MATH 205, Winter 2008, Sample Test

Problem 1: Find the following integrals using an appropriate substitution:

(a) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$ (b) $\int \frac{x^2}{\sqrt{1-x}} dx$ (c) $\int \frac{\sin(x)}{1+\cos^2 x} dx$ (d) $\int \sqrt[3]{x^3+1}\, x^5 dx$

Problem 2: Use integration by parts to find integrals:

(a) $\int x tan^{-1}(x^2) dx$ (b) $\int \cos(x) \ln(\sin x) dx$ (c) $\int \sin(\sqrt{x}) dx$

Problem 3: Calculate the area of the closed regions:

(a) bounded by the graphs of: $y = \sin x$, $y = \sin 2x$, $x = 0$, $x = \frac{\pi}{2}$.

(b) bounded by the graphs of: $y = \sin(\pi x)$, $y = x^2 - x$, $x = 2$.

(c) enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Problem 4: Estimate the area under the graph of: $f(x) = x^3 + 2$

from $x = -1$ to $x = 2$ using: (a) three (b) six rectangles and the right endpoints.

Problem 5: Graph the integrand and use area to evaluate the integral:

$\int_{-1}^{1} (1 + \sqrt{1-x^2}) dx$

Problem 6: Find derivatives of the functions:

(a) $F(x) = \int_1^{1+e^{x^2}} \frac{e^t \cos t}{\sqrt{t^2+t+1}} dt$ (b) $G(x) = \int_{\tan x}^{1} \frac{dt}{1+t^2}$

Problem 7: Evaluate:

(a) $\int_0^1 \frac{e^x}{1+e^{2x}} dx$ (b) $\int_1^{e^2} x^5 \ln(x) dx$ (c) $\int_1^{\pi/2} \sin^2(x) \cos^3(x) dx$ (d) $\int \frac{x^3+x+2}{x^2+1} dx$