COMP 1805
Feb 17, 2017

Assignment 2

1. a)

Let R represent “It is raining”
Let U represent “I bring my umbrella to work”

1.) $R \; -> \; U$
2.) U $\qquad Conclusion \; = \; R$

Both premises are correct as, 1 case is where I bring my umbrella to work and it is raining, which would be true for U. This means that R-> U is true as well if the antecedent is false. Therefore the conclusion is false, so argument is invalid.

b) Let P(x) be “x has a pc”
Let G(x) be “x plays games”
Let C(x) be “x is taking 1501”

1.) $\forall x \, P(x) \; -> \; G(x)$
2.) $\forall x \, C(x) \; -> \; G(x) \qquad Conclusion \; = \; \forall x C(x) \; -> \; P(x)$

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3.) $P(x) \; -> \; G(x) \qquad Universal\ Instantiation\ 1)$
4.) $C(x) \; -> \; G(x) \qquad Universal\ Instantiation\ 2)$

The case where I do not have a PC but I do play computer games and taking 1501. 1) would be true, since antecedent is false and 2) would be true because both antecedent and consequent are true. The conclusion would be false because antecedent is true and consequent is false. Arguments are valid but conclusion is false, so the argument is invalid.

2. Prove that $\sqrt{4} + \sqrt{5}$ is irrational

Proof by Contradiction
Assume that $\sqrt{4} + \sqrt{5}$ is rational

$2 + \sqrt{5} = \; rational\ number$ by Math
$\sqrt{5} = \; rational\ number \; - \; 2$ by Math
$\sqrt{5} = \frac{a}{b}$ by definition, where $\frac{a}{b}$ are in lowest terms and b cannot be 0
$5 \; = \; \frac{a^2}{b^2}$ by Math

$5b^2 = a^2$ by Math
$2(\frac{5}{2}b^2) = a^2$ by Math
$2k = a^2$ by letting $\frac{5}{2}b^2$ be some integer k

Lemma 1

If a^2 is even $\rightarrow a$ is even as well

Proof by contradiction

$a = 2r+1$ by definition of odd numbers, where r is some integer
$a^2 = (2r+1)^2$ by math
$a^2 = 4r^2+4r+1$ by Math
$a^2 = 2(2r^2+2r)+1$ by Math
$a^2 = 2s+1$ by Math, where $2r^2+2r$ is some integer

Therefore since a is odd than a^2 is odd as well, therefore if a^2 is even than a is even as well
Return to original proof

$5b^2 = a^2$ where a is 2z, z being some integer
$5b^2 = (2z)^2$ by Math
$5b^2 = 4z^2$ by Math

Lemma 2

If y is odd ∧ xy is even → x is even
¬(y is odd ∧ xy is even) ∨ x is even by Implication Equivalence
¬ y is odd ∨ ¬ xy is even ∨ x is even by DeMorgan's law
Y is even ∨ xy is odd ∨ x is even by Definition
Proof by contradiction
¬(y is even ∨ xy is odd ∨ x is even) by Negation
¬((y is even ∨ xy is odd) ∨ x is even) by Associativity
¬ (y is even ∨ xy is odd) ∧ x is even by DeMorgan's Law
¬ y is even ∧ ¬ xy is odd ∧ x is even by DeMorgan's law
(y is odd ∧ xy is even) ∧ x is odd
E is odd ∧ (y is odd ∧ xy is even) by Commutativity
(e is odd ∧ y is odd) ∧ xy is even by Associativity
(y is odd ∧ x is odd) ∧ xy is even by Commutativity
Assume y is odd ∧ xy is even ∧ x is even
Y is odd ∧ x is odd by Simplification

Y = 2t + 1 ∧ X = 2w + 1	by definition of odd numbers
Xy = (2t+1)(2w+1)	by math
Xy = 4tw + 2t + 2w + 1	by Math
Xy = 2(2tw + t + w) + 1	by Math
Xy = 2(i) + 1	By math, Let 2tw + t + w be some integer i
Therefore xy is odd	By definition
There xy is even	By simplification of y is odd ∧ xy is even ∧ x is even
Xy is odd ∧ xy is even	By conjunction
Let n become xy is odd	
N ∧ ¬ n	
F	By negation

Return to Original Proof

$5b^2 = 2a^2$

b^2 is Even	by Lemma 2
b is Even	by Lemma 1

Therefore a and b are both even numbers, so they are not in lowest forms.
Since they are not in lowest terms $\sqrt{4} + \sqrt{5}$ are not rational numbers.
Since $\sqrt{4} + \sqrt{5}$ are not rational numbers, they are not irrational numbers

3. Prove, by indirect proof, that if n is an integer and n^5 +7 is odd, then n is even. Show all your work.

1.	n ∈R ∨ $n^5 + 7$ is even,then n is odd	Contrapositive
2.	Assume n is odd:n=2k+1 ,k ∈Z	Assumption
3.	$n^5 + 7$	
4.	$(2k+1)^5 + 7$	Odd numbers =2k+1 Substitution
5.	$k^5 + 4$	By Math
6.	$-4 = k^5$	By Math
7.	$\sqrt[5]{-4} = k$	By Math
8.	$\frac{(n-1)}{2} = \sqrt[5]{-4}$	n=2k+1,so k= (n-1)/2 Substitution
9.	$n = 2\sqrt[5]{-4} + 1$	By Math

Let x represent $\sqrt[5]{-4}$

10. $n = 2x + 1$

∴N is an odd number,so it cannot be an even number and thus in the original statement the antecedent is false making the statement true.

4. (Question is in Assignment Doc)

They are missing the base case, the proof went straight to the inductive hypothesis. By definition they are not following proof by induction, therefore the proof is invalid.

5. For integer x, such that -2≤x≤2, prove that y<0, where y=x4 -4x2 -9x-36.

$$(x \geq -2 \ \wedge x \ \leq 2) \ \rightarrow x^4 - 4x^2 - 9x - 36 \ < 0$$

By Exhaustive Proof

Case 1: x=-2

$y \ = \ (-2)^4 - 4(-2)^2 - 9(-2) - 36$

= 16-16+18-36

$= \ -18$

∴ $(x \geq -2 \ \wedge x \ \leq 2) \ \rightarrow x^4 - 4x^2 - 9x - 36 \ < 0$

$T \ \vee T \ \rightarrow \ T$

Hence, T

Case 2: x=-1

$y \ = \ (-1)^4 - 4(-1)^2 - 9(-1) - 36$

= 1-4+9-36

$= \ -30$

∴ $(x \geq -2 \ \wedge x \ \leq 2) \ \rightarrow x^4 - 4x^2 - 9x - 36 \ < 0$

$T \ \vee T \ \rightarrow \ T$

Hence, T

Case 3: x= 0

$y \ = \ (0)^4 - 4(0)^2 - 9(0) - 36$

= -36

∴ $(x \geq -2 \ \wedge x \ \leq 2) \ \rightarrow x^4 - 4x^2 - 9x - 36 \ < 0$

$T \vee T \rightarrow T$

Case 4: x= 1

$y = (1)^4 - 4(1)^2 - 9(1) - 36$

= 1-4-9-36

$= -48$

$\therefore$ $(x \geq -2 \wedge x \leq 2) \rightarrow x^4 - 4x^2 - 9x - 36 < 0$

$T \vee T \rightarrow T$

Case 5: x= 2

$y = (-2)^4 - 4(-2)^2 - 9(-2) - 36$

= 16-16+18-36

$= -18$

$\therefore$ $(x \geq -2 \wedge x \leq 2) \rightarrow x^4 - 4x^2 - 9x - 36 < 0$

$T \vee T \rightarrow T$

Every case has been proven. $\therefore$ $(x \geq -2 \wedge x \leq 2) \rightarrow x^4 - 4x^2 - 9x - 36 < 0$ is True.

6.

Prove that

1 + 3 + 5 + ... + (2n-1) = n^2 for any integer n ≥1 (6.1)

Proof (strong induction):

1. For n = 1 (6.1) is true, since 1 = 1^2
2. Suppose (6.1) is true for some n = k ≥ 1, that is:

   1 + 3 + 5 + ⋯ + (2k - 1) = k^2
3. From (6.1) it is true for n = k + 1, that is:

   = 1 + 3 + 5 + ⋯ + (2k - 1) + (2k + 1) = $(k+1)^2$

   = 1 + 3 + 5 + ⋯ + (2k - 1) + (2k + 1) = k^2 + (2k + 1) = $(k+1)^2$

7. **Bonus** question

//I tried didn’t get anywhere close.

8. What is a power set of {0, 3, 9} ?

Let A represent {0,3,9}
Subsets of A:
Ø
{0}
{3}
{9}
{0,3}
{0,9}
{3,9}
{0,3,9}
∴ the 𝓟(A)= {Ø , {0} , {3} , {9} , {0,3} , {0,9} , {3,9} , {0,3,9} }

9.

a) {1,2,3,8,10,11,12,13,18}
b) Ø

10.

S = {1, 5, {1,5}, {apples, bananas}, {5,6,7,8,9,10,11,12,13,14,15}} and
T = {{1, 5, 10}, {5}, 1, {fruits}, cats, {dogs}}

a) S has a cardinality of 5, as its 5 elements are:

1. 1
2. 5
3. {1,5}
4. {apples, bananas}
5. {5,6,7,8,9,10,11,12,13,14,15,16}

T has a cardinality of 6, as its 6 elements are:

1. {1,5,10}
2. {5}
3. 1
4. {fruits}
5. Cats
6. {dogs}

Therefore T has a larger cardinality

b) $S \cap T = \{1\}$ **----------**

c) $S \cup T =$ {1, 5, {1,5}, {apples, bananas}, {5,6,7,8,9,10,11,12,13,14,15}, {1, 5, 10}, {5}, {fruits}, cats, {dogs}}

The cardinality of this set is 10.

11. Determine whether the following is valid, use membership tables

Membership table for $(A \cup B \cup C) - B - (C \cap A)$

A	B	C	$A \cup B$	$A \cup B \cup C$	$(A \cup B \cup C) - B$	$C \cap A$	$(A \cup B \cup C) - B - (C \cap A)$
1	1	1	1	1	0	1	**0**
1	1	0	1	1	0	0	**0**
1	0	1	1	1	1	1	**0**
1	0	0	1	1	1	0	**1**
0	1	1	1	1	0	0	**0**
0	1	0	1	1	0	0	**0**
0	0	1	0	1	1	0	**1**
0	0	0	0	0	0	0	**0**

Membership table for $(A - B) \cup (B - A)$

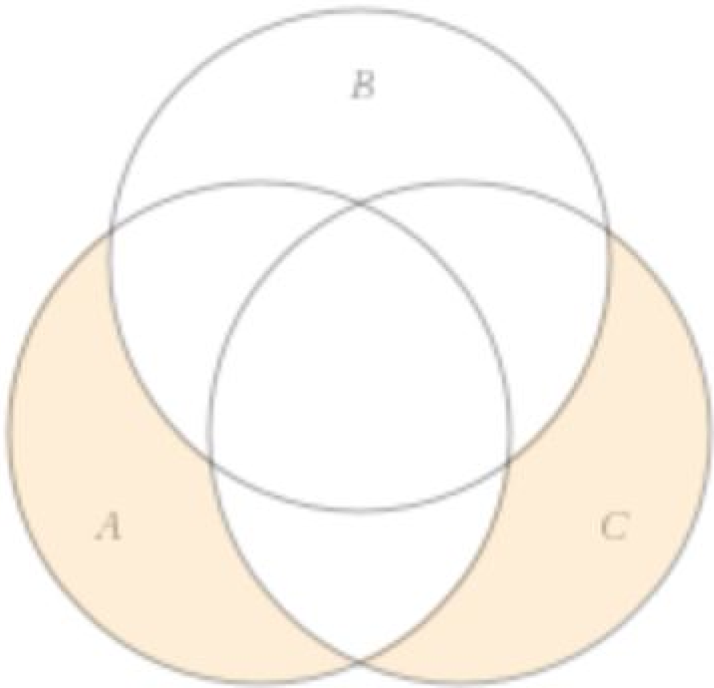
A	B	C	$(A - B)$	$(B - A)$	$(A - B) \cup (B - A)$
1	1	1	0	0	**0**
1	1	0	0	0	**0**

1	0	1	1	0	**1**
1	0	0	1	0	**1**
0	1	1	0	1	**1**
0	1	0	0	1	**1**
0	0	1	0	0	**0**
0	0	0	0	0	**0**

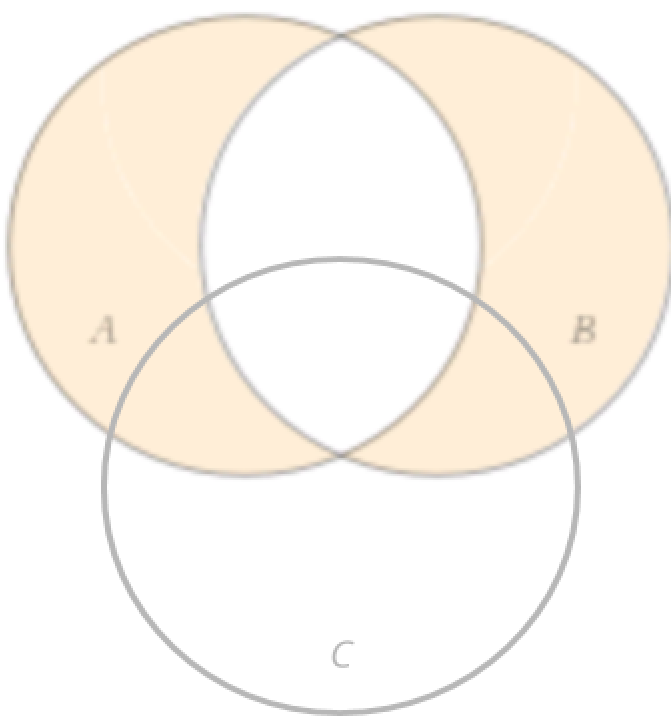
Therefore as shown by the final columns in the above membership table, $(A \cup B \cup C) - B - (C \cap A) = (A - B) \cup (B - A)$ is not a valid statement.

12.

From the first membership table for above the venn diagram for
$(A \cup B \cup C) - B - (C \cap A)$



From the second membership table venn diagram for $(A - B) \cup (B - A)$



13.

Student id: 101000120
B = {0,1,2}
C = {x | x modulo 2}
$A = B \cap C = \{0, 2\}$