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MAT1322/1722

Midterm 2-AID

Mock Exam Solutions

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1. Find the formula for the general term a_n

$$\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \dots\}$$

$$a_n = \frac{(-1)^{n+1} n^2}{n+1}$$

2. Find the limit of the convergent sequence

$$a_n = \ln(2n^2+1) - \ln(n^2+1)$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln(2n^2+1) - \ln(n^2+1) = \lim_{n\to\infty} \ln\left(\frac{2n^2+1}{n^2+1}\right)$$

$$= \ln\left(\lim_{n\to\infty} \frac{2n^2+1}{n^2+1}\right) = ln2$$

3. Express the following repeated decimal as a ratio of integers

$$1.53\overline{42}$$

$$= 1.53 + 0.0042 + 0.000042 + 0.00000042 + \cdots$$

$$= 1.53 + \frac{42}{10000} + \frac{42}{1000000} + \frac{42}{100000000} + \cdots$$

$$= 1.53 + \frac{42}{10000}(\frac{1}{100})^n$$

$\frac{42}{10000}(\frac{1}{100})^n$ is a geometric series where: $\sum_{n=0}^{\infty} \frac{42}{10000}(\frac{1}{100})^n$

$$1.53 + \frac{42}{10000}\left(\frac{1}{100}\right)^n = \frac{153}{100} + \frac{42}{10000}\left(\frac{1}{1-\frac{1}{100}}\right)$$

$$= \frac{5063}{3300}$$

4. Determine if the following series converges and, if so, determine the sum of series.

$$\sum_{n=1}^{\infty} \frac{1}{e^n} - \frac{1}{n^2+n}$$

$\lim_{n\to\infty} \frac{1}{e^n} - \frac{1}{n^2+n} = 0$ Therefore the series does not diverge.

First part of the series is geometric $\sum_{n=1}^{\infty} \frac{1}{e^n} = \frac{\frac{1}{e}}{1-\frac{1}{e}} = \frac{1}{e-1}$

Second part of the series is telescopic $\sum_{n=1}^{\infty} \frac{1}{n^2+n} = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots = 1 - \frac{1}{\infty} = 1$

$$\sum_{n=1}^{\infty} \frac{1}{e^n} - \frac{1}{n^2+n} = \frac{1}{e-1} - 1 = \frac{1}{e-1} - \frac{e-1}{e-1} = \frac{2-e}{e-1}$$

5. Determine if the following series converges.

$$\sum_{n=1}^{\infty} \frac{5+3\sin(n)}{3n^3+n+4}$$

$$\frac{5+3\sin(n)}{3n^3+n+4} \leq \frac{8}{3n^3}$$

$\frac{8}{3n^3}$ is a p-series sequence, with p = 3 > 1, therefore it converges.

Since $\frac{8}{3n^3}$ converges, $\frac{5+3\sin(n)}{3n^3+n+4}$ converges as well.

6. Determine whether a series converges absolutely or conditionally, or simply diverges.

$$\sum_{n=1}^{\infty} \frac{10^n}{(n+1)!\,4^{2n+1}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(10)^{n+1}}{(n+2)!\,4^{2n+2}}\left(\frac{(n+1)!\,4^{2n+1}}{10^n}\right)$$

$$= \lim_{n\to\infty} \frac{10}{4(n+2)} = 0 < 1$$

Therefore converges absolutely.

7. Determine the first four terms of Maclaurin series development of

$$(4 + x^2)^{1/2}$$

Solution: $2 + \frac{x^2}{4} - \frac{x^4}{64} + \frac{x^6}{512}$

8. What is the smallest value of k that would allow for the partial sum $\sum_{n=1}^{k} \frac{1}{n^{5/2}}$ to approximate the series $S = \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ with an error of at most 10^{-3}?

$$\int_{k+1}^{\infty} x^{-\frac{5}{2}} dx \le 10^{-3} \rightarrow \lim_{t\to\infty} -\frac{2}{3}\left[\frac{1}{x^{\frac{3}{2}}}\right]_{k+1}^{t} \le 10^{-3}$$

$$-\frac{2}{3}\left(-\frac{1}{(k+1)^{\frac{3}{2}}}\right) \le 10^{-3}$$

$$k \ge \sqrt[\frac{3}{2}]{\frac{2000}{3}} - 1 \rightarrow k \ge 77$$

9. Find the radius and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{n}{2^n}(x-3)^n$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(x-3)^{n+1}}{2^{n+1}}\left(\frac{2^n}{n(x-3)^n}\right)\right|$$

$$= \lim_{n\to\infty}\left|\frac{n+2}{n}\left(\frac{2^n}{2^{n+1}}\right)(x-3)\right| = \left|\frac{x-3}{2}\right|$$

$$|x-3| < 2$$

Therefore the radius of convergence of the series is 2 .

Center x = 2, the endpoints are x = 1 and x = 5

* x = 1

$$\sum_{n=1}^{\infty} \frac{n}{2^n}(-2)^n = \sum_{n=1}^{\infty} n(-1)^n$$

$\lim_{n\to\infty} n(-1)^n = \infty$

Therefore the series is divergent at x = 1

* x = 5

$$\sum_{n=1}^{\infty} \frac{n}{2^n}(2)^n = \sum_{n=1}^{\infty} n$$

$\lim_{n\to\infty} n = \infty$

Therefore the series is divergent at x = 5.

The interval of convergence of the series is $x \in (1, 5)$.

10. Develop the function $f(x) = \frac{2}{1+3x}$ into a power series. (Maclaurin)

$$\frac{1}{x+1} = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$2\left(\frac{1}{1+3x}\right) = \sum_{n=0}^{\infty} 2(-1)^n (3x)^n$$

$$\frac{2}{1+3x} = \sum_{n=0}^{\infty} 2(-3x)^n$$

11. Given the series $g(x) = \sum_{n=0}^{\infty} \frac{nx^n}{3^n}$

i. **Find the power series for g'(2)**

$$g'(x) = \sum_{n=1}^{\infty} \frac{n^2 x^{n-1}}{3^n}$$

$$g'(2) = \sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{3^n} = \sum_{n=1}^{\infty} \frac{n^2}{2} (\frac{2}{3})^n$$

ii. **Find the power series for $\int_0^2 g(x)dx$**

$$\int g(x)\,dx = \sum_{n=1}^{\infty} \frac{nx^{n+1}}{3^n(n+1)}$$

$$\int_0^2 g(x)dx = \left[\sum_{n=1}^{\infty} \frac{nx^{n+1}}{3^n(n+1)}\right]_0^2 = \sum_{n=1}^{\infty} \frac{2n2^n}{(n+1)3^n}$$

$$\int_0^2 g(x)dx = \sum_{n=1}^{\infty} \frac{2n}{(n+1)} (\frac{2}{3})^n$$