MAT1320A

Practise sheet for the final exam

These are in addition to the practice sheets for the two midterms.

1. Compute the following limits.

1. $\lim_{x\to 4} \frac{\ln(x)-\ln(4)}{x-4}$
2. $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$
3. $\lim_{x\to -\infty} \frac{2x^2+x-1}{x^2+x-2}$
4. $\lim_{x\to -\infty} \frac{2x^3+x-1}{x^2+x-2}$
5. $\lim_{x\to \infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$
6. $\lim_{x\to \infty} \frac{e^{u/10}}{u^3}$
7. $\lim_{x\to \infty} x^3 e^{-x^2}$
8. $\lim_{x\to \infty} (x - \ln(x))$
9. $\lim_{x\to 0} \frac{e^x-e^{-x}-2x}{x-\sin(x)}$
10. $\lim_{x\to 1^+} x^{1/(1-x)}$
11. $\lim_{x\to \infty} x^{1/x}$
12. $\lim_{x\to 0^+} (4x+1)^{\cot(x)}$

2. Find the first and second derivatives of the following functions.

1. $f(x) = x^3 + 3x + 1$
2. $y(t) = \sqrt{3t^2 + \cos(t)}$
3. $f(x) = \frac{\sqrt{3x+1}}{6x^2+1}$
4. $f(x) = \sin(3x^2 e^x)$
5. $f(x) = \frac{\sin(2x)+1}{\cos(4x)}$
6. $f(y) = y^{500} \cdot e^{y^2}$
7. $f(x) = e^{\cos(5x^2)}$
8. $f(t) = \sqrt[4]{\arctan(t^3) \cdot \sin(t)}$
9. $u(y) = \ln(y + e^y)$
10. $s(v) = \cos(v^3 + \tan(v))$
11. $p(x) = \sec(x) \cdot e^{x^3 + 4\cos(x)}$

3. Find the derivative of the following functions:

1. $\int_{3x}^{0} \frac{u^2-1}{u^2+1}$
2. $\int_{\sqrt{x}}^{2x} \arctan(t) dt$
3. $\int_{2}^{t^2} \sqrt{4x+7} dx$
4. $\int_{\sin(t)}^{t^2} \frac{x+3}{\cos(x)}$

4. Differentiate the following and find the equations of the tangent lines at the given points.

1. $x^2-4xy+y^2=4$, tangent line at $(1,1)$
2. $x^y+y=x$, tangent line at $(1,0)$
3. $x\cdot e^y=x-y$
4. $2x^2+xy-y^2=2$, tangent line at $(1,1)$
5. $\arctan(x^2y)=x+xy^2$

5A. Approximate Integration

Set up the Riemann Sums for the following areas. Do the same with the Trapezoidal rule for the given number of steps and Simpson's rule. Recall that the number of approximation values must be even in that case. Double the given value if needed.

1. $f(x)=\frac{2x}{2x^2+1}$ from 1 to 3, take 4 steps.
2. $f(x)=x^2+\sqrt{1+2x}$, $4\le x\le 7$, take 3 steps.
3. $f(x)=\sqrt{sin(x)}$, $-\pi/2\le x\le\pi/2$, take 4 steps.

5B. Also: integrate functions given numerically : Ch 7.7 #29–40.

6. Theory of integration : Ch 5.2 #41–58.

7A. Compute the following integrals:

1. $\int_1^9\sqrt{x}dx$
2. $\int\frac{1+x}{1+x^2}dx$
3. $\int\frac{z^2}{z^3+1}dz$
4. $\int\frac{x}{1+x^4}dx$
5. $\int(x-1)e^{(x-1)^2}dx$
6. $\int x^2\sqrt{2+x}dx$
7. $\int\frac{\sin(2x)}{1+\cos^2(x)}dx$
8. $\int\sec^2(\theta)\cdot\tan^3(\theta)d\theta$
9. $\int\frac{e^u}{(1-e^u)^2}du$
10. $\int\frac{\sin(x)}{1+\cos^2(x)}dx$
11. $\int_0^{\pi/6}\frac{\sin(t)}{\cos^2(t)}dt$
12. $\int\cos^3(\theta)\sin(\theta)d\theta$
13. $\int 5^t\cdot\sin(5^t)dt$
14. $\int\cos(1+5t)dt$
15. $\int_e^{e^4}\frac{dx}{x\cdot\sqrt{\ln(x)}}dx$
16. $\int_0^1\frac{dx}{(1+\sqrt{x})^4}dx$
17. $\int_0^1 x\cdot e^{-x^2}dx$

7B. More integrals.

1. $\int x \cdot \cos(5x)dx$
2. $\int x^2 e^{-x}dx$
3. $\int (\ln(x))^2 dx$
4. $\int t^4 \cdot \ln(t)dt$
5. $\int \ln(\sqrt{x})dx$
6. $\int_0^{2\pi} x^2 \cdot \sin(x)dx$
7. $\int_1^5 \frac{M}{e^M}dM$
8. $\int_1^2 x^4 \cdot (\ln(x))^2 dx$
9. $\int e^{\cos(t)} \cdot \sin(2t)dt$
10. $\int x \cdot \ln(1+x)dx$
11. $\int_e^{e^2} \frac{5(\ln(x))^{1/5}}{x}dx$
12. $\int \frac{x^3}{\sqrt{x^2+4}}dx$
13. $\int \frac{x^3}{\sqrt{x^2-4}}dx$
14. $\int \frac{4+x}{(1+2x)(3-x)}dx$
15. $\int \frac{x^3+1}{x^3-3x^2+2x}dx$
16. $\int \frac{4y^2-7y-12}{y(y+2)(y-3)}dy$
17. $\int \frac{4+x}{(1+2x)(3-x)}dx$
18. $\int \frac{4x}{x^3+x^2+x+1}dx$
19. $\int \frac{x^3+6x-2}{x^4+6x^2}dx$

7C. And more integrals.

1. $\int_0^8 \sin(x)dx$
2. $\int_1^9 \sqrt{x}dx$
3. $\int_0^1 (x^e + e^x)dx$
4. $\int_1^{\sqrt{x}} \frac{z^2}{z^4+1}dz$
5. $\int_1^8 x^{-2/3}dx$
6. $\int_0^4 2^s ds$
7. $\int_1^{18} \sqrt{\frac{3}{z}}dz$
8. $\int_0^1 (1+r)^3 dr$
9. $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}dx$
10. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}dx$

8. (Curve Sketching) Sketch the following curves, using the guidelines discussed in class and in the textbook Chapter 4.5. Identify all asymptotes, limits, points of interest.

1. $y = \frac{x}{\sqrt{x^2+1}}$
2. $y = x - 3x^{1/3}$
3. $y = \sin^3(x)$
4. $y = x + \cos(x)$
5. $y = xe^{1/x}$
6. $y = \frac{x^2}{\ln(x)}$

9. Sketch curves from clues: Ch 4.3 #2–34

10. (Optimisation)

Exercises from Chapter 4.7: 2-18

11. (Newton's method)

Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation.

1. $2x^3 - 3x^2 + 2 = 0, \quad x_1 = -1$
2. $\frac{2}{x} - x^2 + 1 = 0, \quad x_1 = 2$
3. $x^7 + 4 = 0, \quad x_1 = -1$