CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All except EC
Examination	**Date**	**Time**	**Pages**
Final	December 2015	3 Hours	3

Instructors **Course Examiner**

E. Duma, F. Romanelli, H. Greenspan, J. Ortmann, L. Aggoun, L. Dube D. Sen
M. Padamadan, P. Gauthier, U. Tiwari, V. Kalvin

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[10] **1.** At a price of \$2.28 per bushel, the supply of barley is 7,500 million bushels and the demand is 7,900 million bushels. At a price of \$2.37 per bushel, the supply of barley is 7,900 million bushels and the demand is 7,800 million bushels.

(A) Find a price-supply equation of the form $p = mx + b$.

(B) Find a price-demand equation of the form $p = mx + b$.

(C) Find the equilibrium point.

[10] **2.** Solve for x in the following equations:

(A) $9^{2x-1} = 27^x$

(B) $(2)^{3x} = \dfrac{1}{32}$

(C) $\log_2 \sqrt{2x^2} - 1 = \dfrac{3}{2}$

(D) $\log_5(x+2) + \log_5 x = \log_5(x+12)$

(E) $\log_{\frac{1}{3}}(27) = x + 2$

[10] **3.** For $f(x) = 360 - 60x$ and $g(x) = 10^{x-10}$ find the following by only using a proper formula:

(A) $\sum_{k=0}^{49} f(k) = f(0) + f(1) + f(2) + \cdots + f(49)$.

(B) $\sum_{h=1}^{35} g(h) = g(1) + g(2) + g(3) + \cdots + g(35)$.

[10] **4.** A father opened a savings account for his daughter on the day she was born, depositing \$1000. Each year on her birthday he deposits another \$1000, making the last deposit on her 21st birthday.

(A) If the account pays 5.25% interest compounded annually, how much is in the account at the end of the day on his daughter's 21st birthday?

(B) How much interest has been earned?

[10] **5.** A family purchased their home 8 years ago for \$83,000. The home was financed by paying 20% down and signing a 30–year mortgage at 8.4% compounded monthly for the balance. Equal monthly payments were made to amortize the loan over the 30–year period.

(A) What is the monthly payment?

(B) Find the unpaid balance just after the 96th payment is made.

(C) The market value of the house is now \$95,000. After making the 96th payment, the family applied to the loan company for the maximum loan. The loan company will loan up to 60% of the equity in a home. How much will the family receive?

[10] **6.** Solve by using Gauss-Jordan Elimination:

$$3x_1 + 2x_2 + 4x_3 = -1$$

$$-2x_1 + x_2 - 2x_3 = 6$$

$$3x_1 + 3x_2 + 6x_3 = 3$$

No other method of solving this system of equations will be accepted!

[10] **7.** An island economy consists of the sectors of tourism, agriculture and fishing. To produce a dollar's worth of tourism requires an input of \$0.3, \$0.2 and \$0.1 from tourism, agriculture and fishing respectively. A dollar's worth of agriculture requires inputs of \$0.1 from each sector. On the other hand, a dollar's worth of fishing requires inputs of \$0.3, \$0.2 and \$0.1 from the sectors of tourism, agriculture and fishing.

(A) Write the technology matrix M for this island economy.

(B) If a final demand of \$40 million, \$10 million and \$20 million from tourism, agriculture and fishing is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

[10] **8.** Extremize $P(x, y) = 20x - 15y$ subject to

$$x + 2y \geq 14,\ x + 3y \leq 42,\ 2x + y \leq 42,\ x \geq 0,\ y \geq 0\,.$$

[10] **9.** In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 5 barley plants, and 3 rye plants. She wishes to select 4 plants at random.

(A) In how many ways can this be done?

(B) In how many ways can this be done if exactly 2 wheat plants must be included?

[10] **10.** To transfer into a particular technical department, a company requires an employee to pass a screening test. A maximum of 3 attempts is allowed at 6-month intervals between trials. From past record it is found that 40% pass on the first trial; of those that fail the first trial and take the test a second time; 60% pass; and of those that fail on the second trial and take the test a third time, 20% pass. For an employee wishing to transfer:

(A) What is the probability of passing the test on the first or second try?

(B) What is the probability of failing on all 3 attempts?

(C) What is the probability of failing on the first 2 trials and passing on the third?

FINAL EXAM — MATH 208 — DEC 2015

(1) price $2.28 { S = 7500, D = 7900 } price $2.37 { S = 7900, D = 7800 }

A) SUPPLY EQUATION $p = mx + b$ (LINEAR)

SUPPLY (x, p): (7500, 2.28), (7900, 2.37)

(NOTE: PRICE p ⇒ "y-value")

$m = \frac{y_2 - y_1}{x_2 - x_1}$

SLOPE $= \frac{2.37 - 2.28}{7900 - 7500} = \frac{0.09}{400} \Rightarrow$ **$m = 0.000225$**

$p = mx + b$

$2.28 = 0.000225(7500) + b$

$2.28 = 1.6875 + b$

$b = 2.28 - 1.6875$

$b = 0.5925$

SUPPLY EQUATION

$p = 0.000225x + 0.5925$

B) DEMAND EQUATION $p = mx + b$

DEMAND (x, p): (7900, 2.28), (7800, 2.37)

SLOPE $= \frac{2.37 - 2.28}{7800 - 7900} = \frac{0.09}{-100} \Rightarrow$ **$m = -0.0009$**

$p = mx + b$

$2.28 = -0.0009(7900) + b$

$2.28 = -7.11 + b$

$b = 2.28 + 7.11$

$b = 9.39$

DEMAND EQUATION

$p = -0.0009x + 9.39$

MATH 208 DEC 2015

① (CONTINUED)

c) EQUILIBRIUM POINT ⇒ SUPPLY = DEMAND

$0.000225x + 0.5925 = -0.0009x + 9.39$

$0.000225x + 0.0009x = 9.39 - 0.5925$

$0.001125x = 8.7975$

$x = \frac{8.7975}{0.001125}$ ⟹ $\boxed{x = 7820}$ (million)

AND $p = 0.000225(7820) + 0.5925$

$p = 1.7595 + 0.5925$

$p = 2.352$ ⟹ $\boxed{p = \$2.35}$

② A) $9^{2x-1} = 27^x$

$(3^2)^{2x-1} = (3^3)^x$

$3^{4x-2} = 3^{3x}$ → $4x - 2 = 3x$

$4x - 3x = 2$

$\boxed{x = 2}$

B) $(2)^{3x} = \frac{1}{32} = \frac{1}{2^5}$

$2^{3x} = 2^{-5}$ → $3x = -5$ → $\boxed{x = \frac{-5}{3}}$

C) $\log_2 \sqrt{2x^2} - 1 = \frac{3}{2}$

$\log_2 (2x^2)^{1/2} = \frac{5}{2}$

$\frac{1}{2}\log_2 (2x^2) = \frac{5}{2}$

$\log_2 (2x^2) = 5$

$2^5 = 2x^2$

$32 = 2x^2$ → $2x^2 - 32 = 0$

$2(x^2 - 16) = 0$

$2(x+4)(x-4) = 0$

(APPLY DEF. $\log_b x = y \Leftrightarrow b^y = x$)

$\boxed{x = -4}$

$\boxed{x = 4}$

Checked ok ✓

BOTH VALID SOLUTIONS

FINAL EXAM | MATH 208 | DEC 2015

2 (CONTINUED)

D) $\log_5(x+2) + \log_5 x = \log_5(x+12)$

$\log_5 (x+2)x = \log_5(x+12)$

$\log_5(x^2+2x) = \log_5(x+12)$

$x^2+2x = x+12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ Reject

$x=3$ ✓ check

E) $\log_{\frac{1}{3}}(27) = x+2$

$-3 = x+2$

$x=-5$

$\log_{\frac{1}{3}}(27) = y$

$\left(\frac{1}{3}\right)^y = 27 = 3^3 = \left(\frac{1}{3}\right)^{-3}$

$\left(\frac{1}{3}\right)^y = \left(\frac{1}{3}\right)^{-3}$

$\Rightarrow y=-3$

3

$f(x) = 360-60x \qquad g(x) = 10^{x-10}$

A) $\sum_{k=0}^{49} f(k)$

$f(0) = 360-60(0) = 360$

$\vdots$

$f(49) = 360-60(49) = -2580$

$a_1 = 360$ FIRST TERM

$n=50$ $(d=-60)$

$a_{50} = -2580$ LAST TERM

$S_n = \frac{n}{2}(a_1+a_n) \Rightarrow S_{50} = \frac{50}{2}(360+(-2580)) = 25(-2220)$

$\Rightarrow$ **-55 500**

B) $\sum_{h=1}^{35} g(h)$

$g(1) = 10^{1-10} = 10^{-9}$

$a_1 = 10^{-9}$ FIRST TERM

$n=35$ $r=10$

$S_n = \frac{a_1(r^n-1)}{r-1}$

$S_{35} = \frac{10^{-9}(10^{35}-1)}{10-1}$

$\Rightarrow$ **$1.111111\ldots \times 10^{25}$**

MATH 208 DEC 2015

4 A) 1ST DEPOSIT ⇒ DAY SHE WAS BORN ⇒ $t=0$

0, 1, 2, 3, ---21 ⇒ so $t = 22$ years

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

$$FV = 1000\left(\frac{(1+0.0525)^{22} - 1}{0.0525}\right)$$

FV = ? PMT = \$1000
$r = 0.0525$ $m = 1$
$i = \frac{r}{m} = \frac{0.0525}{1} = 0.0525$
$n = mt = (1)(22) = 22$

FV = \$39 664.40

B) TOTAL INTEREST = FV − TOTAL DEPOSITS

= 39664.40 − 22(1000)

INT. = \$17 664.40

5 A) 20% DOWN so $0.8(83000) =$ **\$66 400**

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$66400 = PMT\left(\frac{1-(1+0.007)^{-360}}{0.007}\right)$$

$$66400 = PMT(131.26156606\ldots)$$

PV = 66400 PMT = ?
$r = 0.084$ $m = 12$
$t = 30$
$i = \frac{r}{m} = \frac{0.084}{12} = 0.007$
$n = mt = (12)(30) = 360$

PMT = \$505.86

FINAL EXAM | MATH 208 | DEC 2015

(5) (CONTINUED)

B) UNPAID BALANCE AFTER 96TH PAYMENT = 8 years (8×12)

⇒ 30 − 8 = 22 YEARS LEFT OR 22×12 = 264 PAYMENTS REMAINING

(OR 360 − 96 = 264)

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$PV = 505.86\left(\frac{1-(1+0.007)^{-264}}{0.007}\right) = \$60\,806.57$$

UNPAID LOAN BALANCE AFTER 96 PAYMENTS

C) EQUITY = NET CURRENT MARKET VALUE − UNPAID LOAN BALANCE

Given = $95 000

So EQUITY = 95 000 − 60 806.57 = $34 193.43

EQUITY

So LOAN 60% of EQUITY

60% × 34 193.43 ⇒ $20 516.06

(6)

$$\left(\begin{array}{ccc|c} 3 & 2 & 4 & -1 \\ -2 & 1 & -2 & 6 \\ 3 & 3 & 6 & 3 \end{array}\right) \quad \frac{1}{3}R_3 \to R_3$$

$$\sim \left(\begin{array}{ccc|c} 3 & 2 & 4 & -1 \\ -2 & 1 & -2 & 6 \\ 1 & 1 & 2 & 1 \end{array}\right) \quad R_1 \leftrightarrow R_2 \quad \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -2 & 1 & -2 & 6 \\ 3 & 2 & 4 & -1 \end{array}\right)$$

$2R_1 + R_2 \to R_2$

$-3R_1 + R_3 \to R_3$ →

(6) (CONTINUED)

$$\sim \begin{pmatrix} 1 & 1 & 2 & | & 1 \\ 0 & 3 & 2 & | & 8 \\ 0 & -1 & -2 & | & -4 \end{pmatrix} -1R_3 \to R_3 \sim \begin{pmatrix} 1 & 1 & 2 & | & 1 \\ 0 & 3 & 2 & | & 8 \\ 0 & 1 & 2 & | & 4 \end{pmatrix} R_2 \leftrightarrow R_3$$

$$\sim \begin{pmatrix} 1 & 1 & 2 & | & 1 \\ 0 & 1 & 2 & | & 4 \\ 0 & 3 & 2 & | & 8 \end{pmatrix} \begin{matrix} -1R_2+R_1 \to R_1 \\ -3R_2+R_3 \to R_3 \end{matrix} \sim \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 2 & | & 4 \\ 0 & 0 & -4 & | & -4 \end{pmatrix} -\frac{1}{4}R_3 \to R_3$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 2 & | & 4 \\ 0 & 0 & 1 & | & 1 \end{pmatrix} -2R_3+R_2 \to R_2 \sim \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$$

SOLUTION: $\boxed{x_1 = -3,\ x_2 = 2,\ x_3 = 1}$

(7) TECHNOLOGY MATRIX M

A) $$M = \begin{matrix} \\ T \\ A \\ F \end{matrix} \begin{matrix} \begin{matrix} T & A & F \end{matrix} \\ \begin{pmatrix} 0.3 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \end{matrix}$$

T: TOURISM
A: AGRICULTURE
F: FISHING

B) $$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$$

FINAL DEMAND (IN MILLIONS)

$$D = \begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$$

$X = MX + D \Rightarrow$ X = M X + D

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix} = \begin{matrix} \\ T \\ A \\ F \end{matrix} \begin{matrix} \begin{matrix} T & A & F \end{matrix} \\ \begin{pmatrix} 0.3 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \end{matrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$$

$X = MX + D$

$X - MX = D$

$(I - M)X = D \Rightarrow \boxed{X = (I-M)^{-1}D}$

$I - M$

So— $$I - M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{matrix} \begin{matrix} T & A & F \end{matrix} \\ \begin{pmatrix} 0.3 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \end{matrix} = \begin{pmatrix} 0.7 & -0.1 & -0.3 \\ -0.2 & 0.9 & -0.2 \\ -0.1 & -0.1 & 0.9 \end{pmatrix}$$

FINAL EXAM | MATH 208 | DEC 2015

(7) (CONTINUED)

c) TO FIND $(I-M)^{-1}$ $\quad X = (I-M)^{-1}D$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 0.7 & -0.1 & -0.3 & 1 & 0 & 0 \\ -0.2 & 0.9 & -0.2 & 0 & 1 & 0 \\ -0.1 & -0.1 & 0.9 & 0 & 0 & 1 \end{array}\right) \text{ ....etc..}$$

$I-M$ | I

$$(I-M)^{-1} = \begin{pmatrix} 1.58 & 0.24 & 0.58 \\ 0.4 & 1.2 & 0.14 \\ 0.22 & 0.16 & 1.22 \end{pmatrix} \Rightarrow X = (I-M)^{-1}D$$

$$D = \begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix} = \underbrace{\begin{pmatrix} 1.58 & 0.24 & 0.58 \\ 0.4 & 1.2 & 0.4 \\ 0.22 & 0.16 & 1.22 \end{pmatrix}}_{(I-M)^{-1}} \underbrace{\begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix}}_{D} = \begin{pmatrix} 77.2 \\ 36 \\ 34.8 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$$

(8) EXTREMIZE $P(x,y) = 20x - 15y$

(1) $x + 2y \geq 14$

x	y
0	7
14	0

SOLID TP(0,0) $0 \geq 14$ FALSE

(2) $x + 3y \leq 42$

x	y
0	14
42	0
21	7

SOLID TP(0,0) $0 \leq 42$ TRUE

(3) $2x + y \leq 42$

x	y
0	42
21	0
14	14
7	28

SOLID TP(0,0) $0 \leq 42$ TRUE

$x \geq 0$, $y \geq 0$ ⇒ QUADI ONLY

POINT C: (2) ∩ (3)

(2) $x + 3y = 42$ $(\times -2)$

(3) $2x + y = 42$

$+$ (2)' $\Rightarrow -2x - 6y = -84$

(3) $\Rightarrow 2x + y = 42$

$-5y = -42$

$y = 42/5 = 8.4$

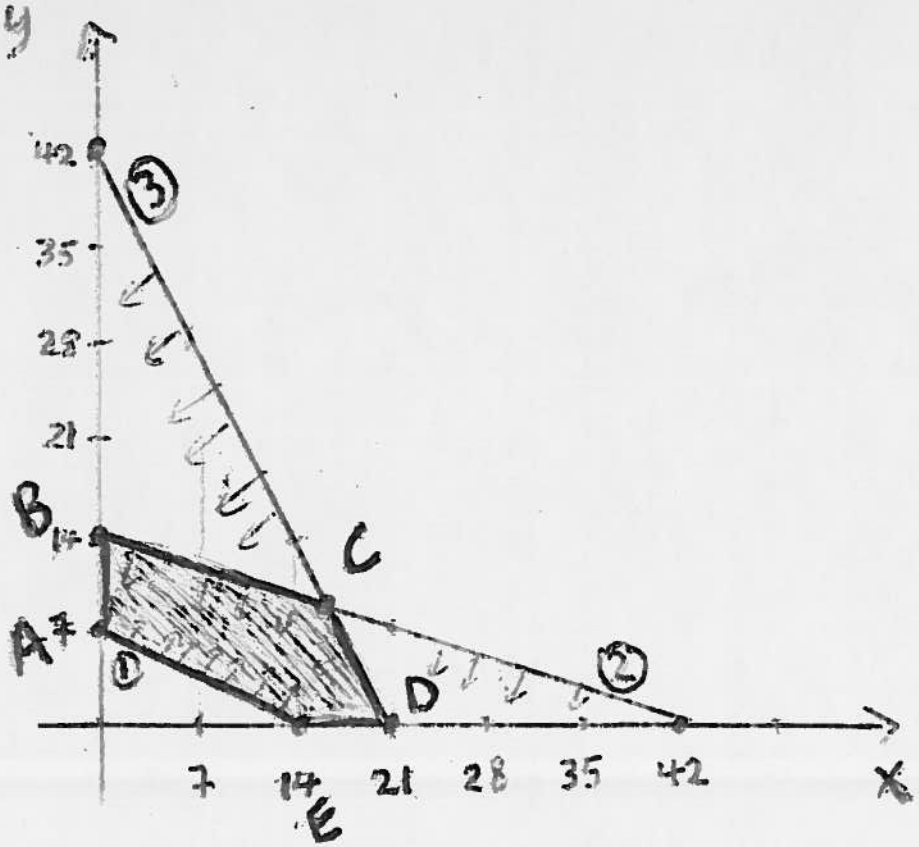
$2x + 8.4 = 42$

$2x = 33.6$

$x = \frac{33.6}{2} = 16.8$

C(16.8, 8.4)

8 (CONTINUED)



CORNER POINTS	$P(x,y) = 20x - 15y$	
A(0,7)	$20(0) - 15(7) = -105$	
B(0,14)	$20(0) - 15(14) = -210$	← MIN
C(16.8, 8.4)	$20(16.8) - 15(8.4) = 210$	
D(21,0)	$20(21) - 15(0) = 420$	← MAX
E(14,0)	$20(14) - 15(0) = 280$	

$\Rightarrow$ MINIMUM VALUE of $P(x,y) = -210$ (at B(0,14))

MAXIMUM VALUE of $P(x,y) = 420$ (at D(21,0))

FINAL EXAM | MATH 208 | DEC 2015 | C

(9) 6 WHEAT PLANTS
5 BARLEY PLANTS
3 RYE PLANTS

A) CHOOSE 4 PLANTS AT RANDOM
OUT OF $6+5+3 = 14$ PLANTS
ORDER NOT IMPORTANT $\Rightarrow$ COMBINATION

$$C_{14,4} = \frac{14!}{10!\,4!} = \boxed{1001}$$

B) CHOOSE 4 PLANTS, EXACTLY 2 MUST BE WHEAT PLANTS

$\Rightarrow$ CHOOSE 2 OUT OF 6 WHEAT PLANTS
AND $\Rightarrow$ CHOOSE 2 OUT OF REMAINING 8 PLANTS

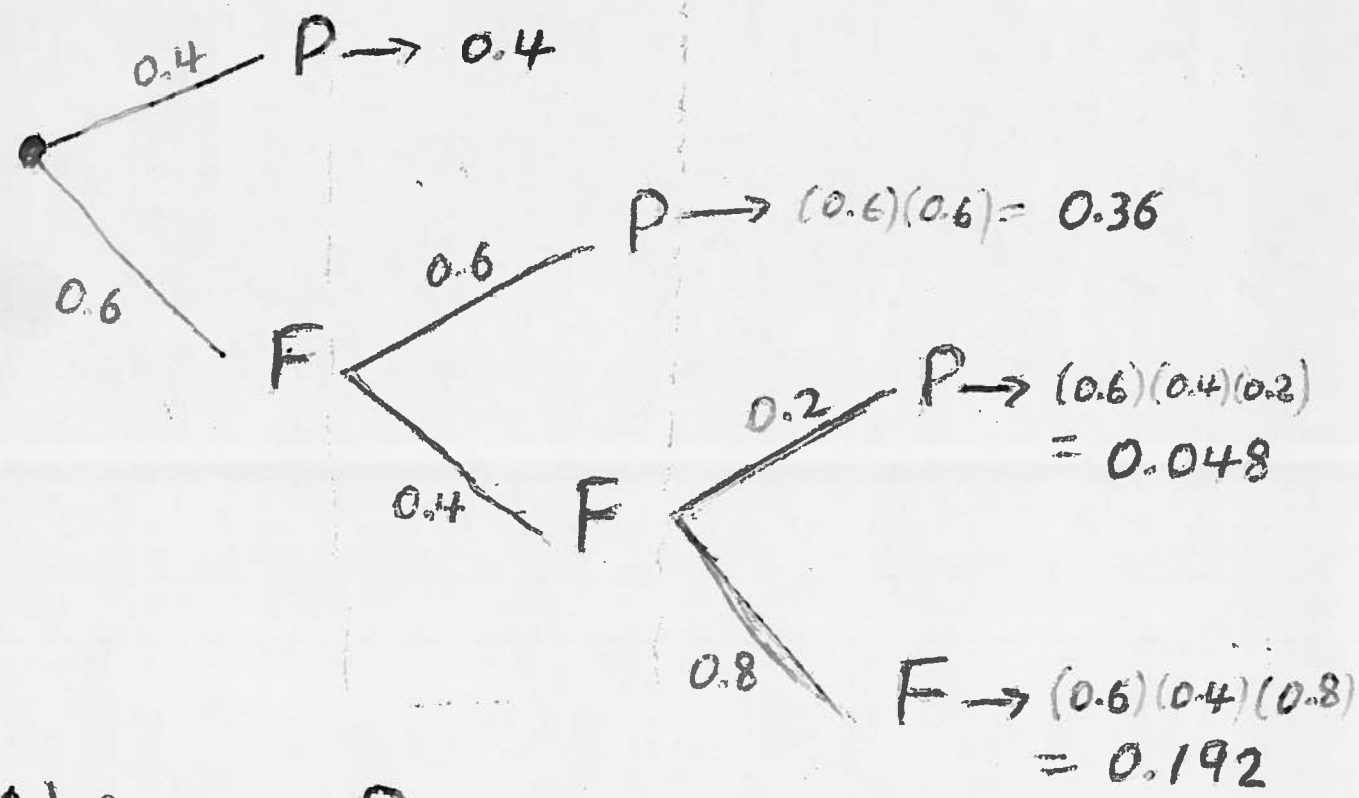
$$C_{6,2} \cdot C_{8,2} = (15)(28)$$
$$= \boxed{420}$$

MATH 208 DEC 2015

(10)

P = PASS F = FAIL

1ST TRIAL | 2ND TRIAL | 3RD TRIAL



A) PROB. of P ON 1ST OR 2ND TRIAL

$0.4 + 0.36 =$ **0.76**

B) PROB of F ON ALL 3 ATTEMPTS

$P(F,F,F) =$ **0.192**

C) PROB of F ON 1ST TWO TRIALS and P ON THIRD

$P(F,F,P) =$ **0.048**