FINAL EXAM-MATH 1300 C Winter 2009

Part 1. Long Answer Questions

Question 1 (5 = 2 + 3 points) *Calculate the derivatives of the following functions (do not need to simplify!):*

(i) $g(x) = 2x^3 + \sqrt{x^4 + x^2 + 1}$, *find* $g'(x)$.

Solution:

$$g' = 6x^2 + \frac{1}{2}(x^4 + x^2 + 1)'(x^4 + x^2 + 1)^{-1/2} = 6x^2 + \frac{1}{2}(4x^3 + 2x)(x^4 + x^2 + 1)^{-1/2}$$

(ii) $f(x) = x^3 \ln(1 + x^2)$, *find* $f'(x)$.

Solution: $f'(x) = 3x^2 \ln(1 + x^2) + \frac{2x^4}{1+x^2}$

Question 2 *(3+3) Find the following indefinite integrals:*

a) $\int 4x \ln x dx$.

Sol: INTEGRATION-BY-PARTS

$$\int 4x \ln x dx = \int 2 \ln x dx^2 = 2x^2 \ln x - \int 2x^2 d \ln x = 2x^2 \ln x - \int 2x dx = 2x^2 \ln x - x^2 + C.$$

b) $\int 3x^2(x^3-1)^7 dx$.

Sol: let $u = x^3 - 1$, then $du = 3x^2 dx$.

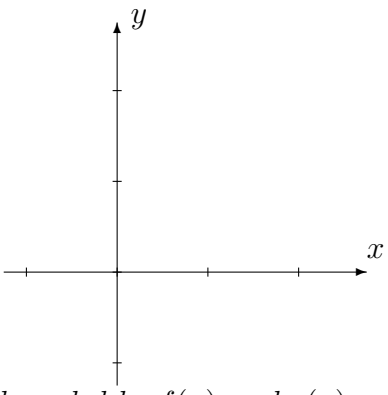
$$\int 3x^2(x^3-1)^7 dx = \int u^7 du = \frac{1}{8}u^8 + C = \frac{1}{8}(x^3-1)^8 + C.$$

Question 3 *(1+1+4) Consider the functions* $f(x) = 4 - x^2$ *and* $g(x) = x + 2$.

a) Find the intersection point(s) of the graphs of these functions.

$4 - x^2 = x + 2 \Rightarrow x = 1, -2.$

b) Sketch the graphs of $f(x)$ *and* $g(x)$ *in the same figure, and shade the region between the two graphs.*



c) Calculate the area of the region bounded by $f(x)$ *and* $g(x)$.

Area =

$$\int_{-2}^{1}(4 - x^2 - x - 2)dx = (2x - \frac{1}{3}x^3 - \frac{1}{2}x^2)|_{-2}^{1} = \frac{7}{6} - (-\frac{10}{3}) = \frac{9}{2}.$$

Question 4 *(5+2+2) Let* $f(x,y) = 42x + 102y - 5x^2 - 2xy - 8y^2$.

a) Find partial derivatives:

$f_x(x,y) = \underline{42 - 10x - 2y}$, $f_y(x,y) = \underline{102 - 2x - 16y}$,

$f_{xx}(x,y) = \underline{-10}$, $f_{xy}(x,y) = \underline{-2}$, $f_{yy}(x,y) = \underline{-16}$.

b) Find all critical points of $f(x,y)$.

$$42 - 10x - 2y = 0, \quad 102 - 2x - 16y = 0, \Rightarrow x = 3, y = 6.$$

c) Using the Second-Partials Test, classify the critical points (local min/max, saddle points).

$$d(3,6) = (-10)(-16) - (-2)^2 > 0, f_{xx}(3,6) = -10 < 0, \Rightarrow$$

$f(3,6)$ is a relative max.

Part 2. Multiple Choice Questions

Question 5 *Solve the equation* $1 + 2\ln(x) = 5$.

A) e^2 **B)** $e - 1$ **C)** $e^2 - 1$ **D)** $2e + 1$ **E)** $e + 1$

A

Question 6 *The function* $y = y(x)$ *is given implicitly by* $x^2 + y^2 + y = 7x$. *Find the slope of the graph of* $y = y(x)$ *at the point* $(1, 2)$.

A) 0 **B)** $-\frac{7}{3}$ **C)** 1 **D)** 2 **E)** -1

C

Question 7 *Let* $f(x) = \ln(x-6)$. *Find the domain of* $f(x)$.

A) $x > e$ **B)** $x \neq 0$ **C)** $x > 0$ **D)** $x > 6$ **E)** $x < e$

D

Question 8 *Let* $f(x) = \frac{x^2-9}{x-3}$. *Find* $\lim_{x \to 3} f(x)$.

A) 0 **B)** ∞ **C)** 9 **D)** 4 **E)** 6

E

Question 9 *Assume $f(x)$ is a certain function with $f'(x) = 2(x+2)^2(x-3)$. Find the set of x where f is increasing.*

A) $(3, +\infty)$ **B)** $(-\infty, -2)$ **C)** $(-2, 3)$ **D)** $(-\infty, 3)$ **E)** $(-2, +\infty)$

A

Question 10 *Assume $f(x)$ is a certain function with $f''(x) = x^2 + x - 12$. Find the set of x where f is concave up.*

A) $(-4, 3)$ **B)** $(-3, 4)$ **C)** $(3, +\infty)$ **D)** $(-\infty, -3) \cup (4, +\infty)$ **E)** $(-\infty, -4) \cup (3, +\infty)$

E

Question 11 *If $f(x)$ is a function such that $f'(x) = \frac{1}{x}$ and $f(e^2) = 4$, find $f(1).$*

A) 4 **B)** 1 **C)** 3 **D)** 2 **E)** 0

D

Question 12 *Find the value of the definite integral $\int_1^3 (2x + 1)\, dx.$*

A) 4 **B)** 10 **C)** 3 **D)** 5 **E)** 9

B

Question 13 *Let* $f(x) = \frac{6x^4-3x^3+x^2}{-2x^4-2x^3+x}$. *Find the equation of the horizontal asymptote, if it exists.*

A) $y = 2$ **B)** $y = -3$ **C)** $y = 0$ **D)** $y = -4$ **E)** does not exist

B

Question 14 *Find the equation of the tangent line to the curve* $y = 3x^2 - 6x + 1$ *at the point* $(2, 1)$.

A) $y = 6x$ **B)** $y = -3x + 4$ **C)** $y = -3x - 8$ **D)** $y = -4x$ **E)** $y = 6x - 11$

E

Question 15 *The profit function for selling x units of a certain product is given by*

$$P(x) = xe^{2-x}.$$

Find the absolute maximum profit on $[0, 2]$.

A) 1 **B)** 2 **C)** e^2 **D)** e **E)** $2e^2$

D

Question 16 *The position of a particle is given by*

$$s = t^3 - 3t^2 + 5t + 11, \quad t \geq 0.$$

where s is measured in meters (m) and t in seconds (s). When is the acceleration 6 m/s^2?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

C

Extra page for additional work