Midterm Exam

(1) The response of a LTI causal system to the input $u_1(t) = u_s(t)\, u_s(2-t)$ is $y_1(t) = (1-e^{-t})\, u_s(t)\, u_s(2-t) + e^{-(t-2)} u_s(t-2)$. Find the response of the system to a new input $u_2(t) = (t-2)\, u_s(t-2)\, u_s(4-t)$ for $t \le 4$. Note that $u_s(t)$ designates a unit step function.

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(2) Consider a LTI system
$$\begin{cases} \dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & -1 \end{bmatrix} x \end{cases}$$

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Let $u(t) = \begin{bmatrix} -3e^{-t} + 5e^{-2t} \\ 3e^{-2t} \end{bmatrix}_{2\times 1}$ for $t \ge 0$, and suppose that $y(t) = -2e^{-t} + 6e^{-2t}$. If $\lim_{t\to\infty} x(t) = 0$, find $x(0)$.

find $y(t)$, $\dot{y}(t)$; $\ddot{y}(t)$ and eliminate them to get $x_1(t)$ and $x_2(t)$

$x(0)$

(3) Verify whether the input-output map

$x(t)$ $\dot{x}(t)$ $x(t)$

$$y(t) = \int_{t_0}^{t} e^{\sin(t-\tau+1)} \cos(u(\tau)-1)\, d\tau \; ; \quad t \ge t_0$$

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(i) is linear or nonlinear, (ii) is time-invariant or time-varying, and (iii) is causal or non-causal. Show all work and provide reasoned arguments.

(4) Find a state space representation (A, B, C, D) for the system

$$\ddot{y}_1 + 2\dot{y}_1 + 3(y_1 - y_2) = 2u_1 + \dot{u}_2$$

$$\dot{y}_2 - 4(y_1 - y_2) = 2u_2 + 4\dot{u}_1$$

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Mid term Exam #1

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ALL PROBLEMS ARE EQUALLY WEIGHTED

(1) Verify whether the input-output map

$$y(t) = \int_{-\infty}^{t} \sin(t-\tau)\, e^{u(\tau)}\, d\tau$$

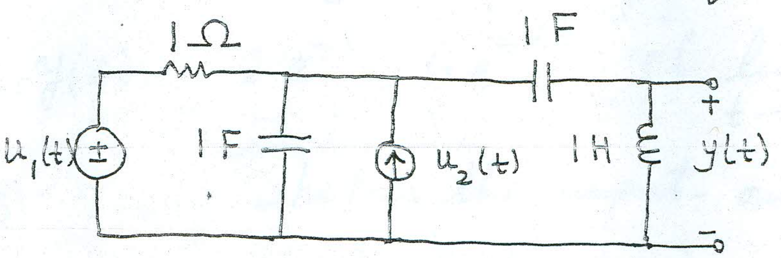
(i) linear or nonlinear

(ii) time-invariant or time-varying

(iii) Causal or non-causal

Show all work and provide reasoned arguments.

(2) Write the state and output equations for the network shown below:



Also, obtain the A, B, C, and D matrices.

(3) The response of a LTI causal system to the input $u_1(t)$ is $y_1(t) = e^{-2t}\sin(t-1)\,u_s(t)$, where $u_s(t)$ is the unit step input. The response to a second input $u_2(t)$ is $y_2(t) = \cos 2t\ u_s(t)$. Compute the response to $u_3(t) = 2u_1(t-1) + \dfrac{du_2(t+1)}{dt}$.

$u_1(t) =$

in LTI system, whatever you apply to an i/p is applied to an o/p

$y_3(t) = 2y_1(t-1) + \dfrac{dy_2}{dt}$

(4) Consider the state equation $\dot{x} = Ax$. It is known that the fundamental matrix at $t = 3$ is $M(3) = \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$ and further $M(0) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. Find the state transition matrix $\longrightarrow \Phi(6,0)$.