MAT1300: Mathematical Methods I
MIDTERM # 1(a)-2009

LASTNAME______________________**Firstname**______________________

Student number:____

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The total value of the midterm exam is 25 points.

- The multiple choice questions are worth 2 points each. Place your answers to the multiple choice questions into the boxes on page 2.
- You have to show your steps clearly on the long answer questions.
- You may use the backs of pages and indicate precisely when doing so.

NO CALCULATORS. Closed BOOKS.

Answers:

1	2	3	4	Total mark/25
E	D	A	B	

Multiple Choice Questions (1-4)

Question 1 *Let*

$$f(x) = \frac{x-1}{x(x-1)(x+2)}.$$

Find all values of x such that the function $f(x)$ are discontinuous.

A) $0, -2$ **B)** $1, -2$ **C)** -2 **D)** $0, -1, 2$ **E)** $0, 1, -2$

Solution: denominator can not be zero.

Question 2 *Solve the inequality $3x^2 + x < 2x^2 + 4x - 2$.*

A) $(-\infty, 0)$ **B)** $(-\infty, 2)$ **C)** $(-\infty, 0) \cup (2, \infty)$ **D)** $(1, 2)$ **E)** $(2, \infty)$

Solution: $3x^2 + x < 2x^2 + 4x - 2 \Rightarrow x^2 - 3x + 2 < 0, \Rightarrow 1 < x < 2.$

Question 3 *Which of the following functions is the inverse of* $f(x) = \frac{2x+5}{3x-4}$*?*

A) $f^{-1}(x) = \frac{4x+5}{3x-2}$ **B)** $f^{-1}(x) = \frac{3x-4}{2x+5}$ **C)** $f^{-1}(x) = \frac{5-2x}{4+3x}$

D) $f^{-1}(x) = \frac{5x-2}{4x-3}$ **E)** $f^{-1}(x) = \frac{3-4x}{5+2x}$

Solution: (A)
In $f(x) = \frac{2x+5}{3x-4}$ we replace x by y and $f(x)$ by x, then

$$x = \frac{2y+5}{3y-4}.$$

Solve y we get

$$y = f^{-1}(x) = \frac{4x+5}{3x-2}.$$

Question 4 *The demand function from selling* x *units is given by* $D(x) = 2x + 5$*. Then the marginal revenue from selling 3 units will be:*

A) 5 **B)** 17 **C)** 21 **D)** 33 **E)** 41

Solution: $R(x) = xD(x) = 2x^2 + 5x, \Rightarrow R'(x) = 4x + 5, \Rightarrow R'(3) = 17.$

Long Answer Questions (5-7)

Question 5 (5 points)

Let $f(x) = 3x^2 - 4$.

a) (3 points) Use <u>*the definition of the derivative*</u> *to compute $f'(2)$.*

b) (2 points) Compute the equation of the tangent line at $(2, f(2))$.

Solution: a) From the definition of the derivative, we have

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{[3(2+h)^2 - 4 - [3(2)^2 - 4]}{h}$$

$$= \lim_{h \to 0} (12 + 3h) = 12.$$

b) Note that $f(2) = 3(2)^2 - 4 = 8$, we get

$$y - 8 = 12(x - 2).$$

Simplify this yields

$$y = 12x - 16.$$

Question 6 (6 = 2 + 4 points) *Calculate the following limits:*
(i) $\lim\limits_{x\to 3} \frac{x^2-9}{x-3}$

Solution:
$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3}(x+3) = 6,$$

(ii) $\lim\limits_{x\to 1} \frac{\sqrt{x+3}-2}{x-1}$.

Solution:
$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x\to 1} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)}$$
$$= \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}.$$

Question 7 (6 = 2 + 2 + 2 points) *Calculate the derivatives of the following functions (do not need to simplify!):*

(i) $f(x) = (\sqrt{x} + 3x)(x^2 + x + 1)$

(ii) $g(x) = \frac{x^3+4x^2}{x^5+x+2}$

(iii) $h(x) = (x^3 - 2x - 1)^{50}$.

(i) Solution:

$$f'(x) = (\sqrt{x}+3x)'(x^2+x+1)+(\sqrt{x}+3x)(x^2+x+1)' = (\frac{1}{2}x^{-1/2}+3)(x^2+x+1)+(\sqrt{x}+3x)(2x+1).$$

(ii) Solution:

$$g'(x) = \frac{(x^3 + 4x^2)'(x^5 + x + 2) - (x^3 + 4x^2)(x^5 + x + 2)'}{(x^5 + x + 2)^2}$$
$$= \frac{(3x^2 + 8x)(x^5 + x + 2) - (x^3 + 4x^2)(5x^4 + 1)}{(x^5 + x + 2)^2}.$$

(iii) Solution:

$$h'(x) = 50(x^3 - 2x - 1)^{49}(x^3 - 2x - 1)' = 50(x^3 - 2x - 1)^{49}(3x^2 - 2).$$