MATH 1007H **Term test #2 – solution** **9 February 2015**

This test was written in room: ________________ Total marks: **34** marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT

First name ____ Last name ____ Student number ____

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

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1. Fill in the blanks:

a. [1] $\log_3 1 =$ 0

b. [1] $\log_3 \frac{1}{3} =$ -1

c. [1] $\log_3 0 =$ d.n.e. / undefined / $-\infty$ / "N/A", etc...

d. [1] Let $f(x) = \arccos(x-1)$. The range of f is $[0, \pi]$

e. [1] Let $f(x) = \arctan(x-2)$. The domain of f is: $\mathbb{R}$

f. [1] Let $f(x) = \ln(x-1)$. The domain of f is $(1, \infty)$

g. [1] $\arctan x = arc\cot\left(\frac{1}{x}\right)$. **TRUE** ✓ **FALSE** ____

2. Determine the following limits:

a. [3] $\lim_{x\to 0} \frac{-5(x-1)^2+5}{x} \xrightarrow{D.S.} \frac{-5(0-1)^2+5}{0} = \frac{0}{0}$

$$\ldots = \lim_{x\to 0}\left[\frac{-5(x^2-2x+1)+5}{x}\right] = \lim_{x\to 0}\left[\frac{-5x^2+10x-5+5}{x}\right] =$$

$$\ldots = \lim_{x\to 0}\left[\frac{-5\cancel{x}(x-2)}{\cancel{x}}\right] \xrightarrow{D.S.} \boxed{10}$$

b. [4] $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2} \xrightarrow{D.S.} \frac{\sqrt{0^2+4}-2}{0^2} = \frac{0}{0}$

$$\ldots = \lim_{x\to 0}\left[\frac{\sqrt{x^2+4}-2}{x^2}\cdot\frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}\right] = \lim_{x\to 0}\left[\frac{x^2\cancel{+4-4}}{x^2\left(\sqrt{x^2+4}+2\right)}\right] =$$

$$\ldots = \lim_{x\to 0}\left[\frac{\cancel{x^2}\ 1}{\cancel{x^2}\left(\sqrt{x^2+4}+2\right)}\right] = \lim_{x\to 0}\left[\frac{1}{\sqrt{x^2+4}+2}\right] \xrightarrow{D.S.}$$

$$\ldots \xrightarrow{D.S.} \frac{1}{\sqrt{0^2+4}+2} = \boxed{\frac{1}{4}}$$

c. [3] $\lim_{x\to 1} \frac{\sin(x-1)}{x^2-1} \xrightarrow{D.S.} \frac{\sin(1-1)}{1^2-1} = \frac{0}{0}$ (b.c. $\sin 0 = 0$)

$$= \lim_{x\to 1}\left[\frac{\sin(x-1)}{(x-1)(x+1)}\right] = \lim_{x\to 1}\left[\frac{1}{x+1}\right]\cdot\left[\lim_{x\to 1}\frac{\sin(x-1)}{x-1}\right]$$

$$= \lim_{x\to 1}\left[\frac{1}{x+1}\right]\cdot\lim_{(x-1)\to 0}\left[\frac{\sin(x-1)}{x-1}\right] = \boxed{\frac{1}{2}}$$

d. [3] $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3} \xrightarrow{D.S.} \frac{\frac{1}{3}-\frac{1}{3}}{3-3} = \frac{0}{0}$

$$\ldots = \lim_{x\to 3}\left[\frac{\frac{3-x}{3x}}{x-3}\right] = \lim_{x\to 3}\left[\frac{-\cancel{(x-3)}}{3x\cancel{(x-3)}}\right] = \lim_{x\to 3}\left[\frac{-1}{3x}\right] \xrightarrow{D.S.}$$

$$\xrightarrow{D.S.} \boxed{-\frac{1}{9}}$$

3. Simplify as much as possible.

a. [3] $\tan\left(\arccos(3/4)\right) = \dfrac{\sin(\arccos(3/4))}{\cos(\arccos(3/4))} = \dfrac{\sqrt{1-\cos^2(\arccos(3/4))}}{\cos(\arccos(3/4))} =$

$$\cdots = \frac{\sqrt{1-(3/4)^2}}{3/4} = \frac{\sqrt{16/16 - 9/16}}{3/4} = \frac{\sqrt{7}/4}{3/4} = \boxed{\frac{\sqrt{7}}{3}}$$

b. [3] $\left(\sqrt[3]{25}\right)^{\log_5 27} = \left[(25)^{1/3}\right]^{\log_5 27} = \left[(5^2)^{1/3}\right]^{\log_5 27} =$

$$\cdots = \left(5^{2/3}\right)^{\log_5 27} = (5)^{2/3 \cdot \log_5 27} = (5)^{\log_5 27^{2/3}} =$$

$$\cdots = 27^{2/3} = \boxed{9}$$

$$\left(\text{i.e. } 27^{2/3} = (3^3)^{2/3} = 3^2 = 9\right)$$

4. [3] Let $\log_2(y-1) - 2\log_2 5 = x - \log_2 x$. Express y as a function of x.

$$\log_2(y-1) - \log_2 5^2 = x - \log_2 x$$

$$\log_2(y-1) - \log_2 25 + \log_2 x = x$$

$$\log_2\left[\frac{x(y-1)}{25}\right] = x$$

$$\frac{x(y-1)}{25} = 2^x$$

$$\to x(y-1) = 25(2^x)$$

$$y-1 = \frac{25}{x}(2^x)$$

$$\boxed{y = \frac{25}{x}(2^x) + 1}$$

5. Let $f(x) = \begin{cases} \sqrt{4-x}+1, & \text{if } x \le 4 \\ 9-ax, & \text{if } x > 4 \end{cases}$

a. [3] Determine $\lim\limits_{x\to 4^-} f(x)$ and $\lim\limits_{x\to 4^+} f(x)$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}\left[\sqrt{4-x}+1\right] \xrightarrow{\text{D.S.}} \boxed{1} \quad (= f(4))$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}\left[9-ax\right] \xrightarrow{\text{D.S.}} \boxed{9-4a}$$

b. [2] For which value of a is f continuous?

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4)$$

$$\therefore\ 9-4a = 1$$

$$\boxed{a = 2}$$