Department of Economics

Carleton University

ECON 2030B – Intermediate Microeconomics II: Consumers and General Equilibrium

ASSIGNMENT 1 - Due Jan 30 at the beginning of the tutorial at 4:05PM

1. (30 points) Joan is on summer break and spends most of her time either playing video games or browsing the internet. Her utility function is $U(q_1, q_2) = 3q_1^{0.2}q_2^{0.8}$, where q_1 represents the hours spent browsing the internet and q_2 hours playing video games. Joan does not have a budget constraint but she has 15 hours to spend on both activities.

   (a) (5 points) Write the equation for one of Joan's indifference curves when the level of utility is equal to 100.

$$\bar{U} = 3q_1^{0.2}q_2^{0.8}$$
$$100 = 3q_1^{0.2}q_2^{0.8}$$
$$\frac{100}{3q_1^{0.2}} = q_2^{0.8}$$
$$\left(\frac{100}{3q_1^{0.2}}\right)^{1/0.8} = q_2 \rightarrow \boxed{q_2 = \left(\frac{100}{3q_1^{0.2}}\right)^{10/8}}$$

   (b) (5 points) Write Joan's time constraint.

   $q_1 + q_2 = 15$

   (c) (10 points) Find the amount of hours that Joan will spend on each activity that maximizes her utility and satisfies her time constraint.

   You can do this using either the Lagrangian or substitution method.

$$\mathcal{L} = 3q_1^{0.2}q_2^{0.8} + \lambda[15 - q_1 - q_2]$$
$$\frac{\partial \mathcal{L}}{\partial q_1} = 0.6q_1^{-0.8}q_2^{0.8} - \lambda = 0 \quad \rightarrow \quad \boxed{0.6q_1^{-0.8}q_2^{0.8} = \lambda} \quad (1)$$
$$\frac{\partial \mathcal{L}}{\partial q_2} = 2.4q_1^{0.2}q_2^{-0.2} - \lambda = 0 \quad \rightarrow \quad \boxed{2.4q_1^{0.2}q_2^{-0.2} = \lambda} \quad (2)$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 15 - q_1 - q_2 = 0 \quad (3)$$

Dividing equations (1) and (2) we get:

$$\frac{0.6q_1^{-0.8}q_2^{0.8}}{2.4q_1^{0.2}q_2^{-0.2}} = 1$$
$$\frac{0.25q_2}{q_1} = 1 \rightarrow \boxed{q_2 = 4q_1}$$

Replacing this result back into equation (3):

$$\begin{aligned} 15 &= q_1 + q_2 \\ 15 &= q_1 + 4q_1 \\ 15 &= 5q_1 \rightarrow \boxed{q_1 = 3} \end{aligned} \quad (4)$$

Replacing back into the equation for q_2:

$$q_2 = 4q_1 \rightarrow \boxed{q_2 = 12} \quad (5)$$

So Joan spends 3 hours browsing and 12 hours playing video games.

(d) (5 points) Can Joan afford any bundle that yields a utility of 50? Explain.

One way to answer this question is to check what is the level of utility that the optimal bundle provides. If it is higher than 50, then Joan can afford bundles that provide a utility of 50. Otherwise, she cannot.

$$U(3, 12) = 3 \times 3^{0.2}12^{0.8} = 27.28 \quad (6)$$

Since the utility-maximizing bundle only provides a utility of 27.28, Joan cannot afford bundles that provide her a utility of 50.

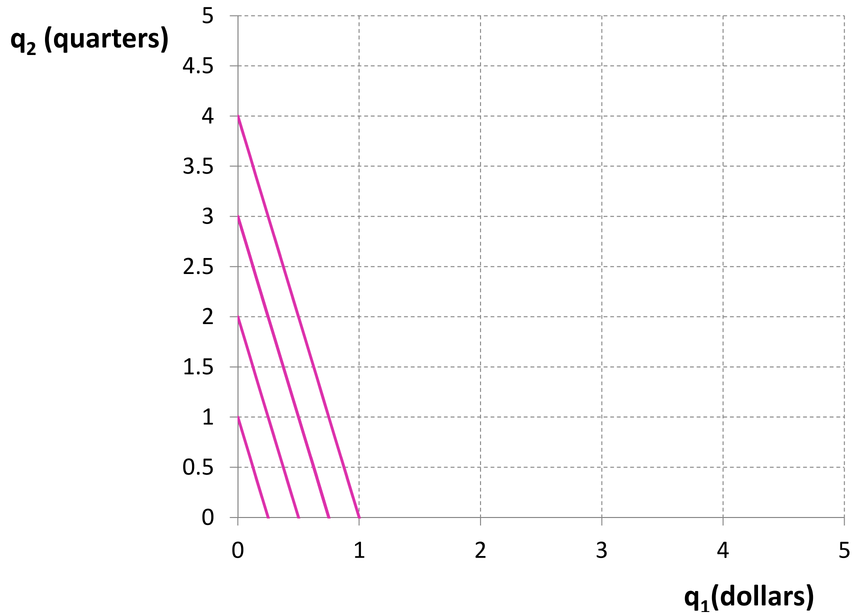
(e) (5 points) How does Joan's optimum amount of time spent on each activity change if her utility function is $U(q_1, q_2) = 5ln(3) + ln(q_1) + 4ln(q_2)$?

Here you can recalculate the maximization problem or you can show that this utility function is a positive monotonic transformation of the original utility and, hence, yields the same optimum.

$$\begin{aligned} U &= 3q_1^{0.2}q_2^{0.8} \\ ln(U) &= ln(3) + 0.2ln(q_1) + 0.8ln(q_2) \quad \text{(take logs)} \\ 10ln(U) &= 10ln(3) + 2ln(q_1) + 8ln(q_2) \quad \text{(multiply by 10)} \\ 5ln(U) &= 5ln(3) + ln(q_1) + 4ln(q_2) \quad \text{(divide by 2)} \end{aligned}$$

2. (15 points) Draw the indifference curves for two types of coins: quarters and dollars. What is their slope? Explain.

Since a dollar coin is worth four quarters, these are perfect substitutes in a 1 to 4 ratio as shown in the graph below. The utility function here is a linear function where the MRS is -4 or -1/4 depending on whether you draw quarter coins or dollar coins on the y-axis.



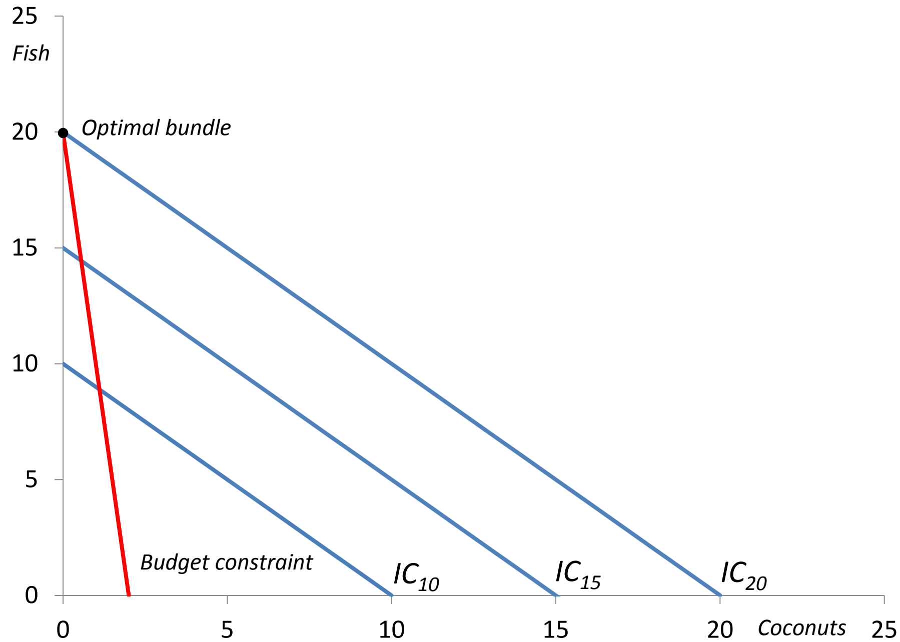
The easiest way to see this is with the highest indifference curve drawn on the graph which shows that having 4 quarters and 0 dollars is on the same indifference curve as 1 dollar and no quarters.

3. (20 points) Robinson Crusoe receives utility from eating coconuts and fish. His utility function is $U(C, F) = C + F$, where C is units of coconuts and F is units of fish. If the price of coconuts is \$10 and the price of fish is \$1, what can you say about the bundle (combination of fish and coconuts) that maximizes Robinson's utility? (Note: you do not need to but you can make additional assumptions if it helps you answer the question).

Given this utility function and prices, Robinson will only buy fish.

Intuitively, Robinson's utility function shows that either one unit of fish or one unit of coconuts increases his utility by the same amount: 1. Therefore, coconuts and fish are perfect substitutes to him. He does not care for a particular combination of coconuts and fish, he cares about getting the greatest amount possible. Because of this, he will only buy the cheapest good, which allows him to get the most units (and utility).

Mathematically, Robinson's utility function is such that the indifference curves are straight lines. Because of this, unless the budget constraint overlaps perfectly with his budget constraint, Robinson's optimal bundle will be a corner solution, i.e. he will choose to buy only fish or only coconuts. In this case, the MRS is 1 because the marginal utility of a unit of coconut is the same as the marginal utility of a unit of fish. But the ratio of prices (negative of the slope of the budget constraint) is not 1. So the budget constraint does not line up with the indifference curves. Note that Robinson derives a constant marginal utility of 1 from each unit of coconut and fish. Since the marginal utility of either good is the same and fish is cheaper than coconuts, Robinson will spend all his income on fish. For example, if Robinson's income is $20, we can graph his indifference curves and budget constraint as below.



Another way of seeing that this problem will have a corner solution is to note that the tangency condition for finding the optimal bundle does not have a solution:

$$\frac{MU_C}{MU_F} = \frac{p_C}{p_F}$$
$$\frac{1}{1} = \frac{10}{1}$$

The above is obviously false, which means that there is no tangency point between the budget constraint and the indifference curves. As shown in the graph, this is because both are straight lines and have different slopes. If they overlapped perfectly, then we would have found a condition that is always true, e.g. 1=1, which would mean that **any** combination of fish and coconuts would maximize Robinson's utility. This would have happened if both prices were the same. But, in this case, because fish is cheaper, Robinson will maximize his utility by spending all his income on fish.

4. (8 points) The automobile makers Mercedes-Benz and Kia were considering entering a developing country. Prior to entering, a survey of consumers in this developing country indicated that consumers prefer Mercedes-Benz automobiles over Kia automobiles. However, after the two automobile makers entered the country and began sales of their vehicles, consumers flocked to Kia dealerships. How can you explain this apparent paradox? (Hint: think about the data needed to infer a consumer's optimal bundle choice and the data provided here.)

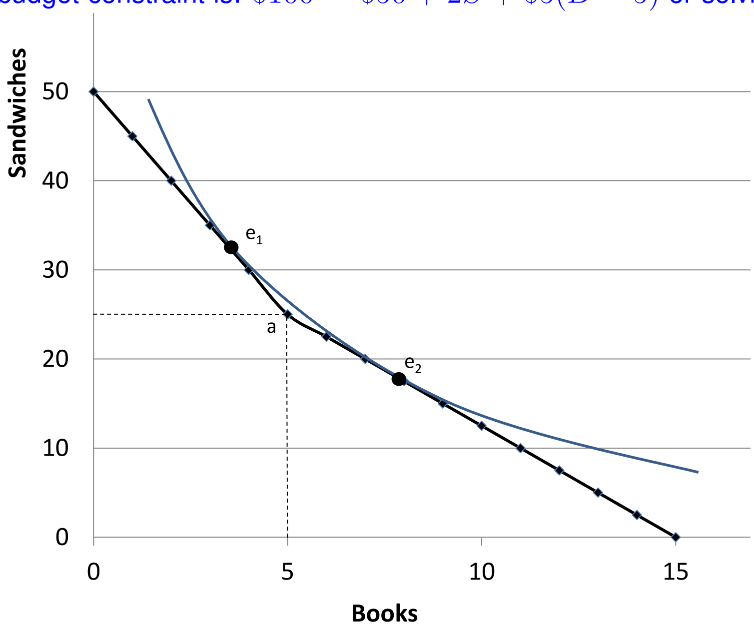
   There is no paradox. Preferences do not involve prices, and consumers in the developing country preferred Mercedes vehicles based solely on product characteristics. However, Mercedes prices are considerably higher than Kia prices. So, even though consumers preferred a Mercedes to a Kia, they either could not afford a Mercedes or they preferred a bundle of other goods plus a Kia to a Mercedes alone. While the marginal utility of consuming a Mercedes exceeded the marginal utility of consuming a Kia, consumers considered the marginal utility per dollar for each good and, for most of them, the marginal utility per dollar was higher for Kia vehicles. As a result, they flocked to Kia dealerships.

5. (20 points) Suppose Carmela's income is \$100 per week, which she allocates between sandwiches and books. Sandwiches cost \$2 each. Books cost \$10 each if she purchases between 1 and 5 books. If she purchases more than 5 books in a week, the price falls to \$5 for the 6th book and all subsequent books. Draw the budget constraint.(15 points) Is it possible that Carmela might have more than one utility-maximizing solution? (5 points)

   Because the price of books falls when Carmela purchases more than 5 books in one week, the budget constraint is nonlinear, i.e. it does not have a constant slope. As the figure shows, the budget line is kinked at a.

   Carmela's budget constraint up to 5 books does not change. $\$100 = \$2S + \$10B$. Or rewriting to solve for Sandwiches which we will draw on the y-axis: $S = 50 - 5B$.

   If Carmela buys more than 5 books, then she spends \$50 on the first five and and has the rest of her budget to spend on sandwiches and books at prices \$2 for sandwiches and \$5 for books. Her budget constraint is: $\$100 = \$50 + 2S + \$5(B - 5)$ or solving for S: $S = 25 - 5/2(B - 5)$.



   This nonlinearity makes it possible that a single indifference curve could be tangent to the constraint in two places. In this figure, the red indifference curve is tangent to the budget constraint at e_1 and e_2.