

MAT 1332, Spring/Summer 2012 Assignment 3
Due June 12, 2012 at the beginning of class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

Instructor: Olga Vasilyeva

Student Name _____ Student Number _____

QUESTION 1. Consider the differential equation

$$\frac{dy}{dt} = y^2 + 9.$$

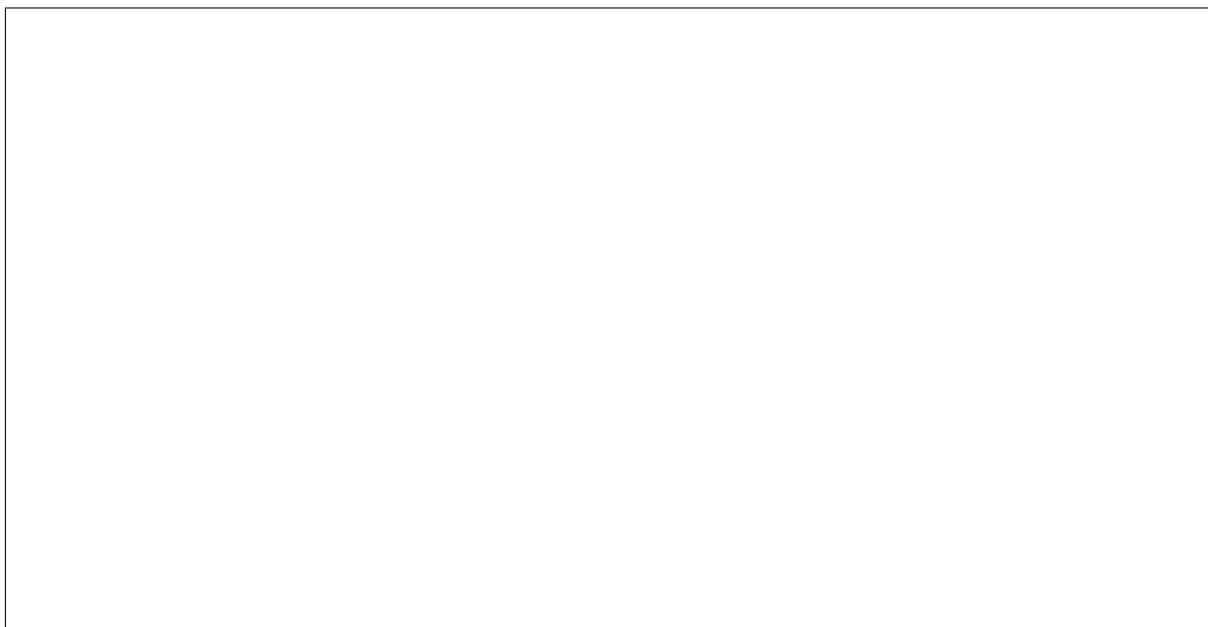
(a) Is this a pure time or an autonomous differential equation? Justify your answer in one sentence.

(b) Find the solution to the equation satisfying the initial condition $y(0) = 3$.

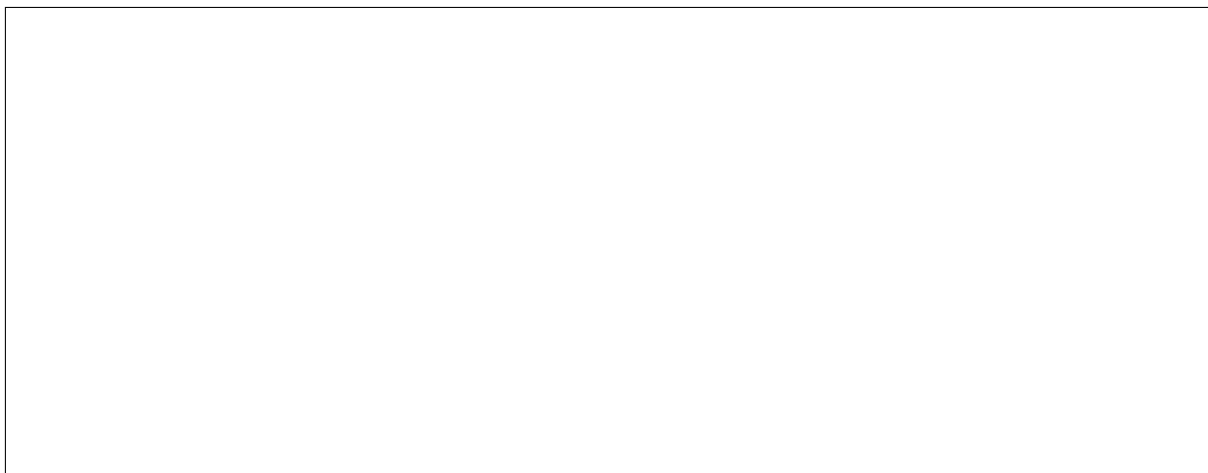
QUESTION 2. Consider the differential

$$\frac{dy}{dx} = (y - 1)e^x.$$

(a) Find the general solution to the equation.



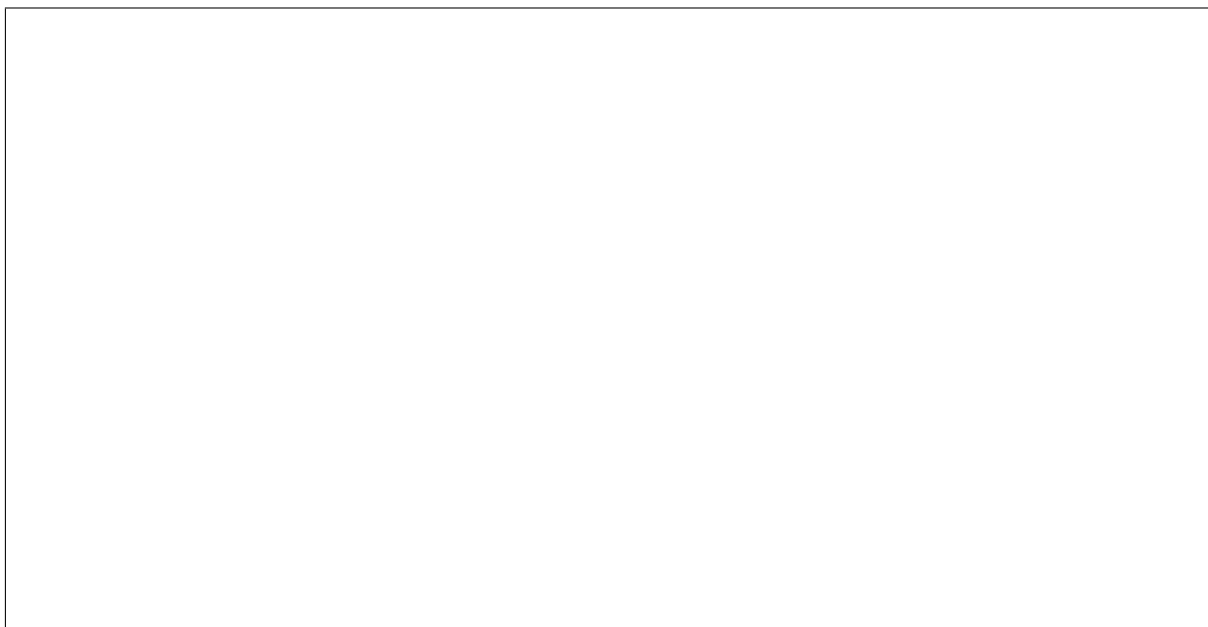
(b) Find the particular solution satisfying the initial condition $y(0) = e$.



QUESTION 3. Consider the differential

$$\frac{dy}{dt} = \cos t \cdot y^2.$$

(a) Find the general solution to the equation.



(b) Find the particular solution satisfying the initial condition $y(\frac{\pi}{2}) = 1$.



QUESTION 4. Consider the differential equation

$$\frac{dy}{dx} = y(y - 1) = f(y).$$

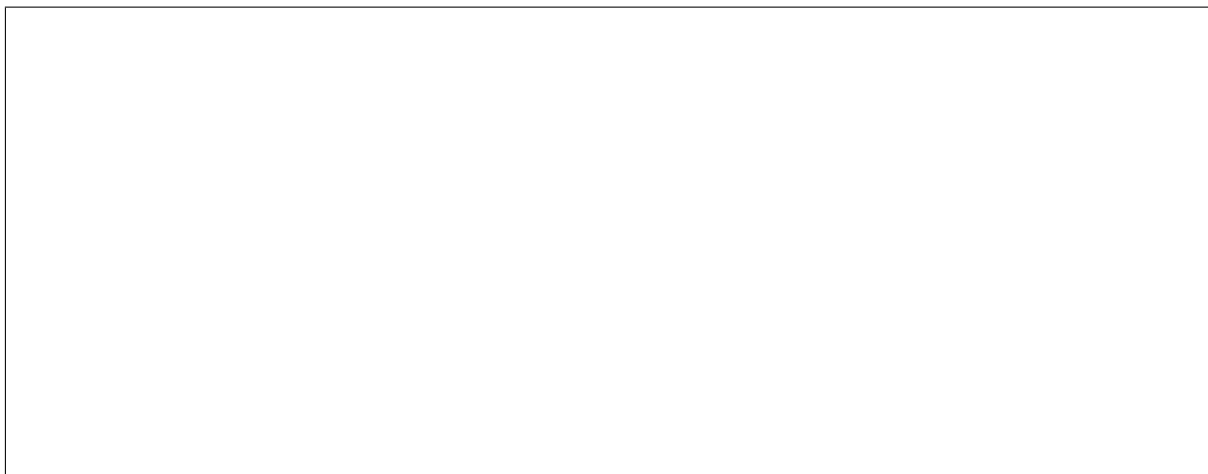
(a) Find all equilibria.

An empty rectangular box with a thin black border, intended for the student's answer to part (a).

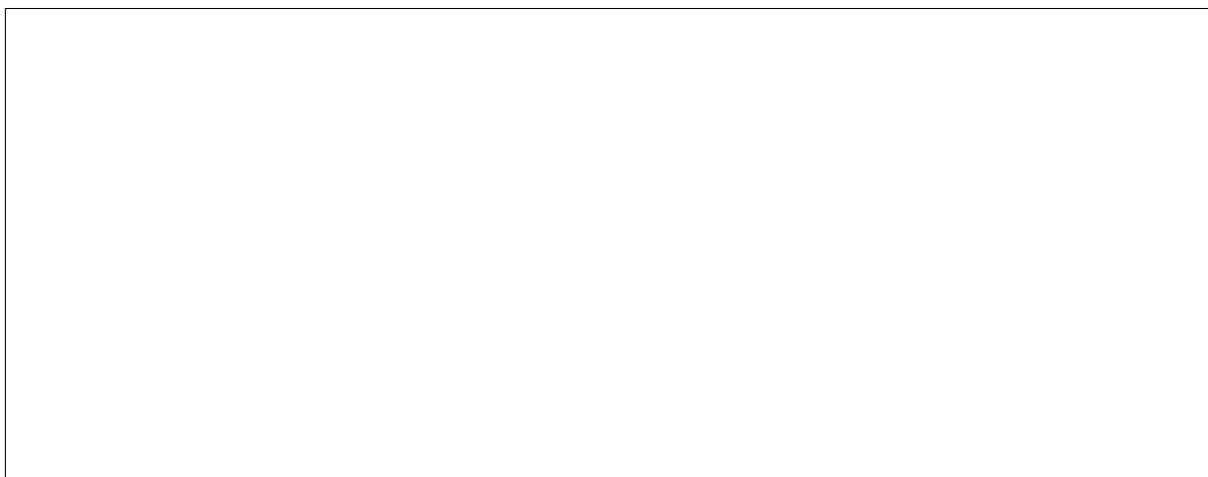
(b) Determine the stability of each equilibrium (you may use the stability criterion or the test point method).

An empty rectangular box with a thin black border, intended for the student's answer to part (b).

(c) Draw the phase-live diagram for the differential equation in the $(y, f(y))$ -plane.



(d) Graph the equilibrium solutions and solutions starting at $y(0) = 0.5$ and $y(0) = 5$ (for $t > 0$). Indicate the behavior of the solution curves as $t \rightarrow \infty$ (use the $(t, y(t))$ -plane).

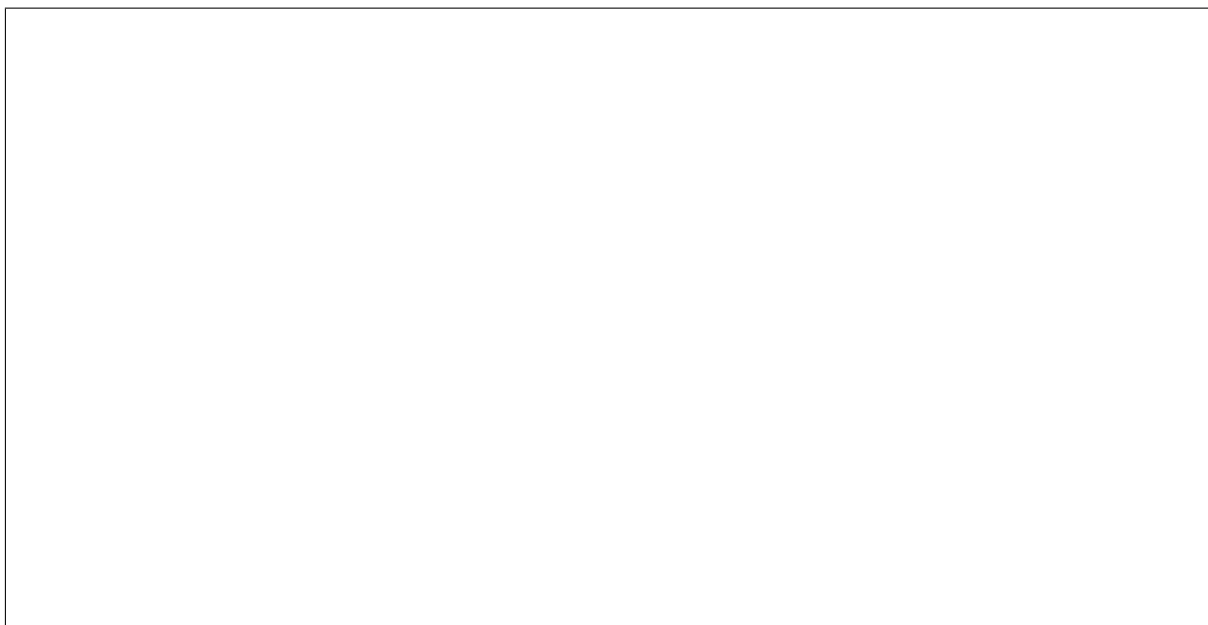


QUESTION 5. Suppose the population of certain organisms follows the model

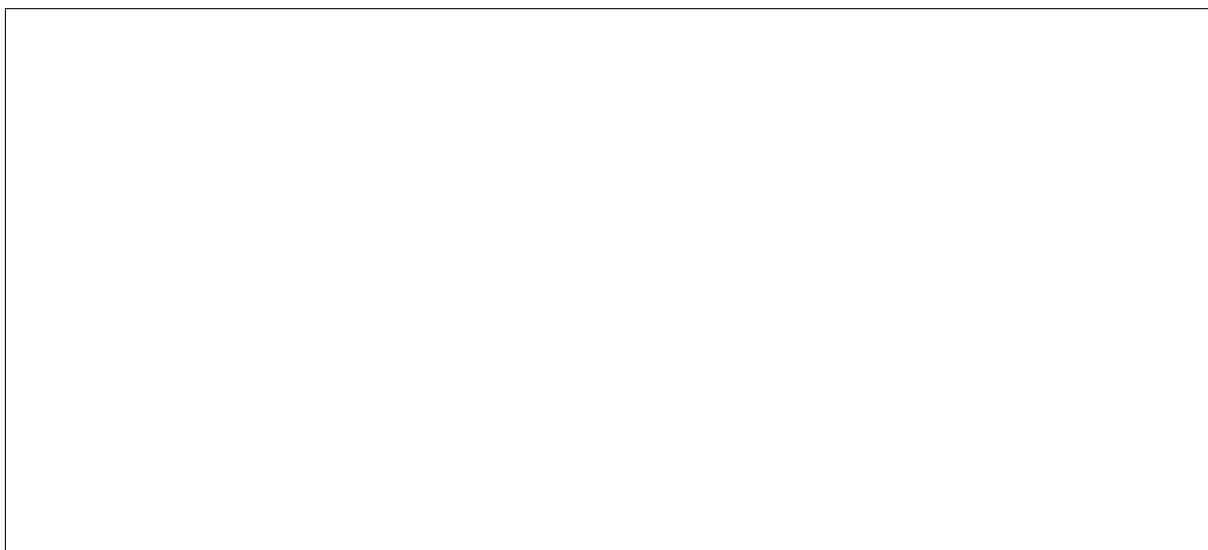
$$\frac{dN}{dt} = \frac{10N^2}{N^2 + 1} - 3N,$$

where N is measured in thousands.

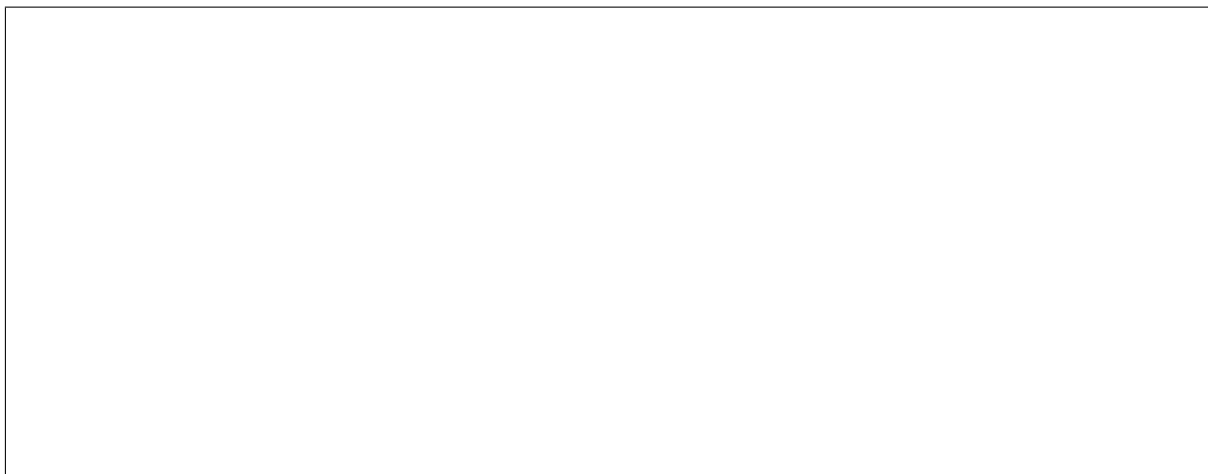
(a) Find the equilibria.



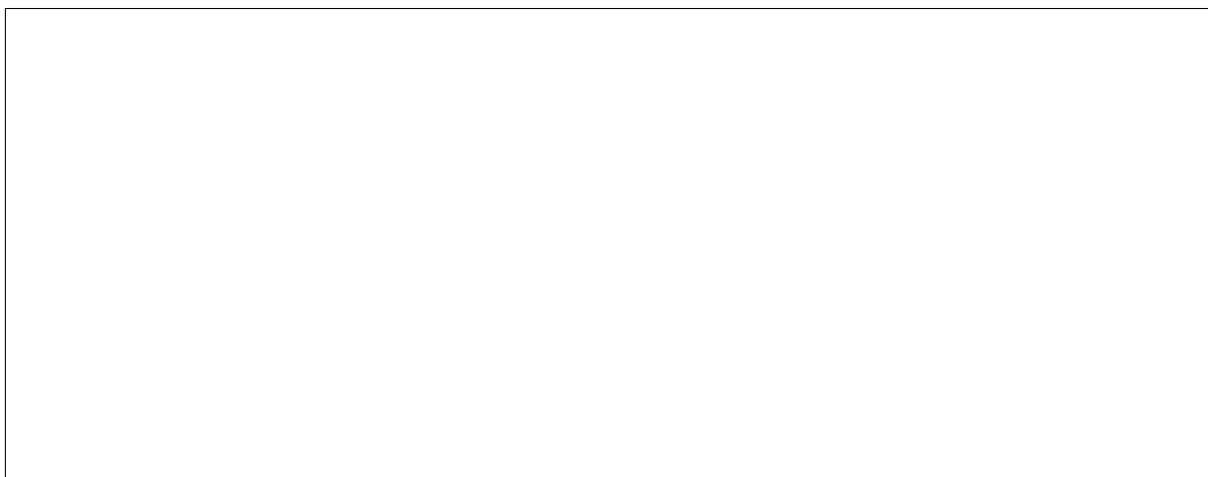
(b) Determine the stability of equilibria using the stability criterion.



(c) Draw the phase-line diagram for the given differential equation.



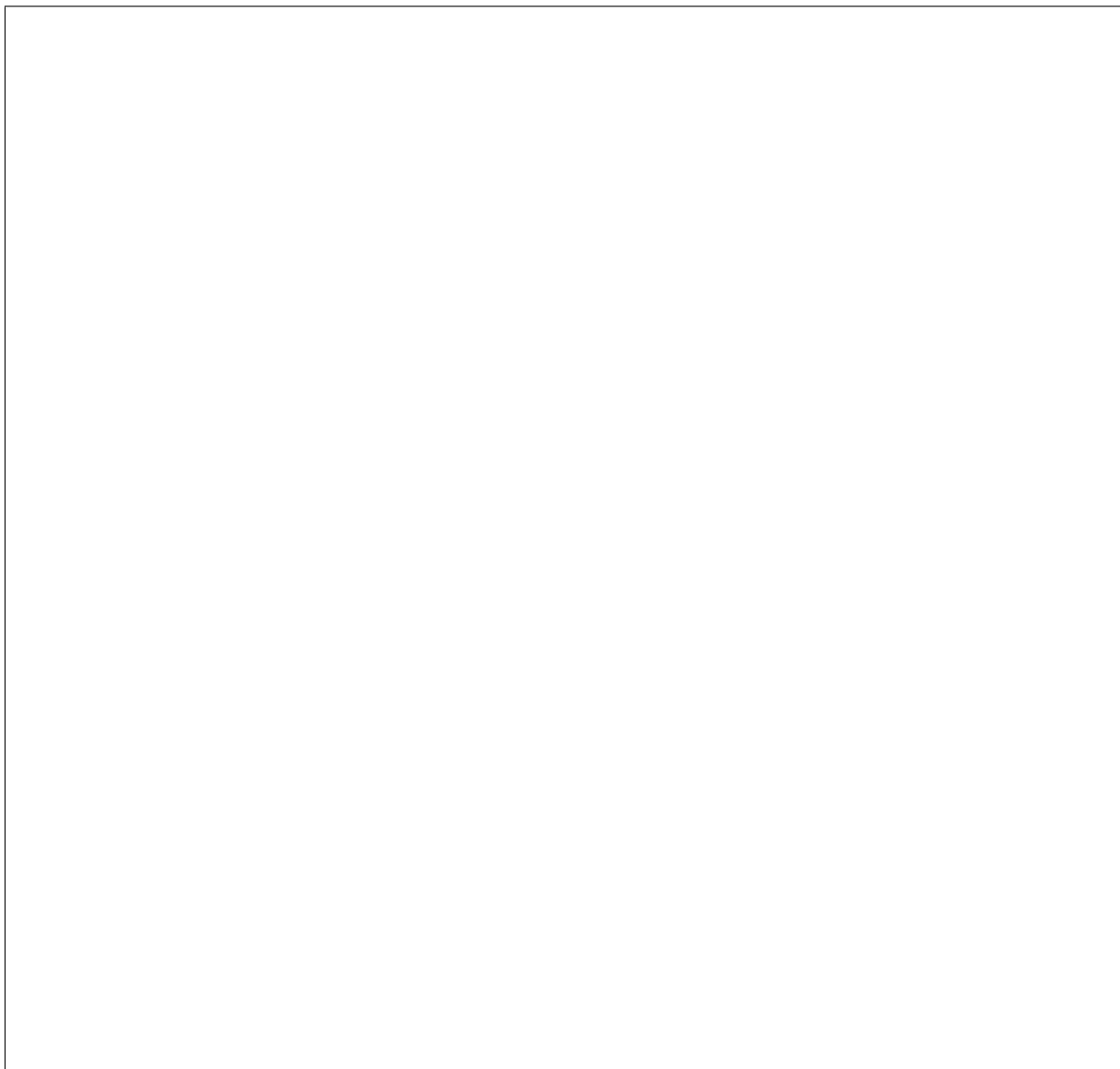
(d) Graph the equilibria and solutions starting at $N(0) = \frac{1}{4}$, $N(0) = 2$ and $N(0) = 5$ (use the $(t, N(t))$ -plane).



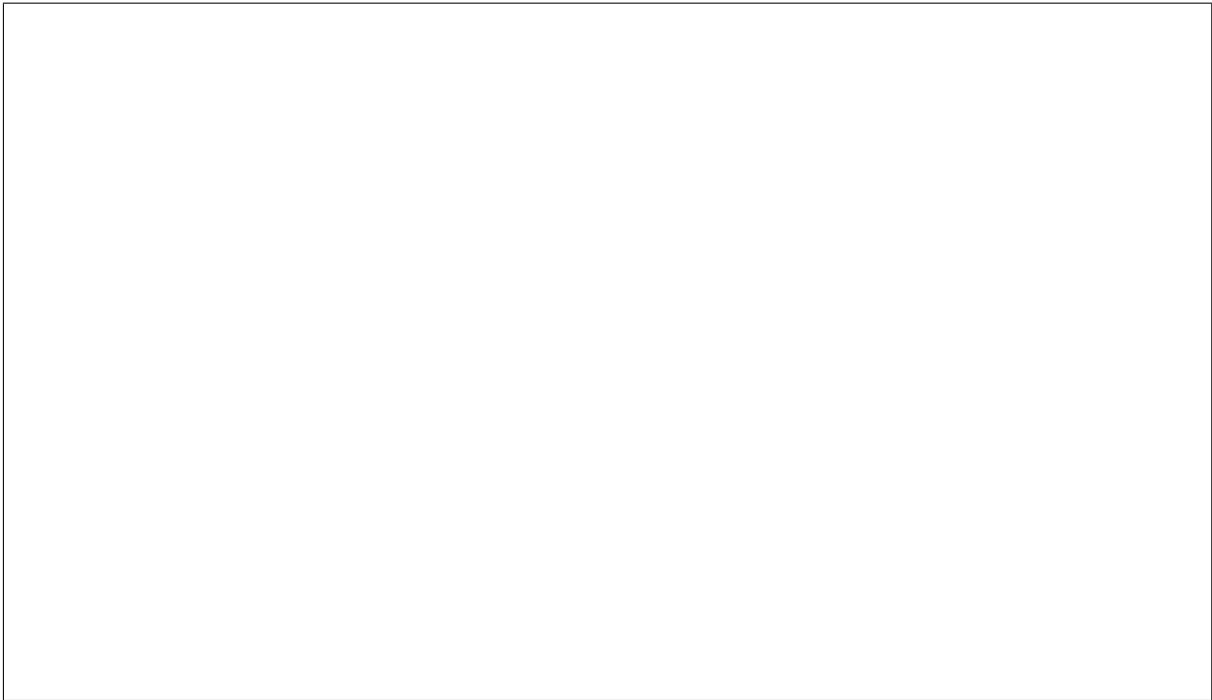
QUESTION 6. A yam is put in a 200°C oven and heats up according to the differential equation

$$\frac{dH}{dt} = \alpha(200 - H).$$

(a) If the yam is at 20°C when it is put in the oven, solve the differential equation. (Your answer should include the constant α .)



(b) Find α using the fact that after 30 min the temperature of the yam is 120°C .



(c) Using the values from part (a) and (b), sketch the graph of the solution $H(t)$.

