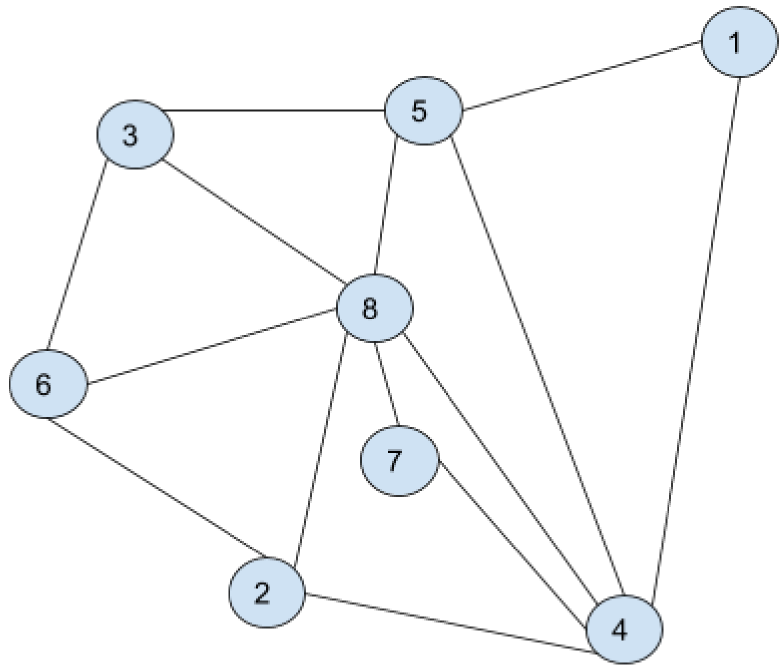
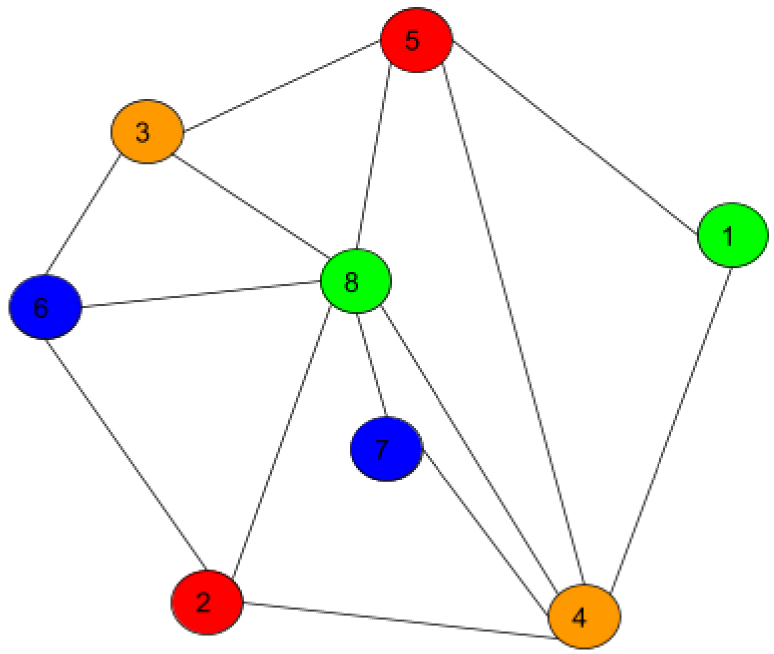
March 25, 2017

Assignment 4

1.



2.



3. Euler Cycle: 1,4,7,8,3,6,7,5,3,2,6,5,4,2,1
(1,4,7,5,4,2,3,5,6,7,8,3,6,2,1)

4. Hamiltonian Path: 1,4,5,6,7,8,3,2 (1,2,3,6,5,4,7,8)
Hamiltonian Cycle: 1,4,5,6,7,8,3,2,1 (1,2,3,8,7,6,5,4,1)

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5.

Let the numeric graph (1-6) be (V1, E1), where
V1 = {1,2,3,4,5,6} and E1 = { {1,2}, {2,3}, {3,4}, {4,5}, {5,6}, {6,1} }

Let the alphabetical graph (a-e) be (V2, E2), where:
V2 = {a,b,c,d,e,f,g} and E2 = { {a,c}, {a,d}, {b,e}, {b,f}, {c,e}, {d,f} }

The vertices of the alphabetical graph were renamed such that:

A was renamed 1
B was renamed 4
C was renamed 2
D was renamed 6
E was renamed 3
F was renamed 5

V2 becomes V2' where V2' = {1,2,3,4,5,6} and E2' = { {1,2}, {2,3}, {3,4}, {4,5}, {5,6}, {6,1} }

V2' = V1 ; E2' = E1
Therefore, the two graphs are isomorphic

6.

a) $(9n-5)^2 \; is \; \Theta(n^2)$

$81n^2 - 90n + 25 < 81n^2 + 90n + 25$

$\leq 81n^2 + 90n^2 + 25n^2 \qquad \forall n \geq 1$

$81n^2 + 90n^2 + 25n^2 = 196n^2$

$(9n-5)^2 < 196n^2$

$c = 196,\; k = 1 \quad f(x) \in O(g(x))$

$81n^2 - 90n + 25 = 41n^2 + (40n^2 - 90n + 25)$

$\geq 41n^2 \qquad \forall\; n > 1.925$ (a zero of $(40n^2 - 90n + 25)$)

$81n^2 - 90n + 25 \geq 41n^2$

$c = 41,\; k = 1.925 \quad f(x) \in \Omega(g(x))$

$f(x) \varepsilon O(g(x)) \wedge f(x) \in \Omega(g(x))$

$\therefore f(x) \in \Theta(g(x))$

b) $2n^2 + n - 6 \; is \; O(n^2)$

$2n^2 + n - 6 < 2n^2 + n + 6$

$\leq 2n^2 + n^2 + 6n^2 \qquad \forall\; n^2 \geq 1$

$2n^2 + n^2 + 6n^2 = 9n^2$

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$2n^2 + n - 6 \leq 9n^2$

$c = 9,\ k = 1$

$f(x)\ \in O(g(x))$

c) $\frac{10log(n+8)}{5}\ is\ O(n^2)$

$2log(n + 8) < 2(n + 8)$

$2(n + 8) = 2n + 16$

$\leq 2n^2 + 16n^2\ = 18n^2$

$\frac{10log(n+8)}{5} \leq 18n^2$

$c\ = 18,\ k = 1\ \ f(x) \in O(g(x))$

d) $\frac{1}{n} - 4\ is\ O(n)$

$\frac{1}{n} - 4 < n^{-1} + 4$

$\leq n + 4n\ = 5n \qquad \forall n \geq 1$

$\frac{1}{n} - 4 < 5n$

$c = 5, k = 1 \quad f(x) \in O(g(x))$

e) $6n^2\ is\ \Omega(n^2)$

$6n^2\ \geq 3n^2 \qquad \forall n\ \geq 1$

$c\ = 3,\ k = 1\ f(x) \in \Omega(g(x))$

f) $4n^2 - 3n\ is\ \Omega(1)$

$4n^2 - 3n\ \geq 4 - 3 = 1 \quad \forall n\ \geq 1$

$4n^2 - 3n \geq 1$

$c = 1, k = 1\ \ f(x) \in \Omega(g(x))$

7.

A linear search would be faster because you would find your element with 4 iterations of the search, since the searched item is in the first 4 elements . A binary search would take 5 iterations. The structure of size 32 must be divided by 2 five times to reach 1 (32,16,8,4,2,1).

8.

a)

In a linear search if the size of the list goes from n to 2n, the number of iterations for the worst case would double. The worst case of this algorithm is if the searched element is the last element in the structure.

b)

In a binary search if the size of the list goes from n to 2n, the number of iterations increases by one. There is no worst case because no matter what happens the element that you want to find in the list will always be found in the last iteration of the algorithm.

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9.

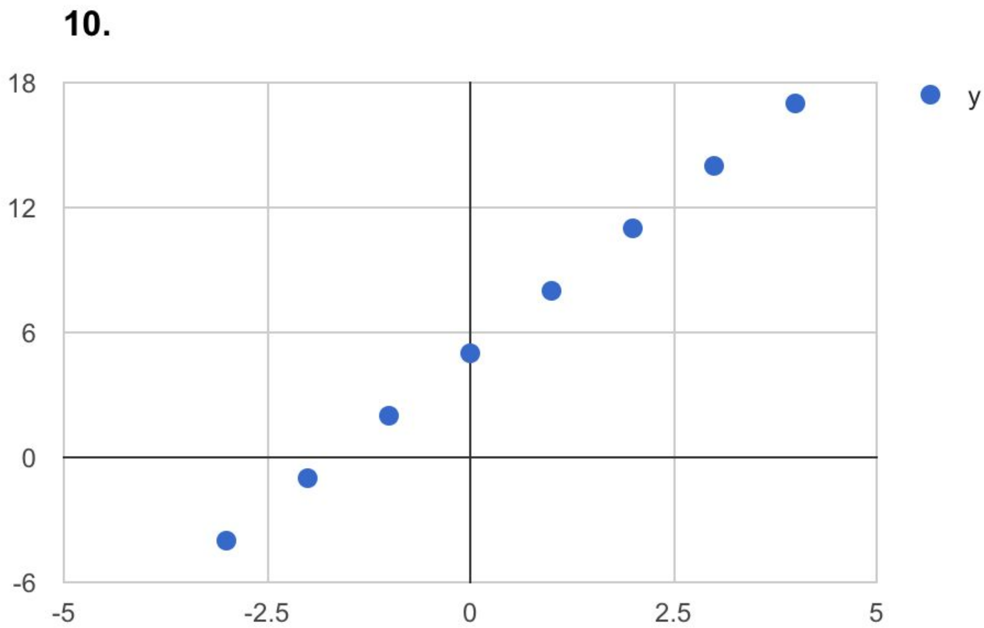
a)

In a bubble sort if the size of the list goes from n to 2n, the number of iterations to find the worst case would remain O(n^2)

b)

In a selection sort if the size of the list goes from n to 2n, the number of iterations to find the element you want to find the worst case would remain O(n^2)

10.



x	y
-3	-4
-2	-1
-1	2
0	5
1	8
2	11
3	14
4	17

11. $a = 0, b = 2$

a) $f \circ g = a((5a)^2 - 2b) + 5a$

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$= 0(5(0)^2 - 2(2)) + 5(0)$

$= 0$

b) $g \circ f = (5a)^2 - 2b$

$= (5(0)^2 - 2(2))$

$= -4$

c) $(f \circ g) \circ g = a((5a)^2 - 2b) + 5a$

$= 0(5(0)^2 - 2(2)) + 5(0)$

$= 0$