MAT 1332, Spring/Summer 2012 Assignment 4

Due July 3, 2012 at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

Instructor: Olga Vasilyeva

Student Name ______________________________ Student Number ______________________________

QUESTION 1. Consider the complex numbers

$$z_1 = 6 + \sqrt{3}i,$$
$$z_2 = -6 + \sqrt{3}i,$$
$$z_3 = -6 - \sqrt{3}i,$$
$$z_4 = 6 - \sqrt{3}i.$$

(a) Graph z_1, z_2, z_3, z_4 on $\mathbb{C}$.

(b) Write down the two pairs of complex conjugates.

(c) Find the inverse of z_1 (hint: $z \cdot \bar{z} = |z|^2$, $z = \frac{|z|^2}{\bar{z}}$, $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{|z|^2}$).

(d) Express z_1, z_2, z_3, z_4 in polar coordinates.

QUESTION 2. Let $z = 2 + 4i$, $w = -3 + 6i$.

(a) Graph the points in $\mathbb{C}$.

(b) Find $z + w$, $z - w$, $i \cdot w$, $z \cdot w$, $\dfrac{z}{w}$.

QUESTION 3. Consider $z_1 = -i$, $z_2 = -3 - 3i$.

(a) Graph the points in $\mathbb{C}$.

(b) Express z_1, z_2 in polar coordinates (write ϕ_1, ϕ_2 as multiples of π).

(c) Write z_1, z_2 using the Euler notation.

(d) Find $z_1^3 = r^3(\cos(3\phi) + i\sin(3\phi))$
(the general formula states $z_1^n = r^n(\cos(n\phi) + i\sin(n\phi))$).

Question 4. For the system of linear equations

$$\begin{aligned} x + 3y + 9z &= 3 \\ 2x + 7y + 23z &= 2 \\ x + ay + a^2 z &= a. \end{aligned}$$

(a) Determine the values of a for which the system has
(i) no solution
(ii) infinitely many solutions
(iii) a unique solution.

(b) In case (ii) above describe all solutions.

(c) If $a = 1$, find the inverse matrix and find the solution to $A\bar{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

QUESTION 5. Consider the following matrices and vectors:

$$A = \begin{pmatrix} 1 & -7 & -9 \\ 6 & 0 & 8 \\ 4 & 4 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -7 \\ 6 & 4 \\ 0 & 1 \end{pmatrix}, \quad \bar{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

$$\bar{v} = \begin{pmatrix} 1 \\ \frac{7}{4} \\ 0 \end{pmatrix}, \quad \bar{w} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{3}{4} \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix}.$$

Compute the following if possible. If not possible, explain in one sentence why.

(a) $3\bar{v} + 2B^T\bar{w}$
(b) $\bar{w}\bar{v}$
(c) $\bar{v}^T\bar{w}$
(d) $A^TB + 2\bar{v}^T\bar{w}$
(e) AB
(f) $B\bar{u}$
(g) BA
(h) D^2, $\det(D)$
(j) $A + D$

Question 6. Consider

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 3x_1 + 2x_2 + 4x_3 &= 4 \\ 2x_1 + x_2 + 2x_3 &= 2. \end{aligned}$$

Reduce the corresponding augmented matrix to the reduced row-echelon form and find the solution to the system.

QUESTION 7. Given $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & -4 & 0 \\ 4 & 0 & 2 \end{pmatrix}$.

(a) Use the characteristic equation to find the eigenvalues.

(b) Find the corresponding eigenvectors.

Question 8. Consider $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$.

(a) Find the eigenvalues of A.

(b) Find the eigenvector corresponding to the eigenvalue from the first quadrant.