Factoring Formulas

$(x+y)(x-y) = x^2 - y^2$

$(x \pm y)^2 = (x^2 \pm 2xy + y^2)$

$(x^3 \pm y^3) = (x \pm y)(x^2 \mp xy + y^2)$

Rationalize the denominator:

FSM 1 (8)-(9)

FSM 2 (2)

$$\frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{5}} = \frac{(\sqrt{3} - \sqrt{5})(\sqrt{3} - \sqrt{5})}{(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})} =$$

$$= \frac{3 - 2\sqrt{3}\sqrt{5} + 5}{(\sqrt{3})^2 - (\sqrt{5})^2} = \frac{8 - 2\sqrt{15}}{3 - 5} =$$

$-4 + \sqrt{15}$

→ Slope - Intercept Form of a Line

$y = mx + b$

m = the slope of the line

b = the y-intercept of the line

→ Point - Slope Form of a Line

$y - y_1 = m(x - x_1)$

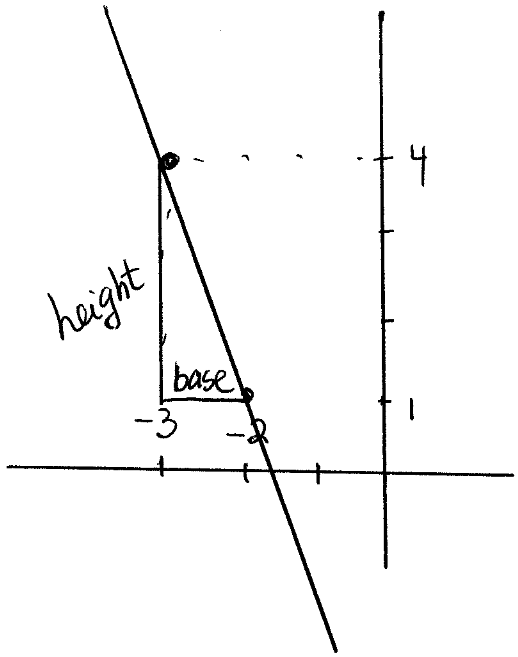
m = the slope of the line

(x_1, y_1) is a point on the line

FSM1 Q17

Find the equation of the line through $(-2,1)$ and $(-3,4)$

$y = mx + b$	$y - y_1 = m(x - x_1)$
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{-3-(-2)} = -3$ $m = \frac{\text{Change in } y}{\text{Change in } x}$	$m = -3$ ~~Th~~ Now, take the second point $(-3, 4)$



Take the first point $(-2,1)$ and plug it in.

$$1 = -3\cdot(-2) + b$$

$$b = -5$$

$$\boxed{y = -3x - 5}$$

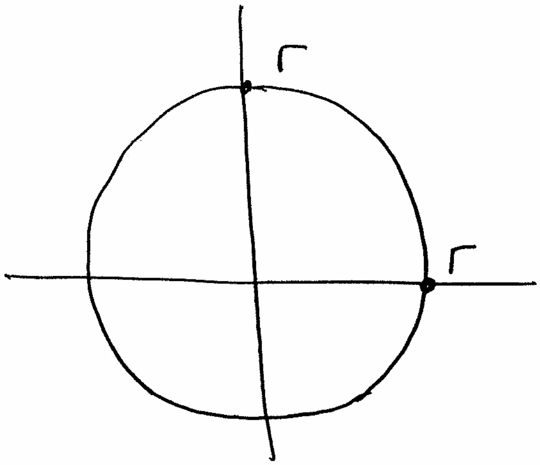
(You could take the second point to find b as well)

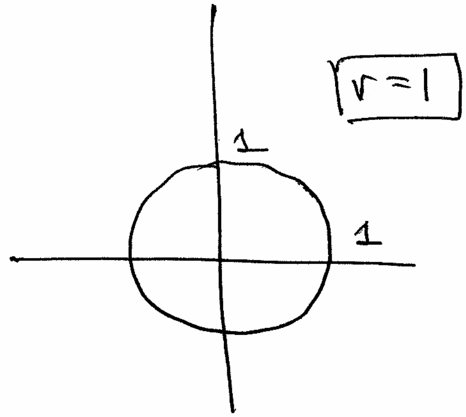
$$y - 4 = -3(x+3)$$

$$\boxed{y = -3x - 5}$$

(The first point will work as well).

Consider a circle of radius r:

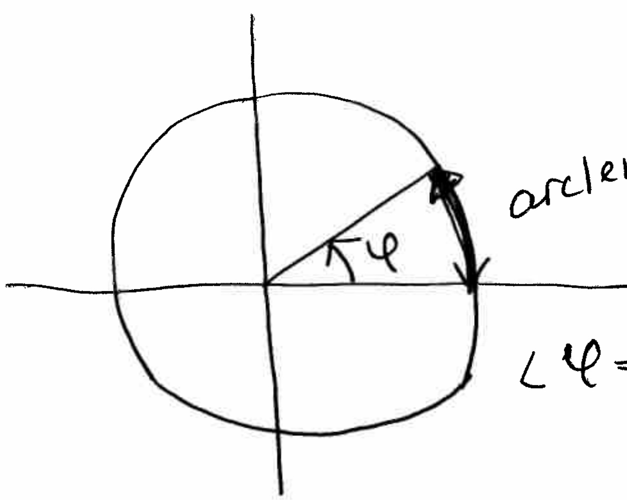




Its arclength is equal to $2\pi r$.

Take a unit circle ($r=1$), then its arclength $= 2\pi$ (radians)

$\boxed{\text{Definition}}$ An angle of 1 radian is defined to be the angle at the center of a unit circle which cuts off an arc of length 1, measured counterclockwise.



arclength = 1 unit

$\angle \varphi = 1$ radian

2π radians $\approx 2 \cdot 3.14 \approx 6.28$ (units)

$360° = 2\pi$ radians

$180° = \pi$ radians

$1° \cdot 180 = 180 = \pi$ radians $\Rightarrow$

$$\boxed{1° = \frac{\pi}{180} \text{ radians}}$$

Example Convert from degrees to radians

$$30° = 1° \cdot 30 = \frac{\pi}{180} \cdot 30 = \frac{\pi}{6} \text{ radians.}$$

$360° = 2\pi$ radians

$180° = \pi$ radians

$1 \text{ radian} \cdot \pi = 180°$, $\pi \approx 3.141$ just a constant.

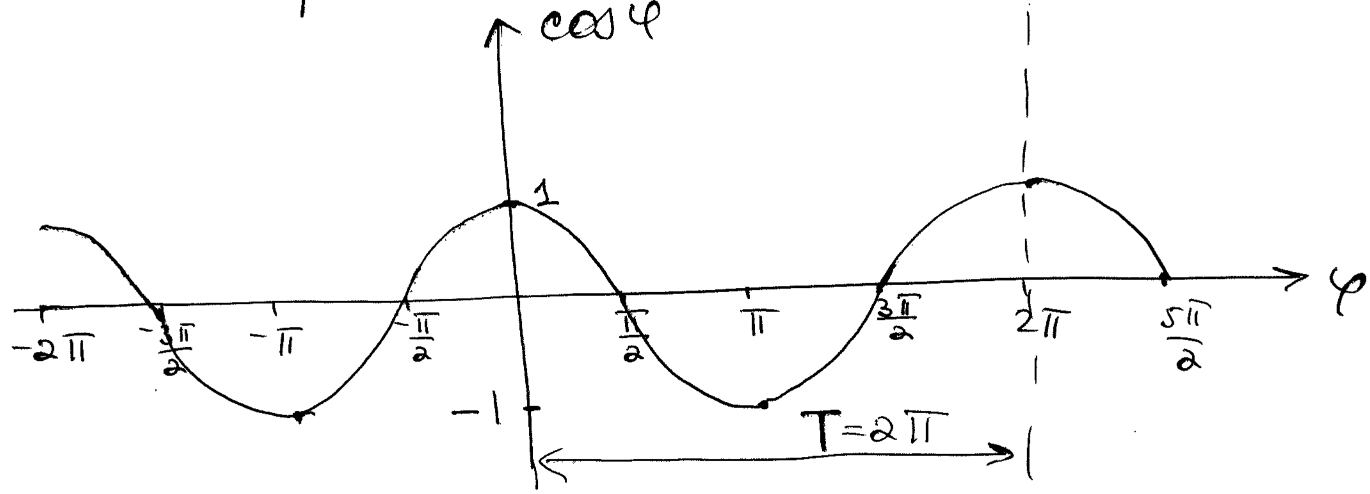
$$\boxed{1 \text{ radian} = \frac{180°}{\pi}}$$

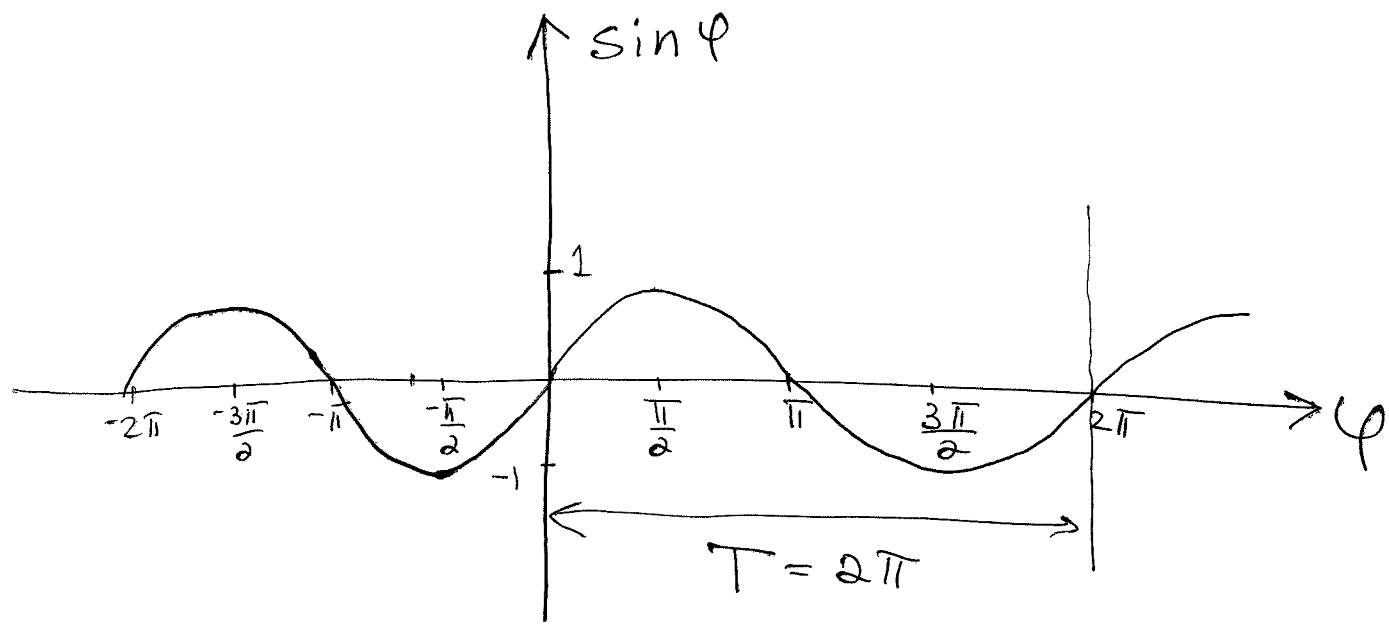
Example Convert from radians to degrees.

$$\frac{\pi}{3} \text{ radians} = 1 \text{ radian} \cdot \frac{\pi}{3} = \frac{180°}{\pi} \cdot \frac{\pi}{3} = 60°$$

Sine, Cosine Functions

$y = \cos\varphi$
$y = \sin\varphi$ are periodic functions with period 2π

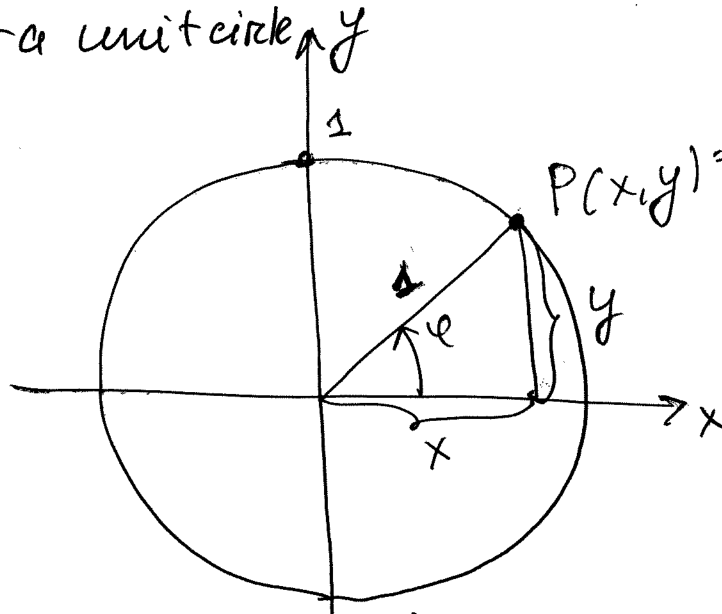




$A = \mathbb{R}$, $\varphi \in (-\infty, +\infty)$

$B = [-1, 1]$, $y \in [-1, 1]$

Consider a unit circle



$P(x,y) = P(\cos\varphi, \sin\varphi)$

$x = \cos\varphi$

$y = \sin\varphi$

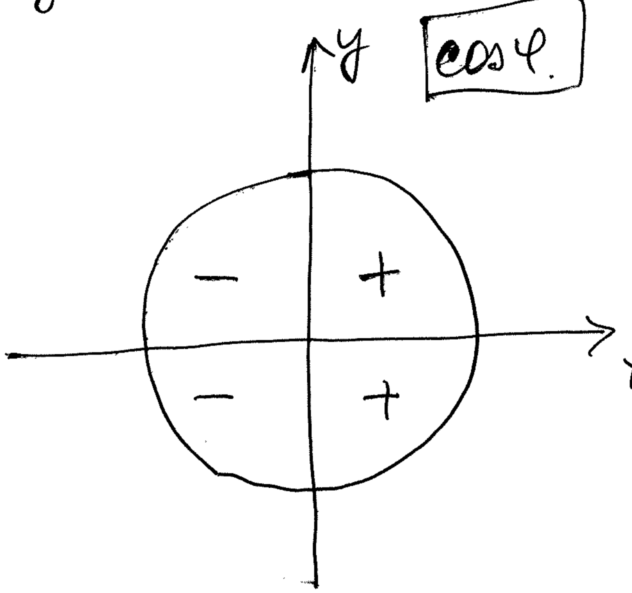
How to tell whether the cosine / sine of an angle is positive or negative

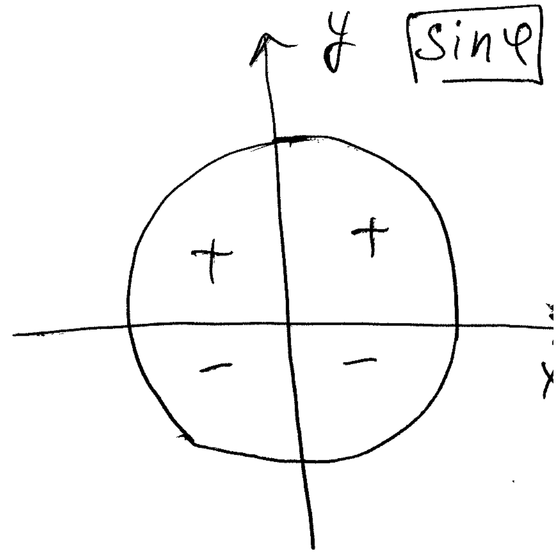
$\cos\varphi$ is the x-value of P.

$\sin\varphi$ is the y-value of P.

Thus, $\cos\varphi$ is negative when the x-value is negative.

$\sin\varphi$ is negative when the y-value of P is negative.





In the general, we would like to understand behaviour of the following functions:

$$y(x) = A + B\cos\Big(\underbrace{\frac{2\pi}{T}}_{=\omega}(x-\varphi)\Big),$$

where A, B, T, ω, φ are some constants.

These functions are solutions to certain differential equations that describe many phenomena in real life.

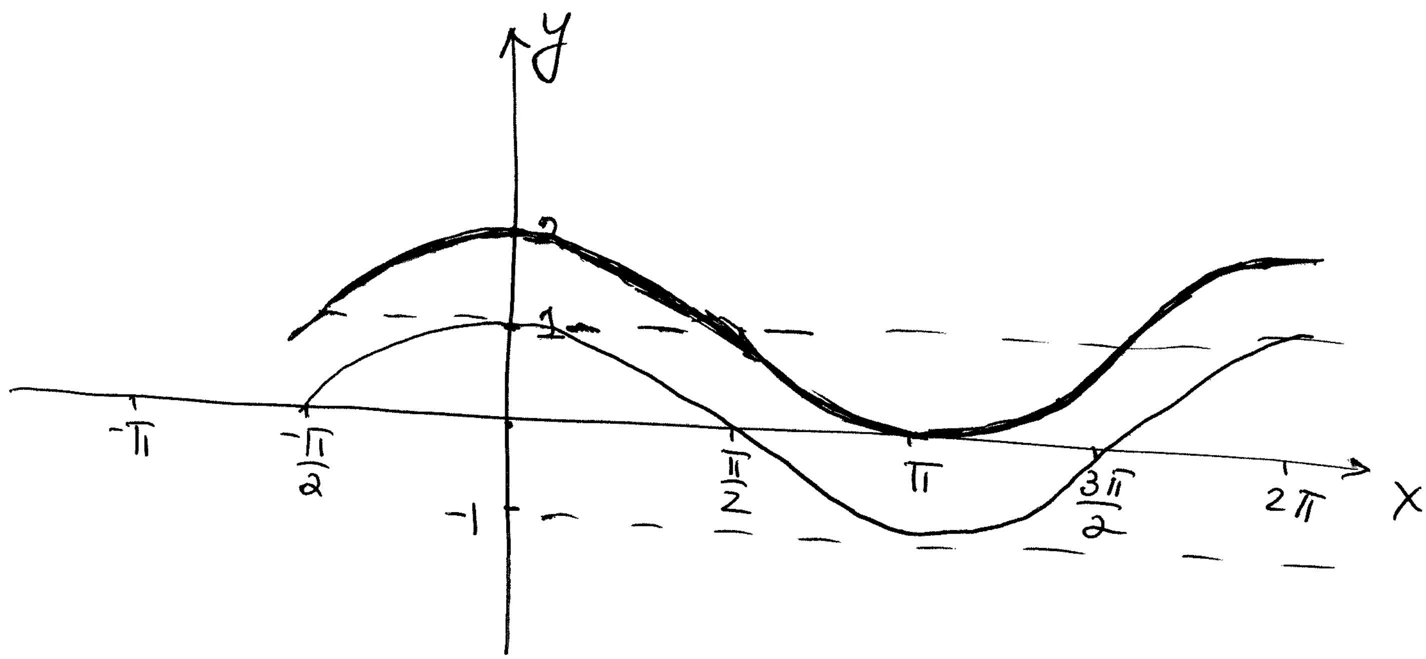
For example, ocean waves, sound waves, daily temperature can be decribed using the above oscillating functions.

What do these functions look like?

→ A is called the average of $y(x)$ and it is a halfway b/n min. and max. values ($A = 0$ for $y(x) = \cos x$ and $y(x) = \sin x$)

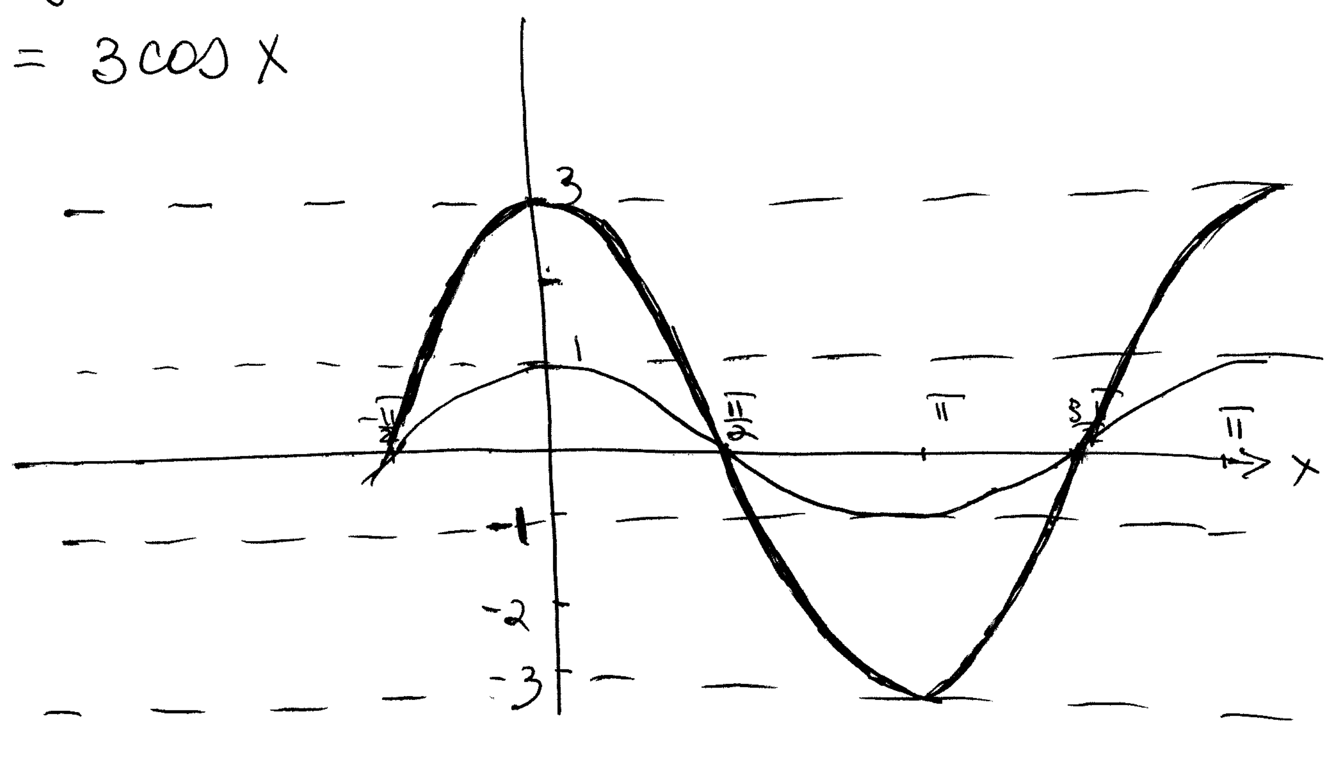
$\boxed{\text{Example}}$ $\quad y(x) = 1 + \cos x \quad , \quad A = 1$

$y = 1 + \cos x$



→ B is called the amplitude, it is the difference b/n max and the average (or min. and the average). Icreasing or decreasing B will vertically stretch or shrink the graph

$y = 3\cos x$

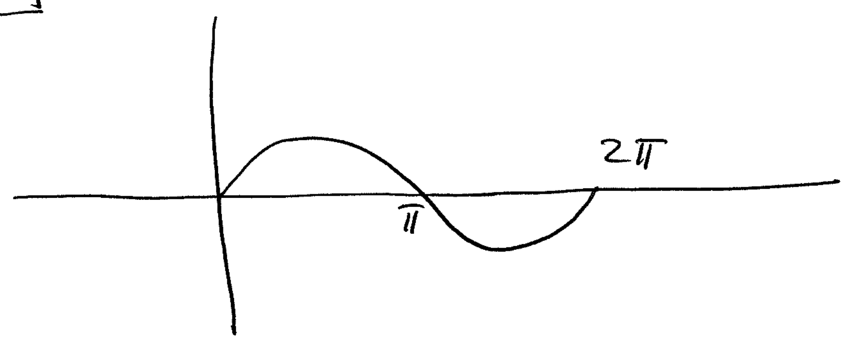


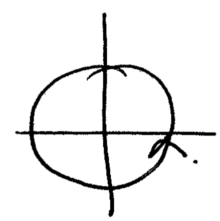
→ T = period, it is the time (distance) b/n successive peaks

→ $w = \frac{2\pi}{T}$ = frequency, it is the number of cycles we complete in "time interval" 2π.

The value of w affects the period T

$\boxed{y = \sin x}$, $w = 1$, $w = \frac{2\pi}{T} \Rightarrow T = 2\pi$



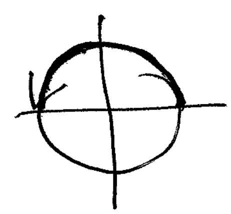


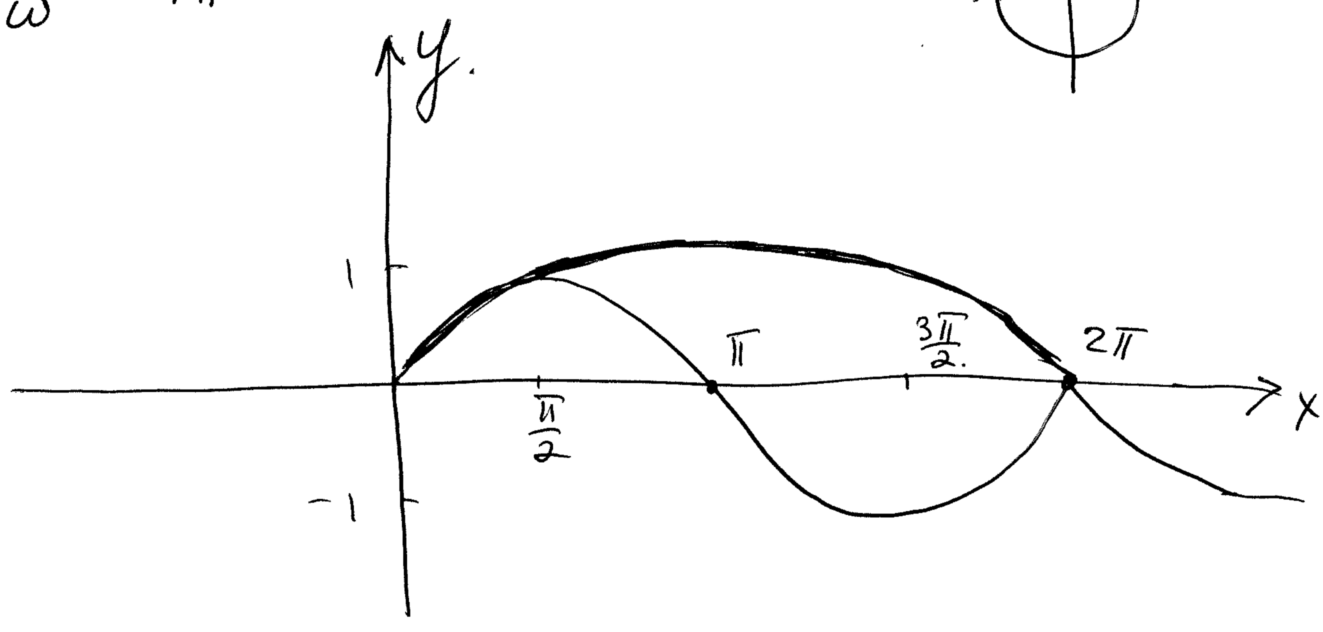
in 2π "time" we make 1 circle.

$y = \sin \frac{x}{2}$, $w = \frac{1}{2} < 1$ – horizontal stretching

In 2π "time", we complete a half of a circle

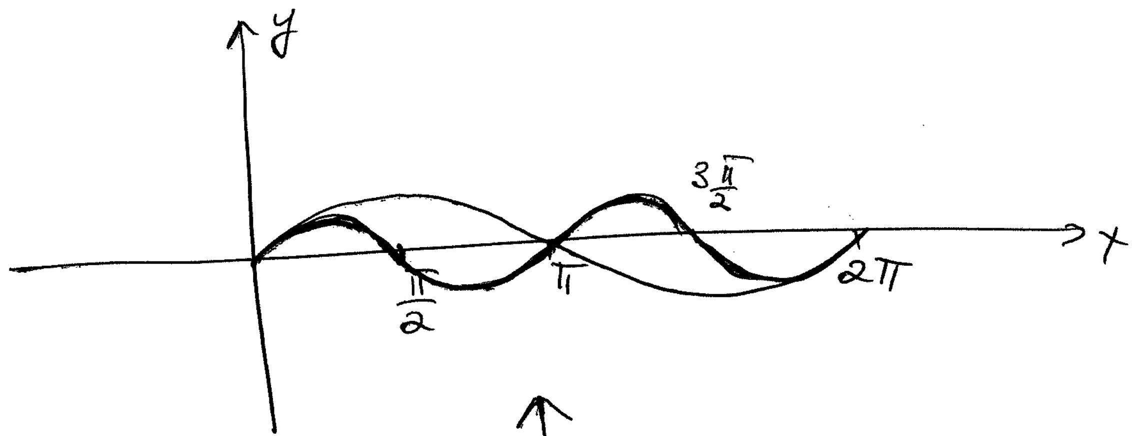
$T = \frac{2\pi}{\omega} = 4\pi$





$y = \sin(2x)$, $w = 2 > 1$ ← horizontal shrinking

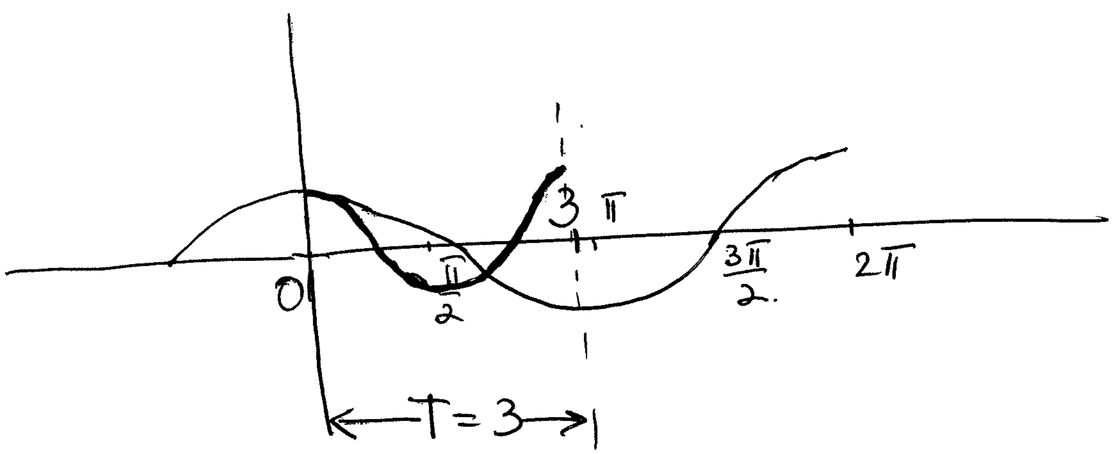
$2 = w = \frac{2\pi}{T} \Rightarrow \boxed{T = \pi}$



In "time" 2π, we complete 2 oscillations (or 2 circles)

$y(x) = \cos\left(\frac{2\pi}{3}x\right)$, $w = \frac{2\pi}{3} > 1$ – horizontal shrinking

$w = \frac{2\pi}{T} = \frac{2\pi}{3} \Rightarrow \boxed{T = 3}$

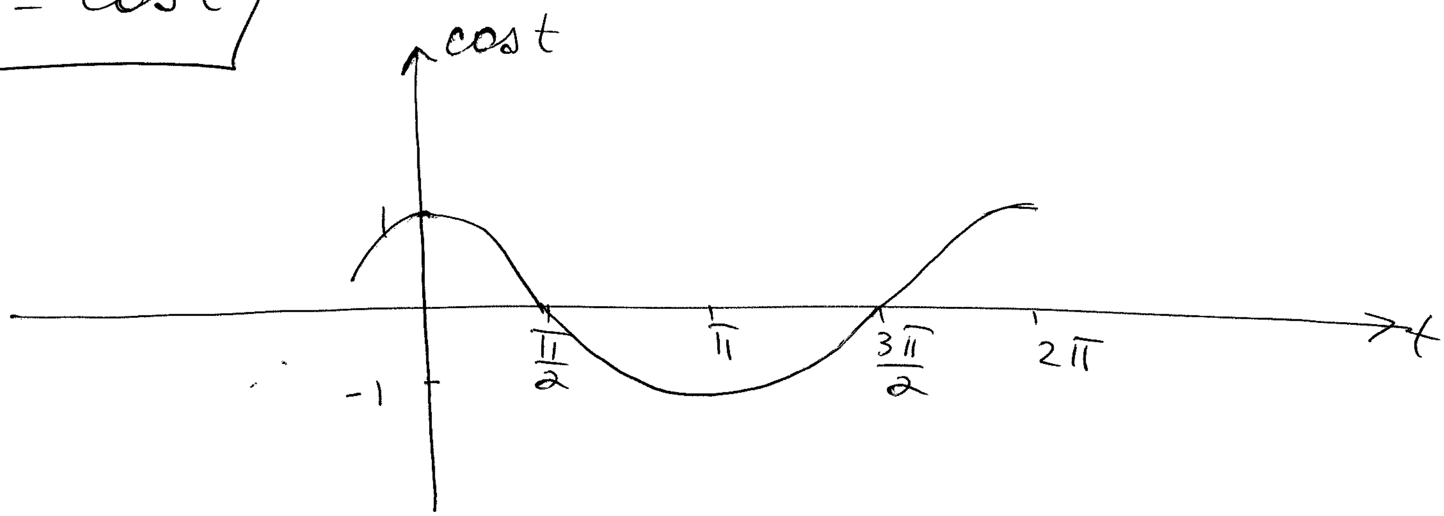


FSM1 (13)

Sketch $y(t) = 1 + 3\cos\left(\frac{2\pi}{3}(t-2)\right) =$

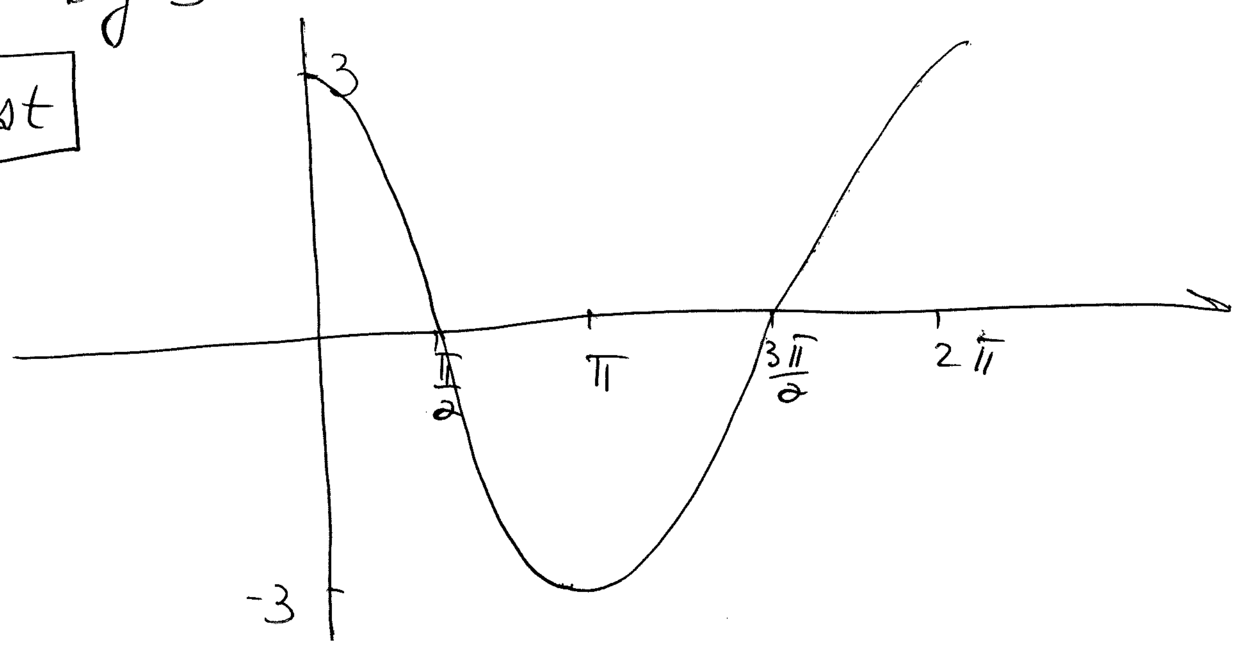
$= A + B\cos(\omega(t-2))$

$\boxed{y = \cos t}$



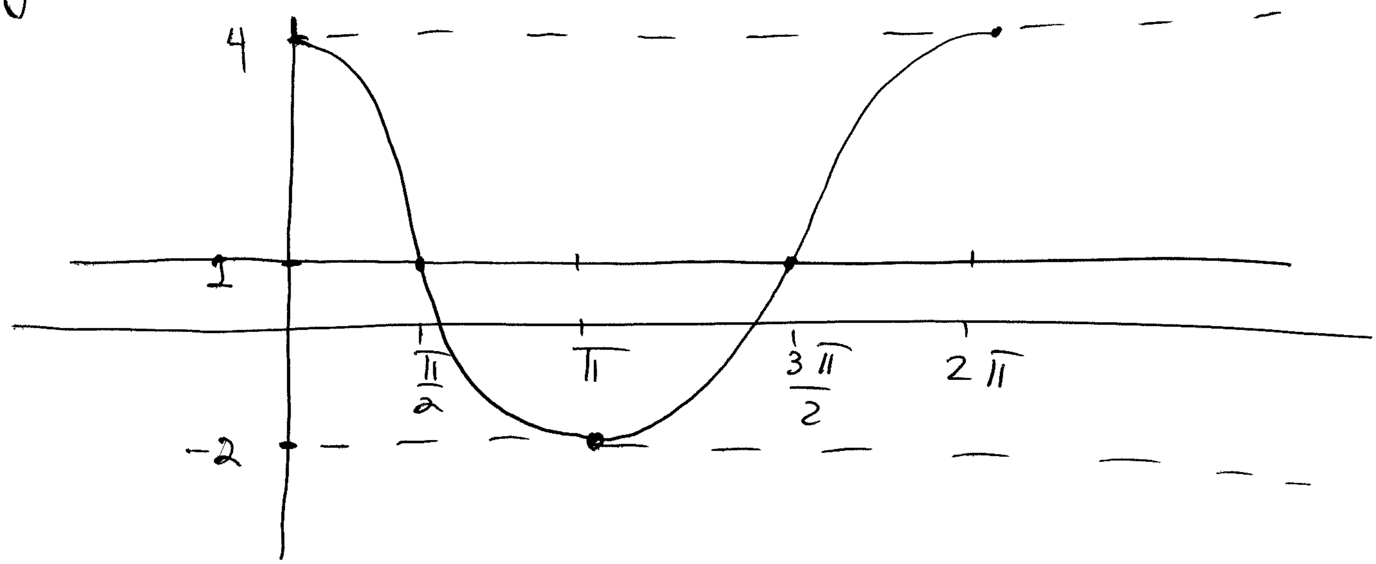
B = 3 is the amplitude, to increase the amplitude by a factor of 3 > 1, we scale vertically ~~[illegible]~~ by multiplying the cosine by 3

$\boxed{y = 3\cos t}$



→ $y = 1 + 3\cos t$.

To increase the average from 0 to 1, we vertically shift the function by adding 1 to the function, making $y = 1 + 3\cos t$

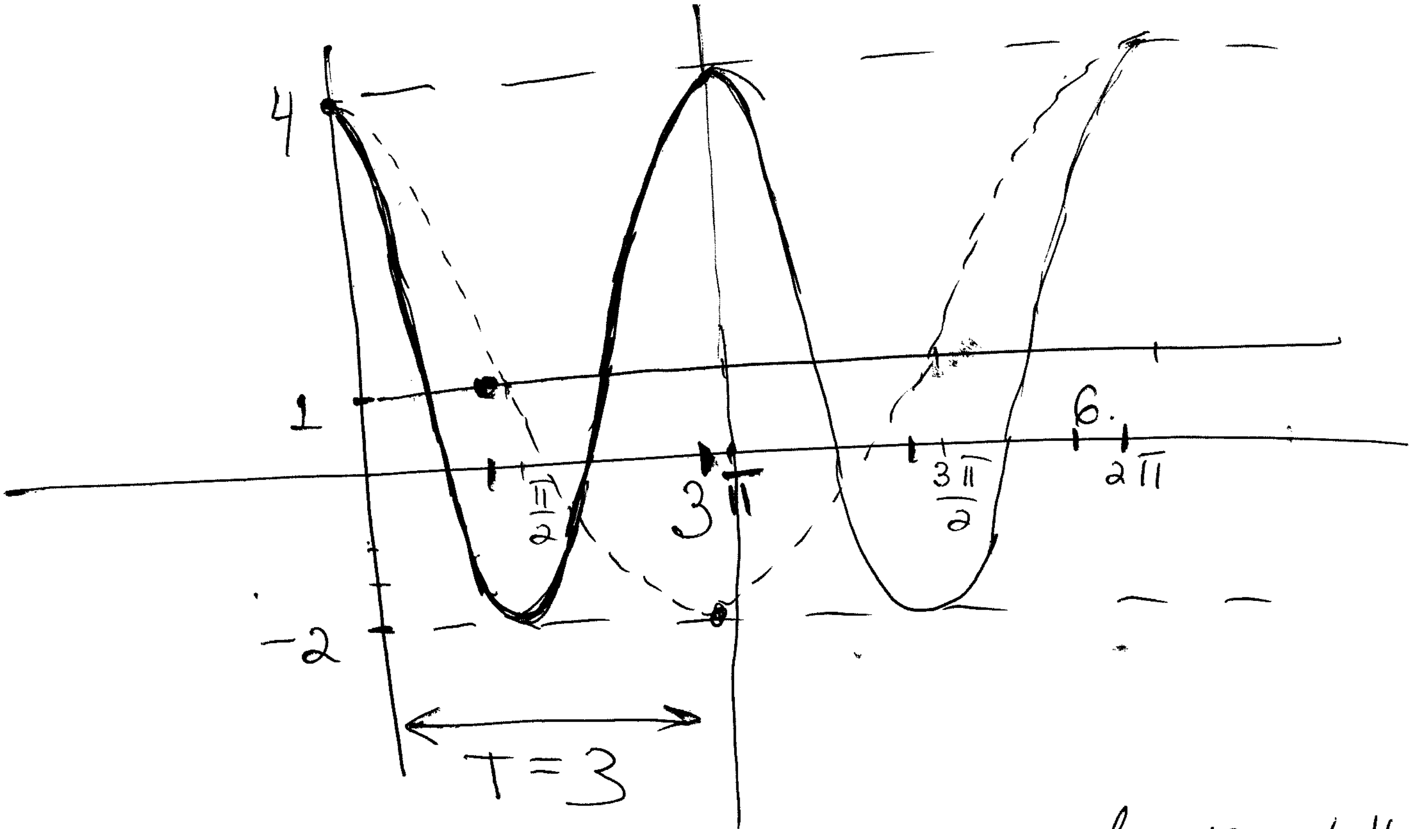


→ $\omega = \frac{2\pi}{3} > 1$

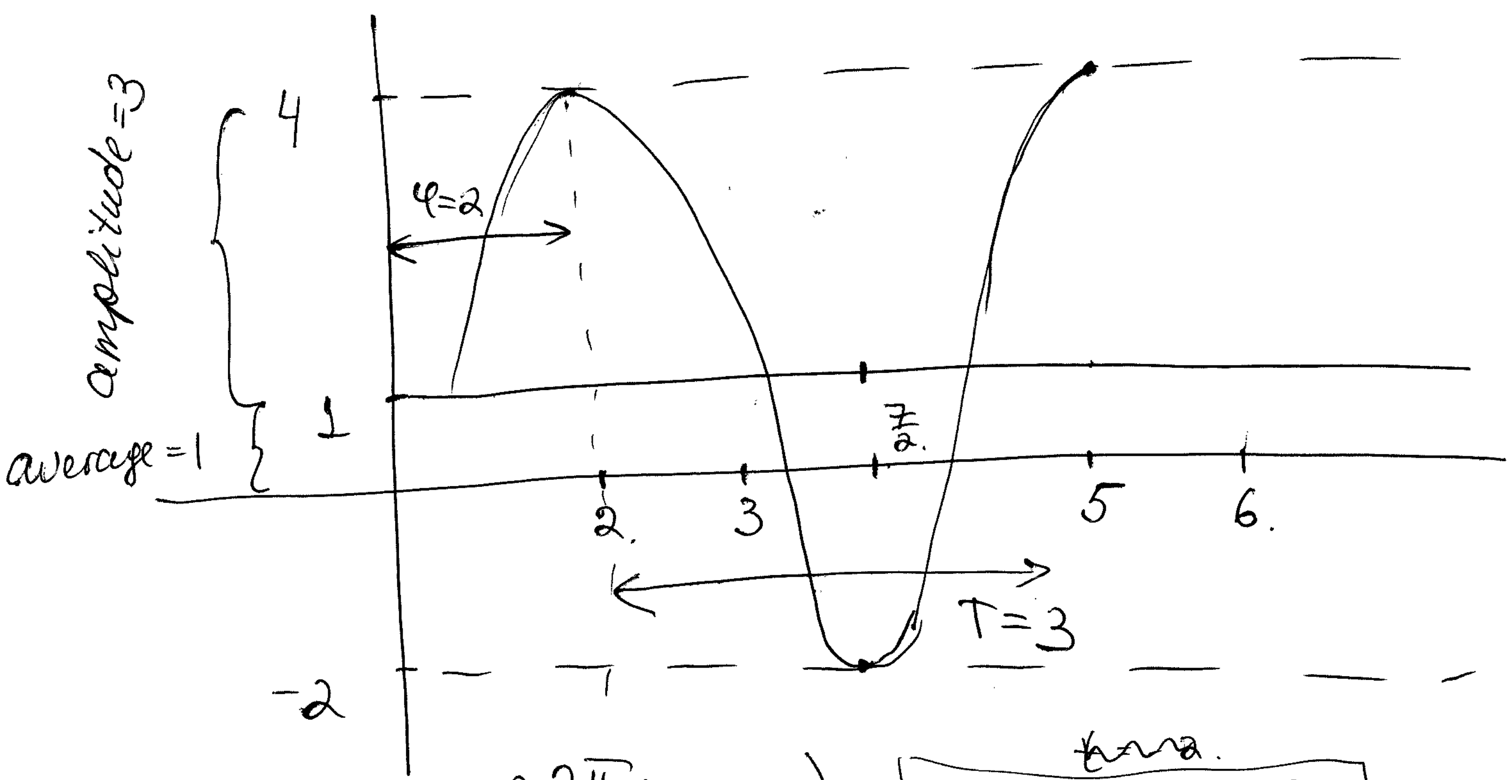
Next, we compress the graph horizontally by a factor of $\omega = \frac{2\pi}{3}$

$\omega = \frac{2\pi}{3} = \frac{2\pi}{T} \Rightarrow T = 3$. (the period)

Thus, we decrease the period from $2\pi \approx 6.28$ units to 3 units.



→ Finally, we shift the curve horizontally to the right, so that the first peak is at 2 instead of 0.

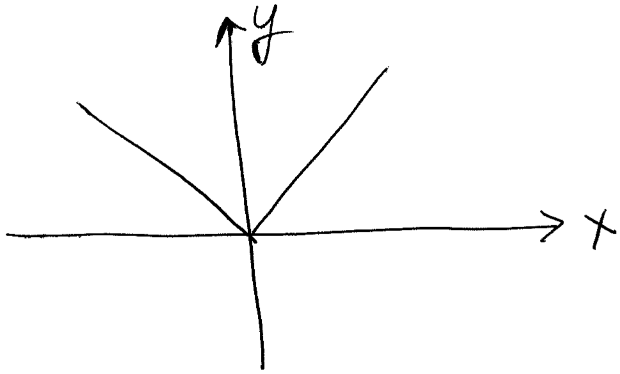


$y = 1 + 3\cos\left(\frac{2\pi}{3}(t-2)\right)$ $\boxed{B = [-2, 4]}$

Absolute values

Definition

$$y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$



Absolute value tells us how far the number x is from the origin (a distance from zero)

Examples FSM I (34) Solve the following

$|x^2 - 9| = 7.$

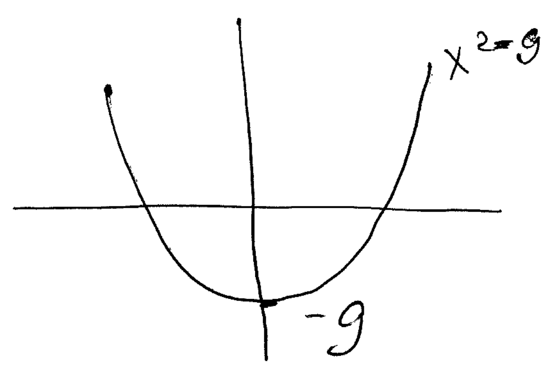
$x^2 - 9 = 7$ or $-(x^2 - 9) = 7$

$x^2 = 16$ or $x^2 - 9 = -7$

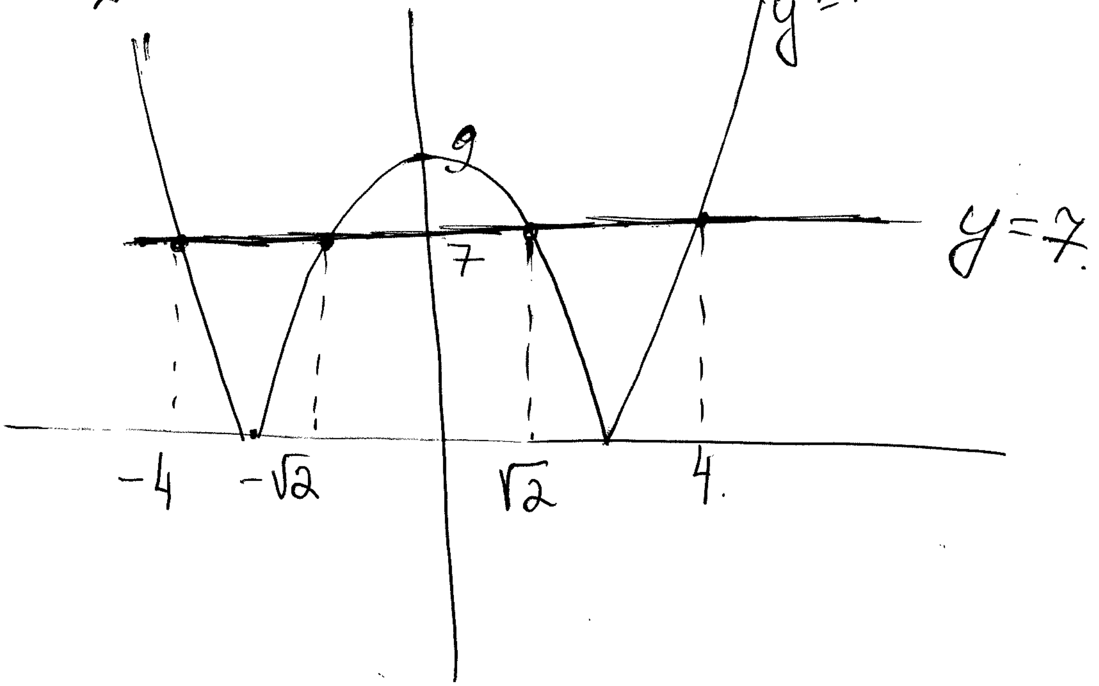
$x^2 = 9 - 7 = 2$

$\boxed{x^2 = 16}$ or $\boxed{x^2 = 2}$

$x = \pm 4$ or $x = \pm\sqrt{2}.$



$\cancel{y}\ 7 = |x^2-9|$



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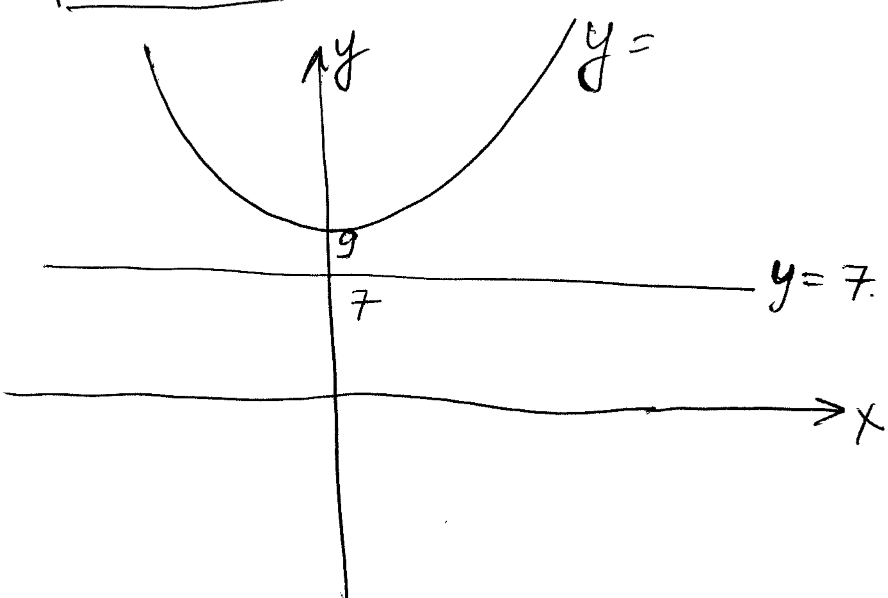
35 $\quad |x^2+9| = 7$

$x^2+9 = 7 \quad$ or $\quad -(x^2+9) = 7$

$\qquad\qquad\qquad\qquad\quad x^2+9 = -7$

$\boxed{x^2=-2} \quad$ or $\quad \boxed{x^2=-16}$



no solution.

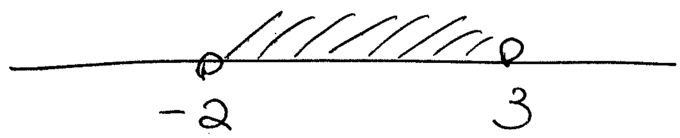
(36) $|2x-1|<5$

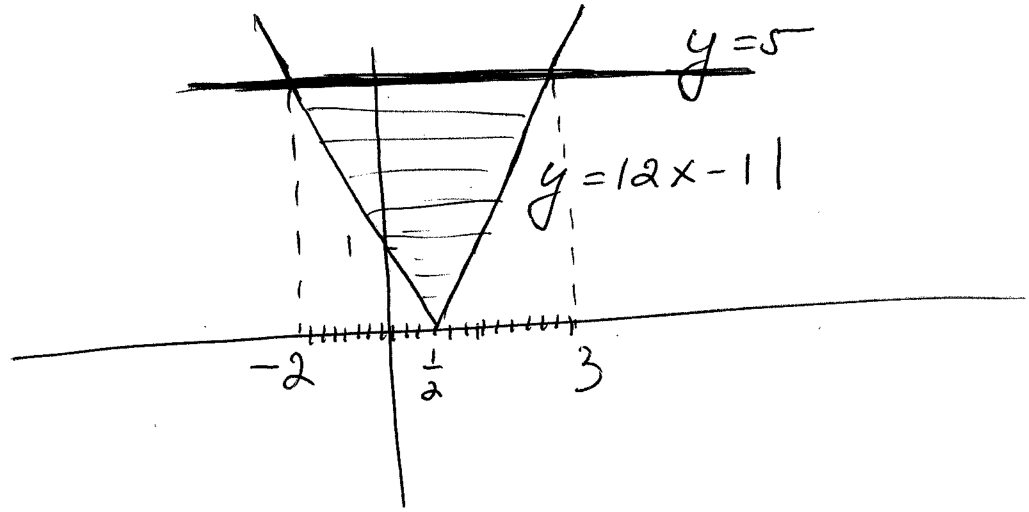
$2x-1<5$ <u>and</u> $-(2x-1)<5$

$2x<6$ and $-2x+1<5$

$x<3$ $\qquad$ $2x>-4$

$x>-2$



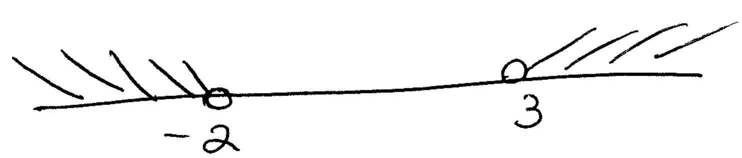


(37) $|2x-1|>5$

$(2x-1)>5$ or $-(2x-1)>5$

$2x>6$

$x>3$ or $x<-2$.



Function inequalities

(43) Solve $\dfrac{3x-2}{4x+1} > 5$

$4x+1$

$$\frac{3x-2}{4x+1} - 5 > 0$$

$$\frac{3x-2-5(4x+1)}{4x+1} > 0.$$

$$\frac{3x-2-20x-5}{4x+1} > 0$$

$$\frac{-17x-7}{4x+1} > 0$$

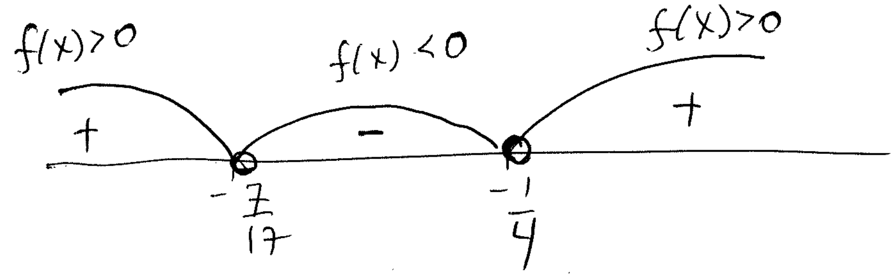
$$-\left(\frac{17x+7}{4x+1}\right) > 0$$

$$f(x) = \frac{17x+7}{4x+1} < 0$$

Critical Points:

$$x = -\frac{1}{4}$$

$$x = -\frac{7}{17}.$$



$\bar{x} = -2 \quad f(-2) > 0 \qquad \bar{x} = 0 \quad f(0) > 0.$

FSM 1, 24

$$3^{x-2} = 27^{x+5} = (3^3)^{x+5} = 3^{3x+15}$$

$$\log_3 3^{x-2} = \log_3 3^{3x+15}$$

$$x-2 = 3x+15$$

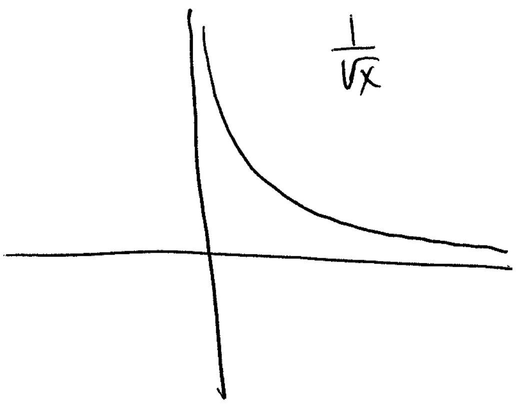
$$x = -\frac{17}{2}.$$

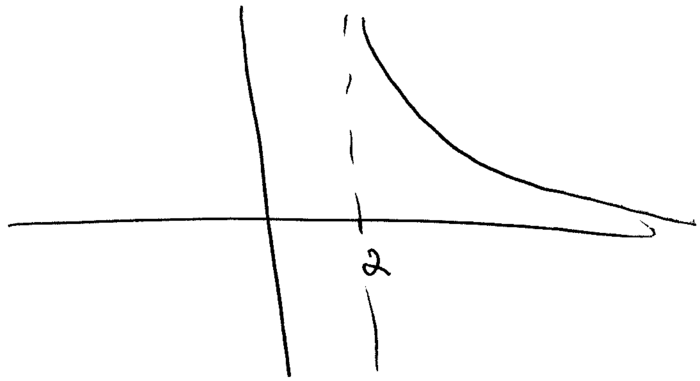
Find the domain and the range of f.

FSM 1 21

$$f(x) = \frac{1}{\sqrt{x-2}}$$

power f-n, $r = \frac{1}{2}$





$x - 2 > 0$

$x > 2$

$A = \{ \cancel{x}\ x \in \mathbb{R} : x > 2 \}$

$B = (0, +\infty)$