Université d'Ottawa
Faculté de génie

Département de
génie civil

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Canada's university

University of Ottawa
Faculty of Engineering

Department of
Civil Engineering

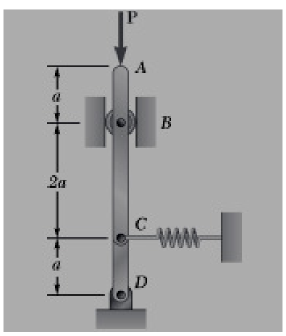
FINAL EXAM **DATE : 7^{TH} DECEMBER, 2010**
Instructor: Elena Dragomirescu **TIME : 14:00 TO 17:00 (180 MIN)**

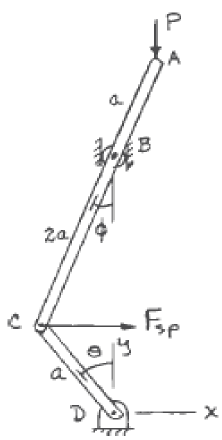
Closed book examination. Calculators are allowed (not programmable).

Solve Question 1, Question 2, Question 3 and Question 4.
Solve Question 5, OR Question 6, at your choice.

Question 1 (20 points)

Two bars are attached to a single spring of constant k = 7.0 N/m that is unstretched when the bars are vertical as shown in the figure below. Determine the range of values of force P for which the system reaches the stable position of equilibrium, knowing that a = 4.0 m. For small values of displacement consider the angle in D is two times the angle in B. Apply the principle of potential energy for particles.





From geometry:

$$x_C = -a\sin\theta = -2a\sin\phi$$

For small values of θ, ϕ

$$\theta = 2\phi$$

or $$\phi = \frac{1}{2}\theta$$

$$y_A = a\cos\theta + 3a\cos\phi$$

$$= a\left(\cos\theta + 3\cos\frac{\theta}{2}\right)$$

For spring:

$$s = x_C = -a\sin\theta$$

Potential Energy: $V = V_{SP} + V_P$

$$= \frac{1}{2}k(-a\sin\theta)^2 + Pa\left(\cos\theta + 3\cos\frac{\theta}{2}\right)$$

$$\frac{dV}{d\theta} = ka^2\cos\theta\sin\theta - Pa\left(\sin\theta + \frac{3}{2}\sin\frac{\theta}{2}\right)$$

$$\frac{d^2V}{d\theta^2} = ka^2\left(-\sin^2\theta + \cos^2\theta\right) - Pa\left(\cos\theta + \frac{3}{4}\cos\frac{\theta}{2}\right)$$

For stable equilibrium: $\frac{d^2V}{d\theta^2} > 0$

Then, with $\theta = 0$

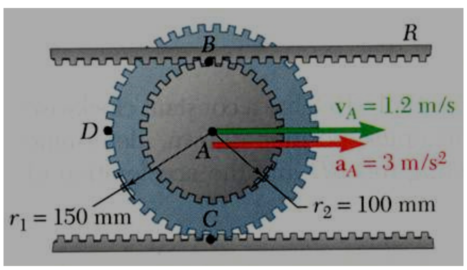
$$ka^2 - Pa\left(1 + \frac{3}{4}\right) > 0 \quad \text{or} \quad P < \frac{4}{7}ka$$

$$P < 16N$$

Question 2 (20 points)

The center of the double gear has a velocity and acceleration to the right of 1.2 m/s and 3 m/s2, respectively. The lower rack is stationary. Determine *(a)* the angular acceleration of the gear, and *(b)* the acceleration of points C, and *D*.

$$\frac{x_A}{2\pi r} = -\frac{\theta}{2\pi} \qquad x_A = -r_1\theta$$



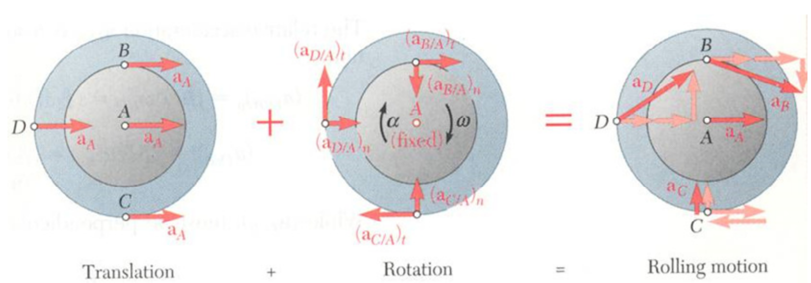
$$x_A = -r_1\theta$$
$$v_A = -r_1\dot{\theta} = -r_1\omega$$

$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\text{m/s}}{0.150\,\text{m}} = -8\,\text{rad/s}$$

$$a_A = -r_1\ddot{\theta} = -r_1\alpha$$

$$\alpha = -\frac{a_A}{r_1} = -\frac{3\,\text{m/s}^2}{0.150\,\text{m}}$$

$$\boxed{\vec{\alpha} = \alpha\vec{k} = -\left(20\,\text{rad/s}^2\right)\vec{k}}$$



$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A} = \vec{a}_A + \alpha\vec{k}\times\vec{r}_{C/A} - \omega^2\vec{r}_{C/A} = \left(3\,\text{m/s}^2\right)\vec{i} - \left(20\,\text{rad/s}^2\right)\vec{k}\times(-0.150\,\text{m})\vec{j} - (8\,\text{rad/s})^2(-0.150\,\text{m})\vec{j}$$
$$= \left(3\,\text{m/s}^2\right)\vec{i} - \left(3\,\text{m/s}^2\right)\vec{i} + \left(9.60\,\text{m/s}^2\right)\vec{j} \qquad \boxed{\vec{a}_c = \left(9.60\,\text{m/s}^2\right)\vec{j}}$$

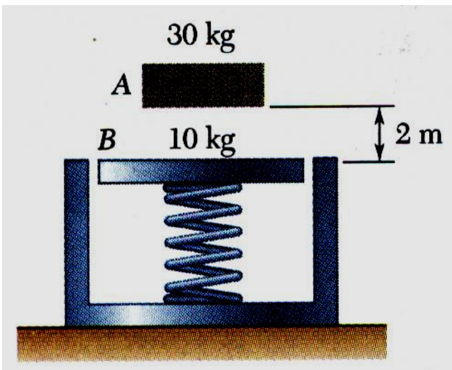
$$\vec{a}_D = \vec{a}_A + \vec{a}_{D/A} = \vec{a}_A + \alpha\vec{k}\times\vec{r}_{D/A} - \omega^2\vec{r}_{D/A} = \left(3\,\text{m/s}^2\right)\vec{i} - \left(20\,\text{rad/s}^2\right)\vec{k}\times(-0.150\,\text{m})\vec{i} - (8\,\text{rad/s})^2(-0.150\,\text{m})\vec{i}$$
$$= \left(3\,\text{m/s}^2\right)\vec{i} + \left(3\,\text{m/s}^2\right)\vec{j} + \left(9.60\,\text{m/s}^2\right)\vec{i}$$

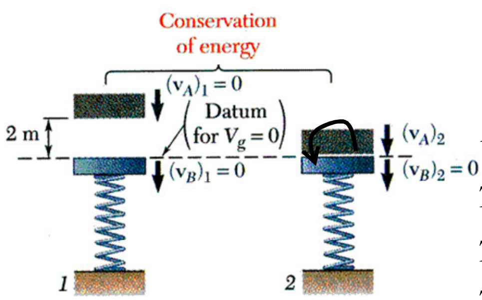
$$\boxed{\vec{a}_D = \left(12.6\,\text{m/}s^2\right)\vec{i} + \left(3\,\text{m/}s^2\right)\vec{j} \qquad a_D = 12.95\,\text{m/s}^2}$$

<u>Question 3 (20 points)</u>

A 30 kg block initially at rest is dropped from a height of 2 m, such that it will rotate counterclockwise, reaching an angular velocity ω = 1.5 rad/s before hitting the 10 kg pan of a spring scale.

Assuming the impact to be perfectly plastic, e = 0, and ignoring any rotation of the pan, determine the maximum deflection of the pan. The constant of the spring is k = 20 kN/m and the mass moment of inertia of the block is $\bar{I} = \frac{1}{6}mb^2$; where b = 1.5 m. Apply the principle of conservation of energy for the rigid bodies and for the particles and also the principle of conservation of linear momentum for particles.





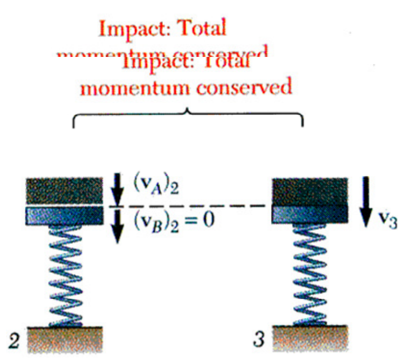
$$I = \frac{1}{6}(30)(1.5)^2 = 11.25$$

$$T_1 = 0 \qquad V_1 = W_A y = (30)(9.81)(2) = 588\ \text{J}$$

$$T_2 = \tfrac{1}{2}m_A(v_A)_2^2 = \tfrac{1}{2}(30)(v_A)_2^2 + \tfrac{1}{2}(11.25)(\omega_A)_2^2 \qquad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 588\ \text{J} = \tfrac{1}{2}(30)(v_A)_2^2 + \tfrac{1}{2}(11.25)(1.5)^2 + 0 \qquad (v_A)_2 = 6.19\ \text{m/s}$$

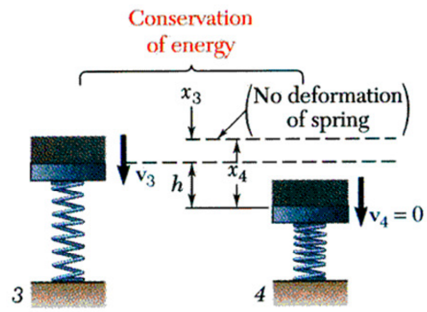


- Determine velocity after impact from requirement that total momentum of the block and pan is conserved.

$$m_A(v_A)_2 + m_B(v_B)_2 = (m_A v_A' + m_B v_B')$$

$$m_A(v_A)_2 + m_B(v_B)_2 = (m_A + m_B)v_3$$

$$(30)(6.19) + 0 = (30 + 10)v_3 \qquad v_3 = 4.64\ \text{m/s}$$



Initial spring deflection due to pan weight:

$$F = kx$$
$$F - W_B = 0$$
$$x_3 = \frac{W_B}{k} = \frac{(10)(9.81)}{20\times10^3} = 4.91\times10^{-3}\,\text{m}$$

- Apply the principle of conservation of energy to determine the maximum deflection of the spring.

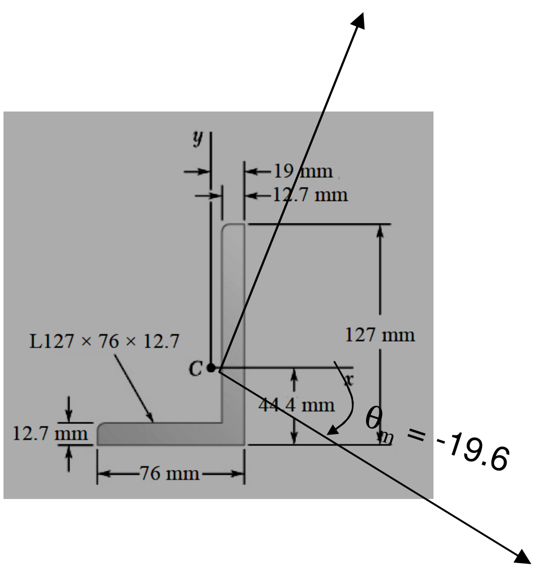
$$T_3 = \tfrac{1}{2}(m_A + m_B)v_3^2 = \tfrac{1}{2}(30+10)(4.64)^2 = 430.6\,\text{J}$$
$$V_3 = V_g + V_e$$
$$= 0 + \tfrac{1}{2}kx_3^2 = \tfrac{1}{2}(20\times10^3)(4.91\times10^{-3})^2 = 0.241\,\text{J}$$
$$T_4 = 0$$
$$V_4 = V_g + V_e = (W_A + W_B)(-h) + \tfrac{1}{2}kx_4^2$$
$$= -392(x_4 - x_3) + \tfrac{1}{2}(20\times10^3)x_4^2$$
$$= -392(x_4 - 4.91\times10^{-3}) + \tfrac{1}{2}(20\times10^3)x_4^2$$

$$T_3 + V_3 = T_4 + V_4$$
$$430.6 + 0.241 = 0 - 392(x_4 - 4.91\times10^{-3}) + \tfrac{1}{2}(20\times10^3)x_4^2$$
$$x_4 = 0.227\,\text{m}$$

$$h = x_4 - x_3 = 0.227\,\text{m} - 4.91\times10^{-3}\,\text{m} \qquad \boxed{h = 0.2227\,\text{m}}$$

<u>Question 4 (20 points)</u>

For the angle cross section shown below a) draw the Mohr circle and b) determine the values of the principal moments of inertia. c) Represent the orientation of the principal axes on the figure and indicate the corresponding angle. Consider: $\bar{I}_x = 3.93\times10^6 mm^4$**,** $\bar{I}_y = 1.06\times10^6 mm^4$ **and** $\bar{I}_{xy} = 1.17\times10^6 mm^4$

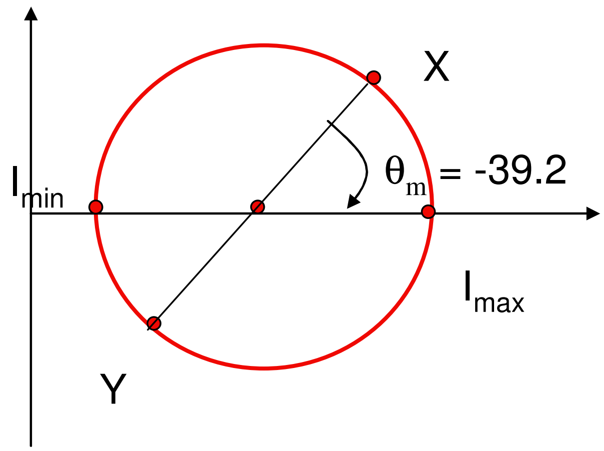


Now $$\bar{I}_{ave} = \frac{\bar{I}_x + \bar{I}_y}{2} = 2.495\times10^6\,\text{mm}^4$$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2}$$
$$= 1.85\times10^6\,\text{mm}^4$$

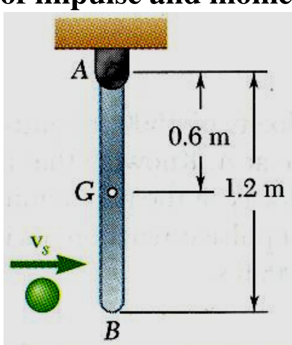
$$2\theta_m = \tan^{-1}\left[\frac{-2(1.17)}{3.93 - 1.06}\right] = -39.2°$$

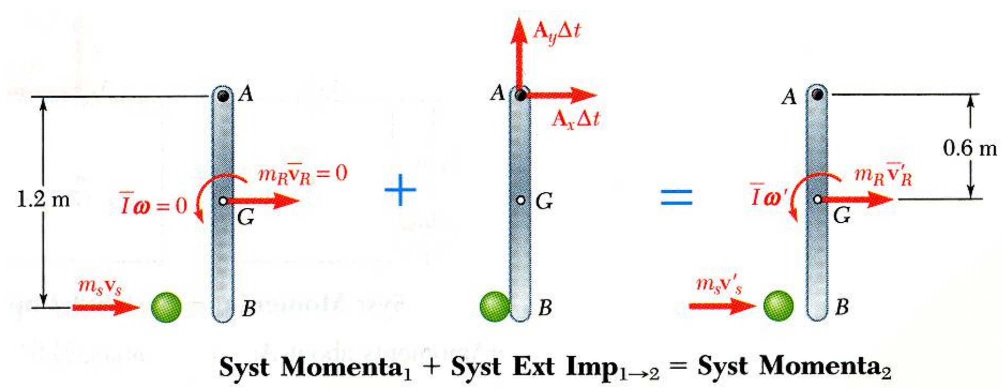
$$\bar{I}_{max} = R + I_{ave} = (1.85 + 2.495)\times10^6 mm^4 = 4.345\times10^6 mm^4$$
$$\bar{I}_{min} = I_{ave} - R = (2.495 - 1.85)\times10^6 mm^4 = 0.645\times10^6 mm^4$$



Question 5 (20 points)

A 2 kg sphere with a velocity of 5 m/s strikes the lower end of an 8 kg rod *AB* = *L*. The rod is hinged at *A* and it is initially at rest. The coefficient of restitution between the rod and sphere is e = 0.8. Determine the angular velocity of the rod and the velocity of the sphere immediately after impact. Consider the mass moment of inertia of the road $\bar{I} = \frac{1}{12}mL^2$. Draw the principle of impulse and momentum for rigid bodies and apply the impact of rigid bodies.





+↺ moments about A:

$$m_s v_s(1.2\,\text{m}) = m_s v'_s(1.2\,\text{m}) + m_R\bar{v}'_R(0.6\,\text{m}) + \bar{I}\omega'$$

$$\bar{v}'_R = \bar{r}\omega' = (0.6\,\text{m})\omega'$$

$$\bar{I} = \tfrac{1}{12}mL^2 = \tfrac{1}{12}(8\,\text{kg})(1.2\,\text{m})^2 = 0.96\,\text{kg}\cdot\text{m}^2$$

$$(2\,\text{kg})(5\,\text{m/s})(1.2\,\text{m}) = (2\,\text{kg})v'_s(1.2\,\text{m}) + (8\,\text{kg})(0.6\,\text{m})\omega'(0.6\,\text{m}) + (0.96\,\text{kg}\cdot\text{m}^2)\omega'$$

$$12 = 2.4v'_s + 3.84\omega'$$

+↺ Moments about A:

$$12 = 2.4v'_s + 3.84\omega'$$

$\overset{+}{\rightarrow}$ Relative velocities:

$$v'_B - v'_s = e(v_s - v_B)$$

$$(1.2\,\text{m})\omega' - v'_s = 0.8(5\,\text{m/s})$$

Solving,

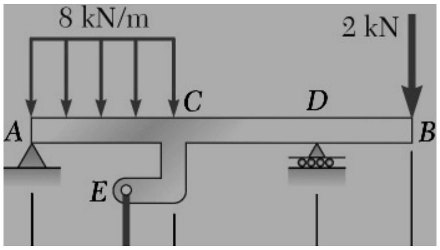
$\omega' = 3.21\,\text{rad/s}$ | $\omega' = 3.21\,\text{rad/s}$ ↺

$v'_s = -0.143\,\text{m/s}$ | $v'_s = 0.143\,\text{m/s} \leftarrow$

OR

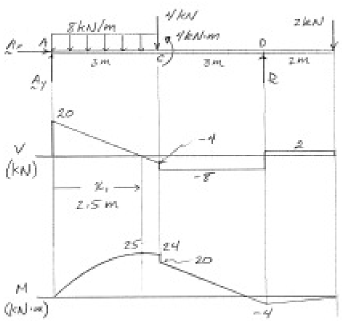
Question 6 (20 points)

Draw the shear and bending moment diagrams for the beam loaded as shown in the figure below.



SOLUTION

(*a*)
Replacing the load at *E* with equivalent force-couple at *C*:



$$\circlearrowleft \Sigma M_A = 0: (6\text{ m})D - (8\text{ m})(2\text{ kN}) - (3\text{ m})(4\text{ kN})$$
$$- (1.5\text{ m})(8\text{ kN/m})(3\text{ m}) - 4\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{D} = 10\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + 10\text{ kN} - 2\text{ kN} - 4\text{ kN} - (8\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{A}_y = 20\text{ kN} \uparrow$$

Shear Diag:

$V_A = A_y = 20\text{ kN}$, then V is linear $\left(\frac{dV}{dx} = -8\text{ kN/m}\right)$ to C.

$$V_C = 20\text{ kN} - (8\text{ kN/m})(3\text{ m}) = -4\text{ kN}$$

$$V = 0 = 20\text{ kN} - (8\text{ kN/m})x_1 \text{ at } x_1 = 2.5\text{ m}$$

At C, V decreases by 4 kN to -8 kN.

At D, V increases by 10 kN to 2 kN.

Moment Diag:

$M_A = 0$, then M is parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$. Max M occurs where $V = 0$.

$$M_{\max} = \frac{1}{2}(20\text{ kN})(2.5\text{ m}) = 25\text{ kN}\cdot\text{m}$$

(*b*) $M_{\max} = 25.0\text{ kN}\cdot\text{m}$, 2.50 m from A ◀