CONCORDIA UNIVERSITY
Department of Economics

ECON 221/4 SECTIONS G, E and FF
STATISTICAL METHODS I
WINTER 2017 – ASSIGNMENT 1
Due: Monday, February 27, before 3:00 pm

Name: **I.D:** **Section:**

Points Total: 100 points

1. **(2 points)** If there are six vacant parking places and six automobiles arrive at the same time, in how many different ways they can park?

2. **(2 points)** A gumball machine has six red gumballs, four blue gumballs, and three yellow gumballs. If you buy three gumballs, what is the probability that you get three different colors?

3. **(10 points)** A general contractor has submitted two bids for two projects; *A* and *B*. The probability of getting project *A* is 0.60. The probability of getting project *B* is 0.75. The probability of getting at least one of the projects is 0.85.
   a. **(2 points)** What is the probability that the contractor will get both projects?

   b. **(2 points)** Are the events of getting the two projects mutually exclusive? Explain using probabilities.

c. **(2 points)** Are the events of getting the two projects independent? Explain using probabilities.

d. **(2 points)** If the contractor gets project *A*, what is the probability that he will get project *B*?

e. **(2 points)** If the contractor gets project *B*, what is the probability that he will get project *A*?

4. **(4 points)** A company wishes to evaluate the effectiveness of a marketing campaign. Seventy five percent of all potential professors were reached in a focused advertising program. Twenty eight percent of those contacted adopted the book while 8% of the adoptions came from professors who did not receive the promotional material. Define the following events of interest:

A_1 = Professor received advertising material

A_2 = Professor did not receive advertising material

B_1 = Professor adopts the book

B_2 = Professor does not adopt the book

a. **(2 points)** What is the probability that a professor who adopts the book received the advertising material?

b. **(2 points)** What is the probability that a professor who received advertising material does not adopt the book?

5. **(4 points)** The following table lists the relative frequency distribution of the number of calls coming into a call center each hour. Each call takes five minutes to process. What is the mean and standard deviation of the number of minutes the operators are answering questions each hour?

x	2	3	4	5	6	7
$P(x)$	0.07	0.11	0.23	0.31	0.18	0.1

   a. **(2 points)** What is the mean of the number of minutes the operators are answering questions each hour?

   b. **(2 points)** What is the standard deviation of the number of minutes the operators are answering questions each hour?

6. **(8 points)** In basketball, one of the possible penalties after a foul is called a free throw. The fouled player gets one shot at the basket from the foul line. If he misses this shot, he is awarded no points and the penalty is over. If he makes the shot, then he gets one point and gets to take another shot. If he makes the second shot he gets an additional point, and the ball is turned over to the other team. If he misses the second shot, the penalty is over. Suppose that the probability of making a shot is 80%, and the likelihood of making one shot is independent of the outcome of any other shot.
   a. **(6 points)** In a table, show the probability distribution function for the points from the penalty, (Hint: attach probabilities to each outcome).

b. **(2 points)** On average, how many points would you expect to get from the penalty?

7. **(8 points)** In a recent survey of 300 teenagers, 62% of the teenagers indicated that they had been to a movie within the past month. 75% of those teenagers who had seen a movie also had gone out to dinner in the past month, while only 64% of the teenagers who had not seen a movie had been out to dinner in the past month. Define the random variables as follows:
   $X = 1$ if teenager had been to movie; $X = 0$ otherwise
   $Y = 1$ if teenager had been out to dinner; $Y = 0$ otherwise

   a. **(4 points)** Construct a joint probabilities table of X and Y (joint probability function).

   b. **(2 points)** Find the conditional probability function of X, given $Y = 1$.

   c. **(2 points)** Find and interpret the covariance between X and Y.

8. **(4 points)** A bank grants mortgages to 87% of all applicants. After the applicant gets approval, the bank sends an appraiser to evaluate the value of the property. The bank pays an appraiser a salary of $2000 a month plus $200 for each appraisal. Assume the bank gets 10 loan applications next month.
   a. **(2 points)** What is the variance of the amount of money the bank will have to pay its appraiser?

b. **(2 points)** What is the expected value of the amount of money the bank will have to pay its appraiser?

9. **(4 points)** As a purchasing agent, you are responsible for selecting sources of supply for manufactured components to use in your firm's production process. The salesman for a certain supplier has indicated that they can supply an electronic sub-assembly that has a defect rate of 1.1%–well below your current supplier's defect rate. You accept 100 sub-assemblies for evaluation, and find that there were four defects.
   a. **(2 points)** Using the Poisson approximation to the binomial, how likely is it to get four or less defects out of 100?

   b. **(2 points)** Using the Poisson approximation to the binomial, how likely is it to get exactly four defects out of 100?

10. **(8 points)** The quality of computer disks is measured by sending the disks through a certifier which counts the number of missing pulses. A certain brand of computer disks averages 0.1 missing pulse per disk. Let the random variable X denote the number of missing pulses.
    a. **(2 points)** What is the distribution of X, and what is the mean of X?

    b. **(2 points)** Find the probability the next inspected disk will have no missing pulse.

c. **(2 points)** Find the probability the next disk inspected will have more than one missing pulse.

d. **(2 points)** Find the probability neither of the next two disks inspected will contain any missing pulse.

11. **(16 points)** The joint probability distribution of variables X and Y is shown in the table below, where X is the number of tennis racquets and Y is the number of golf clubs sold daily in a small sports store.

Y/X	1	2	3
1	*0.30*	*0.18*	*0.12*
2	*0.15*	*0.09*	*0.06*
3	*0.05*	*0.03*	*0.02*

a. **(2 points)** Calculate $E(XY)$.

b. **(4 points)** Determine the marginal probability distributions of X and of Y (two tables).

c. **(2 points)** Are X and Y independent? Explain.

d. **(2 points)** Calculate the conditional probability $P(Y = 2 \mid X = 1)$.

e. **(2 points)** Calculate the expected values of X and Y.

f. **(2 points)** Calculate the variances of X and Y.

g. **(2 points)** Calculate $\text{Cov}(X,Y)$. Did you expect this answer? Why?

12. **(6 points)** The amount of time you have to wait at a dentist's office before you are called in is uniformly distributed between zero and twenty minutes.
    a. **(2 points)** What is the probability that you have to wait more than 8 minutes?

    b. **(2 points)** What is the probability that you have to wait between 10 and 15 minutes?

    c. **(2 points)** Seventy percent of the time, you will be called in before you have to wait how long?

13. **(10 points)** Let the random variable X follow a normal distribution with mean $\mu = 48$ and variance $\sigma^2 = 60.84$.
    a. **(2 points)** Find the probability that X is greater than 58.

    b. **(2 points)** Find the probability that X is greater than 36 and less than 60.

c. **(2 points)** Find the probability that X is less than 52.

d. **(2 points)** The probability is 0.2 that X is greater than what number?

e. **(2 points)** The probability is approximately 0.05 that X is in the symmetric interval about the mean between which two numbers?

14. **(2 points)** As manager of a pizza shop, you are responsible for placing the food orders. You currently have enough anchovies for 8 pizzas. You expect to have orders for 60 pizzas tonight. If 8% of all pizzas are ordered with anchovies, what is the probability that you run out of anchovies before the evening is over? Use the normal approximation for the binomial.

15. **(2 points)** A wire-spinning machine will spin, on average, 12.3 miles before needing maintenance. Assume the time between maintenance is exponentially distributed. What is the probability that a spinning machine just placed back in service will need maintenance before it produces 4 miles of wire?

16. **(4 points)** A random variable X is normally distributed with mean of 50 and variance of 50, and a random variable Y is normally distributed with mean of 100 and variance of 200.

    a. **(2 points)** Given the random variables X and Y have a correlation coefficient equal to 0.50, find the mean and variance of the random variable W = 4X+3Y.

    b. **(2 points)** Given the random variables X and Y have a correlation coefficient equal to 0.50, find the mean and variance of the random variable W = 4X - 3Y.

17. **(6 points)** A sample of 33 students was asked to rate themselves on whether they were outgoing or not using this five point scale: 1 = extremely extroverted, 2 = extroverted, 3 = neither extroverted nor introverted, 4 = introverted, or 5 = extremely introverted. The results are shown in the table below:

Rating x_i	1	2	3	4	5
Frequency f_i	1	7	20	5	0

    a. **(2 points)** Calculate the sample mean.

    b. **(2 points)** Calculate the median.

    c. **(2 points)** Calculate the sample standard deviation.