To be Corrected by Merlin (514)568-3394

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	**Date**	**Duration**
Midterm Test	19 October, 2014	1 h 30 min
Special Instructions:	Only approved calculators are allowed **Show your work for full marks**	

1. (10 marks): **(a)** Let $f(x) = \sqrt{3-x}$ and $g(x) = x^2 - 1$ Find the composite functions $f \circ g$ and $g \circ f$, and determine their domains.

(b) Find the inverse function $f^{-1}(x)$ of $f(x) = \dfrac{2}{e^x + 1}$ and determine the domain and the range of $f(x)$ **and** of $f^{-1}(x)$.

2. (8 marks) Find the limit or explain why the limit does not exist:

(a) $\lim\limits_{x\to\infty} \dfrac{(10+x)\sqrt{x^6+4x^3}}{1+4x^2+2x^4}$

(b) $\lim\limits_{x\to 2} \dfrac{6x-12}{|x-2|}$

3. (5 marks) Find **(a)** all horizontal and **(b)** all vertical asymptotes of the graph

$$y = \frac{3x^2-3}{x^2-2x-3}$$

4. (4 marks) Find the second derivative of the function

$f(x) = \sin(x^2 - 1)$.

(continued on the other side)

5. (16 marks) Find the derivatives of the following functions. (You don't need to simplify the final answer, but you must show how you calculate it):

(a) $f(x) = x^{5/2}\, x^{-2} \tan x$

(b) $f(x) = (x^3 - 3x)\cos x + \sin^2 x$

(c) $f(x) = \dfrac{e^{2x}}{e^{-2x}+1} + \sec x$

(d) $f(x) = \cos\left(x\sqrt{x^3+5}\right)$

6. (8 marks) Given the function $f(x) = \sqrt{2x+5}$,

(a) Calculate $f'(x)$ using its definition as a limit of difference quotient.

(b) Write equation of the tangent line to the curve $y = f(x)$ at the point $(2, f(2))$.

Bonus Question (3 marks). Consider the function

$$f(x) = \begin{cases} x+1 & \text{if } x \le -1 \\ ax^2 - 1 & \text{if } x > -1 \end{cases}$$

where a is a parameter. Find the value of a that makes $f(x)$ differentiable everywhere, or explain why this is impossible.