Assignment 1

Question 1

1. It is cold outside or I am Canadian, or both
2. It is not January and it is not cold outside
3. I am Canadian and if it is January then it is not cold outside
4. Either it is cold outside or I am Canadian, not both

Question 2

a)

- Let c be "I like computer Science"
- Let t be "I like teaching"
- Let e be "I like email"

$$c \land t \land \neg e$$

b)

- Let e be "It is early"
- Let c be "I need to drink coffee"

$$e \rightarrow c$$

c)

- Let b be "I was born in Canada"

$$b \oplus \neg b$$

d)

- Let a be "I am your instructor".
- Let b be "You are a Carleton University Student"

$$a \leftrightarrow b$$

Question 3

a) $(2 < 4) \land (4 < 6) \land (6 < 2)$
$\mathrm{T} \land (4 < 6) \land (6 < 2)$

$T \wedge F \wedge (6 < 2)$
$T \wedge F \wedge F$
$T \wedge F$ Idempotent
F Negation

b) $(1 = 2) \vee (2 > 1)$
$F \vee (2 > 1)$
$F \vee T$
T Negation

c) If 9 = 8 then your instructor is actually Nyarlathotep
$F \rightarrow$ your instructor is actually Nyarlathotep
$F \rightarrow F$
$T \vee F$ Conditional Equivalence
T Idempotent

d) If 8 =8 then either $8 = 2^3$ or $8 = 2^2$
$T \rightarrow (T \oplus F)$
$T \rightarrow T$
$F \vee T$ Conditional Equivalence
T Idempotent

Question 4

$(p \vee q) \leftrightarrow (q \vee \neg r) \equiv \neg[\neg(\neg p \wedge \neg q) \wedge (\neg q \wedge r)] \wedge [(\neg p \wedge \neg q) \rightarrow (\neg q \wedge r)]$

Question 5

$\neg[\neg(\neg p \wedge \neg q) \wedge (\neg q \wedge r)] \wedge [(\neg p \wedge \neg q) \rightarrow (\neg q \wedge r)]$

$\neg[\neg\neg(p \vee q) \wedge (\neg q \wedge r))] \wedge [(\neg p \wedge \neg q) \rightarrow (\neg q \wedge r)]$ De Morgan's Law

$\neg\neg[\neg(p \vee q) \vee (q \vee \neg r)] \wedge [(\neg p \wedge \neg q) \rightarrow (\neg q \wedge r)]$ De Morgan's Law

$\neg\neg[(p \vee q) \rightarrow (q \vee \neg r)] \wedge [(\neg p \wedge \neg q) \rightarrow (\neg q \wedge r)]$ Implication Equivalence

$[(p \vee q) \rightarrow (q \vee \neg r)] \wedge [(\neg p \wedge \neg q) \rightarrow (\neg q \wedge r)]$ Double Negation

$[(p \vee q) \rightarrow (q \vee \neg r)] \wedge [\neg(p \vee q) \rightarrow \neg(p \vee \neg r)]$ De Morgan's Law

$[(p \vee q) \rightarrow (q \vee \neg r)] \wedge [(p \vee q) \leftarrow (p \vee \neg r)]$ Contrapositive

$(p \vee q) \leftrightarrow (q \vee \neg r)$ Bi-conditional Equivalence

Question 6

Truth table for: $(p \vee q) \leftrightarrow (q \vee \neg r)$

p	q	r	$\neg r$	$p \vee q$	$q \vee \neg r$	$(p \vee q) \leftrightarrow (q \vee \neg r)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	F	T
F	F	F	T	F	T	0

Truth Table for $\neg[\neg(\neg p \wedge \neg q) \wedge (\neg q \wedge r)] \wedge [(\neg p \wedge \neg q) \rightarrow (\neg q \wedge r)]$

P	**Q**	**R**	$\neg \boldsymbol{P}$	$\neg \boldsymbol{Q}$	$(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q})$	$(\neg \boldsymbol{q} \wedge \boldsymbol{r})$	$\neg(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q})$	$\neg(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q}) \wedge (\neg \boldsymbol{q} \wedge \boldsymbol{r})$	$\neg[\neg(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q}) \wedge (\neg \boldsymbol{q} \wedge \boldsymbol{r})]$	$(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q}) \rightarrow (\neg \boldsymbol{q} \wedge \boldsymbol{r})$	$\neg[\neg(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q}) \wedge (\neg \boldsymbol{q} \wedge \boldsymbol{r})] \wedge [(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q}) \rightarrow (\neg \boldsymbol{q} \wedge \boldsymbol{r})]$
T	T	T	F	F	F	F	T	F	T	T	T
T	T	F	F	F	F	F	T	F	T	T	T
T	F	T	F	T	F	T	T	T	F	T	F
T	F	F	F	T	F	F	T	F	T	T	T
F	T	T	T	F	F	F	T	F	T	T	T
F	T	F	T	F	F	F	T	F	T	T	T
F	F	T	T	T	T	T	F	F	T	T	T
F	F	F	T	T	T	F	F	F	T	F	F

Question 7

a) Tautology – All final values are true

P	Q	$\neg\neg P$	$\neg Q$	$(\neg q \wedge q)$	$\neg(\neg q \wedge q)$	$P \wedge (\neg(\neg q \wedge q))$	$\neg(P \wedge (\neg(\neg q \wedge q)))$	$\neg\left(P \wedge (\neg(\neg q \wedge q))\right) \wedge \neg\neg P$	$\neg(\neg\left(P \wedge (\neg(\neg q \wedge q))\right) \wedge \neg\neg P)$
T	T	T	F	F	T	T	F	F	T
T	F	T	T	F	T	T	F	F	T
F	T	F	F	F	T	F	T	F	T
F	F	F	T	F	T	F	T	F	T

b) Contradiction – All final values are false

P	Q	R	P + R	$(P + R) \wedge R$	$Q \leftrightarrow ((P + R) \wedge R)$	$(P > P)$	$\neg(P > P)$	$Q \leftrightarrow \left((P + R) \wedge R\right) \wedge \neg(P > P)$
T	T	T	T	T	T	T	F	F
T	T	F	T	F	F	T	F	F
T	F	T	T	T	F	T	F	F
T	F	F	T	F	T	T	F	F
F	T	T	T	T	T	T	F	F
F	T	F	F	F	F	T	F	F
F	F	T	T	T	F	T	F	F
F	F	F	F	F	T	T	F	F

c) Contingency - Final values are not all true nor all false

P	Q	R	$\neg R$	$(P \wedge R)$	$(P \wedge R) \vee Q$	$(P \wedge \neg R)$	$((P \wedge R) \vee Q) \vee (P \wedge \neg R)$	$(R \vee P)$	$(((P \wedge R) \vee Q) \vee (P \wedge \neg R)) \wedge (R + P)$
T	T	T	F	T	T	F	T	T	T
T	T	F	T	F	T	T	T	T	T
T	F	T	F	T	T	F	T	T	T
T	F	F	T	F	F	T	T	T	T
F	T	T	F	F	T	F	T	T	T
F	T	F	T	F	T	F	T	F	F
F	F	T	F	F	F	F	F	T	F

F	F	F	T	F	F	F	F	F	F

Question 8

a)

$\neg(\neg(p \land \neg(\neg q \land q)) \land \neg\neg p)$

$\neg(\neg(p \land \neg(F)) \land \neg\neg p)$ Negation

$\neg(\neg(p \land T) \land \neg\neg p)$

$\neg(\neg p \land \neg\neg p)$ Idempotent

$\neg(\neg p \land p)$ Double Negation

$\neg(F)$ Negation

T

Therefore, it is a tautology

b)

$\left(q \leftrightarrow ((p \lor r) \land r)\right) \land \neg(p \to p)$

$\left(q \leftrightarrow ((p \lor r) \land r)\right) \land \neg(\neg p \land p)$ Implication Equivalence

$\left(q \leftrightarrow ((p \lor r) \land r)\right) \land \neg(T)$ Negation

$\left(q \leftrightarrow ((p \lor r) \land r)\right) \land F)$

F Absorption

Therefore, it is a contradiction

c)

$\left((p \land r) \lor q \lor (p \land \neg r)\right) \land (r \lor p)$

$\neg\neg\big((p \wedge r) \vee q \vee (p \wedge \neg r)\big) \wedge (r \vee p)$	Double negative
$\neg\big(\neg(p \wedge r) \wedge \neg q \wedge \neg(p \wedge \neg r)\big) \wedge (r \vee p)$	De Morgan's Law
$\neg\big((\neg p \vee \neg r) \wedge \neg q \wedge (\neg p \vee r)\big) \wedge (r \vee p)$	De Morgan's Law
$\neg\big((\neg p) \vee (\neg r \wedge r) \wedge \neg q\big) \wedge (r \vee p)$	Distributive Law
$\neg\big((\neg p) \vee (F) \wedge \neg q\big) \wedge (r \vee p)$	Negation
$(\neg(\neg p) \wedge \neg(F) \vee \neg\neg q) \wedge (r \vee p)$	De Morgan's Law
$(p \wedge \neg(F) \vee \neg\neg q) \wedge (r \vee p)$	Double Negation
$(p \wedge \neg(F) \vee q) \wedge (r \vee p)$	Double Negation
$(p \vee q) \wedge (r \vee p)$	Idempotent
$p \vee (r \wedge q)$	Distributive Law

No more applicable laws to simplify the equation. It is therefore a contingency.

Question 9

a) There exists an animal that is a penguin if and only if it eats squid.

b) There exists an animal that is a penguin and does not eat squid

c) For all animals, if it is a bird, then it is a penguin or it is not a bird

Question 10

a) Let a(x) be "x is aquatic"

Let f(x) be "x can fly"

Let p(x) be "x is a penguin"

The Universe of Discourse is all animals.

The statement "Every penguin is aquatic and flightless" can be written as:

$\forall x(p(x) \rightarrow (a(x) \wedge \neg f(x))$

Negating the statement gives the following: “NOT every penguin is aquatic and flightless” as follows:

$\neg\forall x(p(x) \rightarrow (a(x) \wedge \neg f(x))$

This can be rewritten as: “There exists a penguin that is not aquatic, and can fly”

This new statement can be written as:

$\exists x\ \neg(p(x) \rightarrow (a(x) \wedge \neg f(x))$

$\exists x\ \neg(\neg p(x) \vee (a(x) \wedge \neg f(x))$ Implication Equivalence

$\exists x\ (p(x) \wedge \neg(a(x) \wedge \neg f(x))$ De Morgan’s Law

$\exists x\ (p(x) \wedge (\neg a(x) \vee f(x))$ De Morgan’s Law

b) Let s(x) be “x eats squid”

Let f(x) be “x can fly”

Let p(x) be “x is a penguin”

The Universe of Discourse is all animals.

The statement “There is at least one penguin who doesn’t eat squid and can fly” can be written as:

$\exists x\ (p(x) \wedge \neg s(x) \wedge f(x)$

Negating the statement gives the following:

“There is not at least one penguin that doesn’t eat squid and can fly. ” This gives:

$\neg\exists x\ (p(x) \wedge \neg s(x) \wedge f(x))$

This new statement can be rewritten as the following:

“There are no penguins that don’t eat squid and fly”. This can be written as:

$\forall x\ \neg(p(x) \wedge \neg s(x) \wedge f(x))$

$\forall x\ (\neg p(x) \vee s(x) \vee f\neg(x))$ De Morgan’s Law

$\forall x\ (p(x) \rightarrow (s(x) \vee f\neg(x)))$ Implication Equivalence