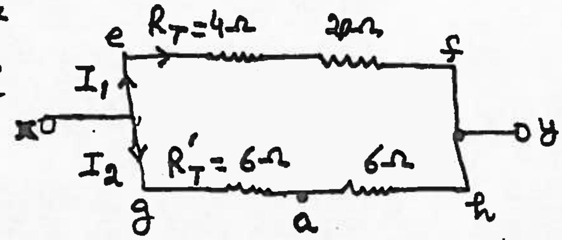


Fig. 2

$$R_T'' = 4 + 20 = 24\Omega$$

Similarly, $R_T' = 6\Omega$ and 6Ω are in series in the branch gh of the circuit and the equivalent resistance R_T''' is

$$R_T''' = 6 + 6 = 12\Omega.$$

The equivalent circuit is given in Fig. 3

Fig 3

$\therefore$ Equivalent resistance between x, y is,

$$\frac{1}{R} = \frac{1}{24} + \frac{1}{12} = \frac{1+2}{24} = \frac{3}{24} \quad \therefore R = 8\Omega$$

(b) Current through 8Ω resistor in Fig 1 is 0.5 A

$\therefore$ Current through 16Ω resistors is $\frac{0.5 \times 8}{16} = 0.25$ A

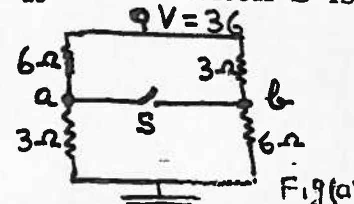
$$\therefore I_1 = 0.5 + 0.25 + 0.25 = 1A.$$

From Fig. 3, Current $I_2 = \frac{24 \times 1}{12} = 2\,A.$

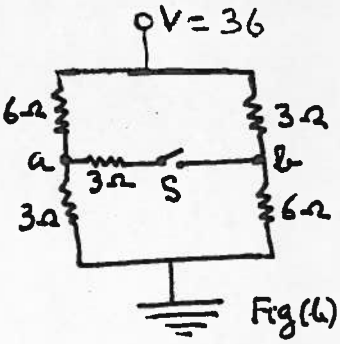
From Fig. 2, the potential difference between x and a can be easily calculated

$V_{xa} = 6 \times I_2$

Q12 In the figures (a), what is the potential difference V_{ab} when switch S is open. (b) What is the current through switch S, when it is closed? (c) In figure (b) what is the potential difference V_{ab} when the switch S is open? (d) What is the current through switch S when it is closed? What is the equivalent resistance in Fig. (b), (e) when switch S is open? (f) When switch S is closed?

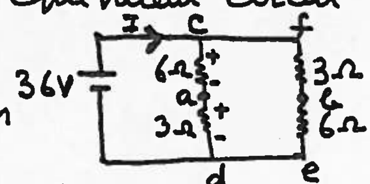


Fig(a)



Fig(b)

Solution: (a) when Switch S in Fig(a) is open, a simplified equivalent circuit is given in Fig(a_1) →



Current, I drawn from 36V-Source is

$I = \frac{36}{R_T}$ where $\frac{1}{R_T} = \frac{1}{9} + \frac{1}{9}$ Fig(a_1)

$= \frac{36}{4.5} = 8A$ or $R_T = \frac{9}{2}\,\Omega$

∴ Current through the branch cd, $I_{cd} = \frac{8}{2} = 4A$

Current through the branch fe, $I_{fe} = 4A$

Potential of point a in Fig(a_1), $V_a = 36 - 6 \times 4 = 12V$

Potential of the point b in Fig(a_1), $V_b = 36 - 3 \times 4 = 24V$

∴ Potential difference between points a and b; V_{ab} is,

$$V_{ab} = V_a - V_b = 12 - 24 = -12V$$

(b) When the Switch S in Fig(a) is closed, the simplified equivalent circuit in Fig(a_1) is modified to the circuit in Fig(a_2)

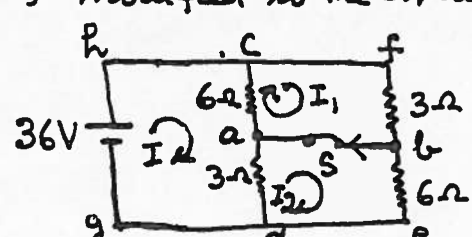


Fig (a_2)

□ abfc: $3I_1 + 6(I_1 - I) = 0$

or $I_1 = 2/3\, I_1$ (1)

□ adeb: $6I_2 + 3(I_2 - I) = 0$

$I_2 = I/3$ (2)

□ cdgh: $6(I - I_1) + 3(I - I_2) = 36$ (3)

The loop currents I, I_1 and I_2 are given by the equations

$$I_1 = \tfrac{2}{3} I \quad (1)$$
$$I_2 = I/3 \quad (2)$$
$$9I - 6I_1 - 3I_2 = 36 \quad (3)$$

Putting I_1 and I_2 in eq. (3) we get

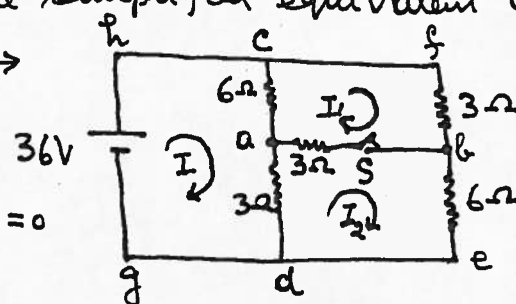
$$9I - 6\times\frac{2I}{3} - 3\times I/3 = 36$$
or $$9I - 4I - I = 36$$
$$\therefore I = 9A \quad (4)$$

Current through ab $= I_2 - I_1 = \frac{I}{3} - \frac{2I}{3} = -\frac{I}{3}$

From (4) $I_{ab} = -\frac{9}{3} = -3A$

(c) when the Switch S is open in Fig(b), there will be no current in the branch ab and the potential difference between a, b will be the same as in the solution of (a) part i.e $V_{ab} = -12V$

(d) when S in Fig(b) is closed, the simplified equivalent circuit in Fig(b) is given in Fig(b_2) ⟶



Fig(b_2)

□abfc: $3I_1 + 3(I_1 - I_2) + 6(I_1 - I) = 0$
or $4I_1 - I_2 = 2I$ (1)

□abed: $6I_2 + 3(I_2 - I) + 3(I_2 - I_1) = 0$
or $4I_2 - I_1 = I$ (2)

□cdgh $6(I - I_1) + 3(I - I_2) = 36$
or $3I - 2I_1 - I_2 = 12$ (3)

From eq (1) and (2) we get $I_2 = \frac{2}{5}I$, $I_1 = 3/5\, I$

$\therefore$ Eq(3) gives, $3I - \frac{6}{5}I - 2/5\, I = 12$

or $7/5\, I = 12$ or $I = \frac{60}{7}A$

Current through ab, $I_{ab} = I_2 - I_1 = \frac{2}{5}I - \frac{3}{5}I = -\frac{I}{5}$

$\therefore I_{ab} = -\frac{I}{5} = -\frac{60}{7}\times\frac{1}{5} = \frac{12}{7}A$

(e) when Switch S is open, equivalent resistance R can be easily calculated by $\frac{1}{R} = \frac{1}{9} + \frac{1}{9}$ $\therefore R = 9/2 = 4.5\Omega$

(f) when the switch in Fig(b) is closed, the equivalent resistance can be calculated from Fig(b_2) in part (d) of the problem.

$$R = V/I = \frac{36}{(60/7)} = \frac{36\times 7}{60} = 4.2\Omega$$

Q13 (a) What is the potential difference between a and b in the figure qhen switch S is open? (b) Which point a or b is at higher potential? (c) What is the final potential of point b when switch S is closed? (d) How much charge flows through switch S when it is closed? (18V, a is at higher V, 6V, 36μc)

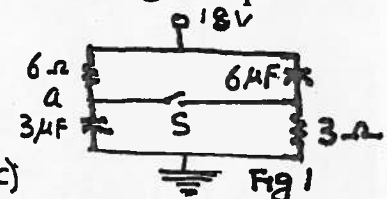


Fig 1

Solution

(a) When Switch S is open (Fig 2) no Current flows in any branch of the Circuit, but both Capacitors C_1 and C_2 are charged to a potential difference of 18V.

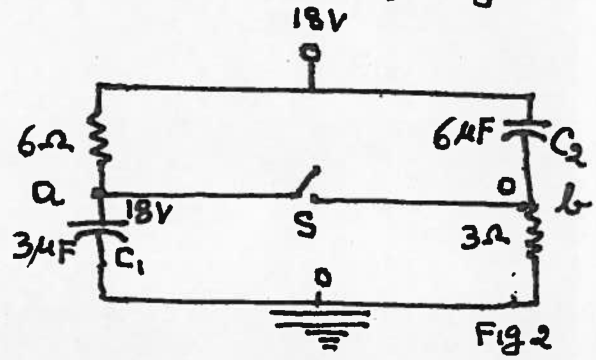


Fig 2

$V_a = 18\ V, \quad V_b = 0\ V$

$\therefore V_{ab} = 18 - 0 = 18\ V$

Charge on C_1, $q_{C1} = (18-0)(3\times10^{-6}) = 54\mu c$

Charge on C_2, $q_{C2} = (18-0)(6\times10^{-6}) = 108\mu c$

(b) $V_a = 18V$, $V_b = 0V$ $\therefore$ a is at higher potential.

(c) When switch S is closed, the current flows along the path shown in Fig 3.

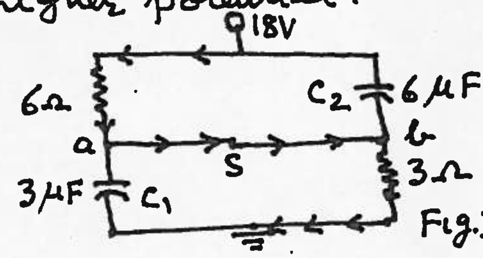


Fig.3

$\therefore I = \frac{18}{9} = 2A$

(c) Now: $V_a = 18 - 6\times2 = 6V$ $\quad V_b = 3\times2 = 6V$

(d) $\therefore$ Charge on $C_1 \rightarrow q'_{C1} = 6\times3\times10^{-6} = 18\mu c$

Charge on $C_2 \rightarrow q'_{C2} = (18-6)(6\times10^{-6}) = 72\mu c$

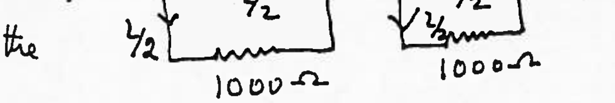
$\therefore$ Amount of charge flowing through S when the switch is closed

is $= q_{C1} - q'_{C1} = 54\mu c - 18\mu c = 36\mu c$

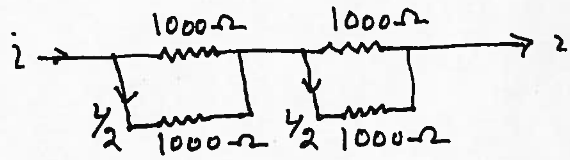
or $= q_{C2} - q'_{C2} = 108\mu c - 72\mu c = 36\mu c$

Q14 A 1000Ω, 2W resistor is needed, but only several 1000Ω ,1 W resistors are available. (a) How can the required resistance and power rating be obtained by a combination of the available units? (b) What power is then dissipated in each resistor? (0.5W)

Solution: (a) The Combination shown in the figure gives the required resistance



(b) In this Combination the the Current is halved in each resestor



The maximum Current, i_{max}, that the requred resistor Can take is,

$$i_{max}^2 \times (1000) = 2$$

$$\therefore i_{max} = \sqrt{2\times 10^{-3}} \text{ A}$$

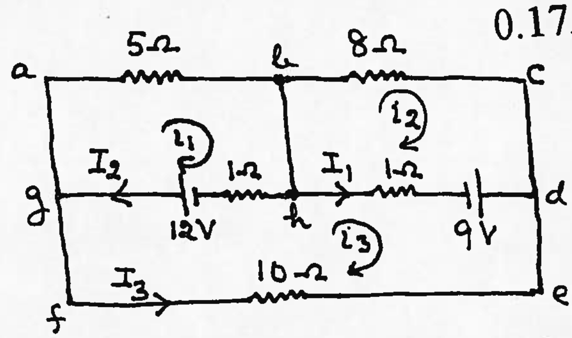
In the Combination, each resestor has $\frac{i_{max}}{2}$ current passing through.

$\therefore$ The total power dissipated in each resestor is

$$P = \left(\frac{i_{max}}{2}\right)^2 \times 1000 = \frac{(2\times10^{-3})(1000)}{4} = \frac{W}{2}.$$

Q15 Calculate the three currents in the circuit diagram. (0.85A, 2.14A, 0.17A)

Let i_1, i_2, i_3 be the three loop currents as shown. Then



$I_2 = i_1 - i_3$, $I_3 = -i_3$

$I_1 = i_3 - i_2$

using Kirchhoff's Laws

□ abhg: $5i_1 + 1(i_1 - i_3) = 12$

or $6i_1 - i_3 = 12$ (1) or $i_1 = 2 + \frac{i_3}{6}$ (1A)

□ bcdh: $8 \times i_2 + 1(i_2 - i_3) = -9$

or $9i_2 - i_3 = -9$ (2) or $i_2 = -1 + \frac{i_3}{9}$ (2A)

□ gdef: $1\times(i_3 - i_1) + 1\times(i_3 - i_2) + 10 i_3 = -12 + 9$

or $12 i_3 - i_2 - i_1 = -3$ (3)

Putting i_1 and i_2 from (1A) and (2A) into eq. (3)

$$12 i_3 + 1 - \frac{i_3}{9} - 2 - \frac{i_3}{6} = -3$$

$$\frac{211}{18} i_3 = -2 \quad \text{or} \quad i_3 = -0.171 \text{ A}$$

From (1A) $i_1 = 1.971$ A and from (2A) $i_2 = -1.019$

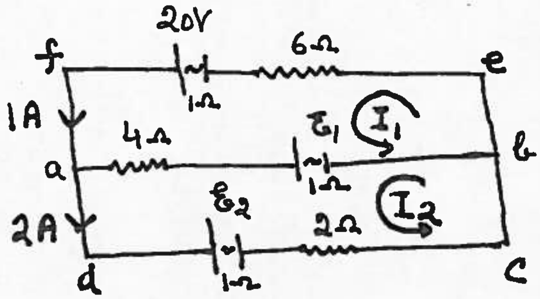
$\therefore I_1 = i_3 - i_2 = -0.171 + 1.019 = 0.85$ A

$I_2 = i_1 - i_3 = 1.971 + 0.171 = 2.14$ A

$I_3 = -i_3 = 0.171$ A

Q.16 Find the emf's E_1 and E_2 in the circuit of the figure and the potential difference between the points a, b. (!8V, 7V, 13V)

Solution:



It is obvious

$I_1 = 1\,A$

$I_2 = 2\,A$

Applying Kirchhoff's Laws

Loop $\boxed{abef}$:

$$4(I_1 - I_2) + 1(I_1 - I_2) + 6\,I_1 + 1\,I_1 = 20 - \mathcal{E}_1 \quad (1)$$

$$\text{or } 4(1-2) + 1(1-2) + 6\times 1 + 1\times 1 = 20 - \mathcal{E}_1$$

$$-4 - 1 + 6 + 1 = 20 - \mathcal{E}_1$$

$$\text{or } E_1 = 18\,V$$

Loop $\boxed{acdb}$ $\quad 1(I_2 - I_1) + 4(I_2 - I_1) + 1.I_2 + 2\,I_2 = \mathcal{E}_1 - \mathcal{E}_2$

$$1(2-1) + 4(2-1) + 1.2 + 2.2 = \mathcal{E}_1 - \mathcal{E}_2$$

$$1 + 4 + 2 + 4 = 18 - \mathcal{E}_2$$

$$\therefore\ \mathcal{E}_2 = 7V$$

Potential Difference between points a, b (V_{ab})

$$V_{ab} = 4(I_1 - I_2) + 1(I_1 - I_2) + \mathcal{E}_1$$

$$= 4(1-2) + 1(1-2) + \mathcal{E}_1$$

$$= -4 - 1 + 18$$

$$V_{ab} = 13V.$$