MAT1332, University of Ottawa, Summer 2012
Instructor: Olga Vasilyeva
Assignment 5, due July 19, 19:00 at the beginning of class.
Late assignments will not be accepted.

Student Name _____ Student Number _____

1. Consider the function $f(x,y) = \sqrt{9-x^2-y^2}$. Find its domain, range, and describe its level sets. Sketch several level curves.

2. Find the tangent plane to the graph of the function $f(x,y) = \tan(2x-3y^2) + 2\pi$ at $(0,0)$.

3. Find the Jacobian matrix of the function $f(x,y) = \begin{bmatrix} x^2e^y + 2x\sin(x^y) \\ \sin(x^2) - 3ye^{-x} \end{bmatrix}$.

4. Consider the following system of linear differential equations:

$$\frac{dx}{dt} = -3x + y$$

$$\frac{dy}{dt} = 4x - 3y$$

(a) Find the eigenvalues and eigenvectors associated with the system.

(b) Write down the general solution formula for the system.

(c) Give the particular solution for the initial values $x(0) = 4$, $y(0) = 4$.

(d) Draw the $x-$ and $y-$nullclines and the direction arrows in the phase plane.

(e) Sketch the solution curve for the initial condition in part (c) into the phase plane.

(f) Is the point (0,0) stable or unstable? Classify this equilibrium.

5. Consider a disease that propagates according to the system

$$\frac{dx}{dt} = 16 - 0.2xy - 0.4x$$

$$\frac{dy}{dt} = 0.1xy - 8y$$

where x represents susceptible individuals and y represents infected individuals.

(a) Find all biologically meaningful steady states.

(b) Find the Jacobian matrix of this system.

(c) For the biologically meaningful steady states from (a), find the eigenvalues of the Jacobian matrix.

(d) Determine the stability of the biologically meaningful steady states.