Old Exam for Practice

Answers for the 10 multiple choice questions:

☐ #1	☐ #2	☐ #3	☐ #4	☐ #5
☐ #6	☐ #7	☐ #8	☐ #9	☐ #10

1. **[2 points]** Consider the following table of values for two functions f and g, and their derivatives f' and g'.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	3	-1	10	1
1	4	0	1	e
2	2	1	π	7
3	18	6	5	-1

If $h(x) = f(g(x))$, which of the following equals $h'(1)$?

A: $3e$ **B**: $10e$ **C**: 0 **D**: e **E**: $4e$ **F**: $-e$

2. **[2 points]** *Use the linear approximation* of the function $f(x) = \tan(x)$ at $x_0 = \frac{\pi}{4}$ to approximate the number $\tan(\frac{\pi}{4} + 0.1)$.

A. 1.10 **B.** 1.16 **C.** 1.20 **D.** 1.22 **E.** 1.29 **F.** 1.63

3. **[2 points]** If we apply Newton's method with initial estimate $x_1 = 1$ to find a solution to $x = \cos(x)$, what is our second estimate x_2?

A. 1.253 **B.** 1.020 **C.** 0 **D.** 0.636 **E.** 0.739 **F.** 0.750

4. **[2 points]** Consider the curve defined by $x^2 - xy + y^2 = 27$. Using implicit differentiation, determine at which of the following points the curve has a vertical tangent line.

A. $(3, -3)$ **B.** $(6, 3)$ **C.** $(0, 3\sqrt{3})$ **D.** $(3, \sqrt{3})$ **E.** $(-3, 3)$ **F.** $(3, 6)$

5. **[2 points]** Suppose f, g and F are continuous functions on the interval $[a, b]$. Which of the following statements are correct? Choose from the options below.

(i) $\int_a^b f(x)\,dx$ is an antiderivative of f.

(ii) $\int_a^b f(x)\,dx = F(b) - F(a)$ if $F' = f$.

(iii) $\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

(iv) If $g(x) = \int_a^x f(x)\,dx$ then $g'(x) = f(x)$.

(v) $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

A. (ii) only **B.** (iii) and (iv) only **C.** (i) and (ii) only
D. (ii), (iv) and (v) only **E.** (i), (ii) and (v) only **F.** all are correct

6. [**2 points**] Some values of a function f are listed in the following table

x	0	2	4	6	8
$f(x)$	10.0	9.5	9.0	8.0	6.0

Use these data to approximate $\int_0^8 f(x)\,dx$ by Rightpoint Rule.

A. 32.0 **B.** 36.5 **C.** 65.0 **D.** 73.0 **E.** 85.0 **F.** 42.5

7. [**2 points**] Evaluate $\displaystyle\int_1^4 \left(\frac{1}{2}x - \sqrt{x}\right) dx.$

A. -0.92 **B.** 1.50 **C.** -0.50 **D.** -1.67 **E.** 2.00 **F.** 2.25

8. [**2 points**] Evaluate

$$\int_0^1 x\sqrt{x^2+1}\,dx.$$

A. 1.07 B. -0.75 C. 2.33 D. 0.93 E. 0.50 F. 0.61

9. [**2 points**] Suppose that the *growth rate* of a certain fungus is expressed by the function

$$g(t) = \frac{\ln(t)}{t} \text{ (grams/month)} \qquad \text{for } t \geq 1.$$

What is the total change in weight (in grams) of the fungus between $t = 1$ and $t = 6$ months?

A. 1.61 **B.** 1.72 **C.** 6.39 **D.** 0.03 **E.** 2.79 **F.** 1.41

10. [**2 points**] Suppose that $f(x)$ is a function so that

$$\int_0^5 f(x)\,dx = 17 \qquad \text{and} \qquad \int_3^5 f(x)\,dx = 20.$$

Evaluate:

$$\int_0^3 f(x)\,dx.$$

A. 14 **B.** 17 **C.** -3 **D.** 3 **E.** -17 **F.** impossible to say

11. [**2 points**] Compute $f'(x)$ where $f(x) = \tan(\ln(2x+1))$.

12. [**2 points**] Find

$$\int \sin 4(x)\,dx.$$

13. [**2 points**] Let $f(x) = \sin(x)$. Compute its tenth derivative, $f^{(10)}(x)$.

14. [**2 points**] Find

$$\int \sqrt{x}\ln(x)\,dx.$$

15. [**3 points**] Find the absolute maximum and the absolute minimum values of the function

$$f(x) = x^2e^{-x}$$

on the interval $[-3, 3]$.

16. [**3 points**] A particle moves along a straight line and its position at time t is given by $s(t) = 2t^3 - 18t^2 + 48t$ where $s(t)$ is measured in metres and t in seconds.
(a) Find $v(t)$, the velocity of the particle at time t.
(b) At what time(s) does the particle stop moving?
(c) Evaluate $\int_0^4 |v(t)|\,dt$ **(Notice the absolute value!)**, the total distance travelled by the particle between 0 and 4 seconds.

17. [**3 points**] Sand is poured into a conical pile at a constant rate of $36\pi\ m^3/min$. At each moment t, measured in minutes, the radius r of the base of the pile is two times the height h of the pile. Recall that the volume of a cone in given by the formula $\frac{1}{3}\pi r^2 h$.
(a) At what rate is the height of the cone increasing, when $h = 12$ m?
(b) *(BONUS (1 point). Answering this part is optional; please be careful not to waste your time here.)* At what time t will the height of the pile be 12 m, assuming the pile started to form at time $t = 0$?

18. [**3 points**] Evaluate $\displaystyle\int_0^{1/\sqrt{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx.$

19. [**5 points**] A piece of wire 10 metres long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. Where should the wire be cut so that the total area enclosed (square + circle) is a (a) maximum and (b) minimum? (You do not need to prove that your answer is a maximum, respectively, minimum.) Write your solution clearly, showing all your steps, and put a box around your final answer.

20. [**5 points**] Let

$$f(x) = \frac{e^x}{1-x}$$

Answer each of the following questions on this page and/or the next. Identify your answer to each part by putting it in a box and writing the letter of the question next to the box.
(a) Give the domain of f. Find all x-intercepts (if any) and all vertical asymptotes (if any).
(b) Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. Does f have horizontal asymptotes? If so, give their equation.
(c) Calculate $f'(x)$. You may verify that

$$f''(x) = \frac{(x^2-4x+5)e^x}{(1-x)^3}$$

if you wish; this is one way for you to check your answer.
(d) Find the coordinates of the critical points of f (if any) and its intervals of increase and decrease.
(e) Find the coordinates of the inflection points of f (if any) and its intervals of concavity.
(f) Calculate $f(0)$. Sketch the graph of f.