UNIVERSITY OF CALGARY

NOS SHUILE TOGAM SUAS

FINAL EXAMINATION
(Fall, 2002-2003)
ENGG 201 - Behaviour of Liquids, Gases and Solids

December 14, 2002 Time Allowed: 180 minutes.

1. Attempt all ten (10) questions. Weighting as noted. Total marks = 100.
2. Electronic calculators are permitted. No books or notes are allowed.
3. A summary of useful formulas is provided on the last pages of the exam paper.
4. Print your name clearly in the space provided on the cover page. Write your ID No. on the last page and all other pages in the provided space.

SOLUTION

*__L01__ for MWF Lectures (Dr. Maini);
__L02__ for T R Lectures (Dr. Azaiez).

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student l.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an l.D. card who can produce an **acceptable** alternative l.D. e.g. one with a printed name and photograph, are allowed to write the examination.

A student without acceptable l.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A **Student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.**

EXAMINATION RULES

(1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
(2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
(3) All inquiries and requests must be addressed to supervisors only.
(4) **Candidates are strictly cautioned against:**
(a) speaking to other candidates or communicating with them under any circumstances whatsoever: (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
(c) making use of calculators and/or portable computing machines not authorized by the instructor: (d) leaving answer papers exposed to view;
(e) attempting to read other student's examination papers.
The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
(5) Candidates are requested to write on both sides of the page, unless the examiner has asked that the left hand page be reserved for rough drafts or calculations.
(6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
(7) Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.
(8) The candidate is to write his/her name on each answer book as directed and is to number each book.
(9) A candidate must report to a supervisor before leaving the examination room.
(10) Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
(11) If during the course of an examination a student becomes ill or receives word of domestic affliction, the student should report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred examination is supported by a completed Physician/Counsellor Statement form. Students can consult professionals at University Health Services or university Counselling Services during normal working hours or consult their physician/counsellor in the community.
Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such a request will be denied.
(12) Smoking during examinations is strictly prohibited.

RO 93-02

Question No.	Maximum Mark	Mark Obtained
I		
II		
III		
IV		
V		
VI		
VII		
VIII		
IX		
X		
Total (out of 100)		

Problem I:
Air is approximately 80mol% Nitrogen (M=28kg/kmol) and 20mol% Oxygen (M=32kg/kmol). Consider a rigid container of volume 112litre that contains air at P=1atm.Throughout this problem, the temperature of the container is maintained fixed and equal to T=0^{o}C. Use the ideal gas law to answer the following questions:

(a) Determine the number of moles of N_2 and of O_2 in the container.
(b) A certain amount of Hydrogen (M=2kg/kmol) is added to the container such that the pressure becomes equal to 1.1atm. Determine the mass of Hydrogen added.

Answer:

(a) Pv=nRT➔ n= (P.v)/(R.T) = 101325(Pa) * 112x10^{-3} (m3)/8.314(J/mol.K)*273.15(K)
= 4.997mol

n_{N2} = 0.8*n= **3.997 mol ≈4mol**
n_{O2} = 0.2*n= **0.999 mol ≈1mol**

(b) The new pressure after adding Hydrogen is,
P=nRT/v ➔ n=P.v/RT
= (1.1*101325*112x10^{-3})/8.314*273.15
=5.497 mol

Therefore, the number of moles of Hydrogen added is,
n_{H2} =5.497-4.997=0.5mol

The mass of Hydrogen added is,
m_{H2} = n_{H2} * M_{H2}=0.5 (mol)*(2g/mol) =**1g.**

Problem II:

The interactions between two non-polar molecules can be described by the Mie potential function given below.

$$\phi(r) = \alpha\left[\left(\frac{\sigma}{r}\right)^{d} - \left(\frac{\sigma}{r}\right)^{c}\right]$$

In the above expression, α, σ, d and c are constants and r is the distance between the centres of the molecules.

Use this model to derive a relationship between the equilibrium distance r_o between two non-polar molecules and the parameters of the Mie potential function (α, σ, d and c).

Answer:

$$\phi(r) = \alpha\left[\left(\frac{\sigma}{r}\right)^{d} - \left(\frac{\sigma}{r}\right)^{c}\right]$$

$$\frac{d\phi(r)}{dr} = \alpha\left[-\frac{d}{r}\left(\frac{\sigma}{r}\right)^{d} + \frac{c}{r}\left(\frac{\sigma}{r}\right)^{c}\right] = 0$$ at minimum potential. Therefore,

$$\frac{d\sigma^{d}}{r^{d+1}} = \frac{c\sigma^{c}}{r^{c+1}}$$, or

$$\left(\frac{\sigma}{r}\right)^{d-c} = \frac{c}{d}$$, or

$$\left(\frac{r}{\sigma}\right)^{d-c} = \frac{d}{c}$$, or

$$r = \sigma\left(\frac{d}{c}\right)^{1/(d-c)}$$

Problem III:

A porous packing contains uniform glass spheres of 5 mm diameter arranged in a face centred cubic arrangement (FCC). The density of solid glass is given as 2650 kg/m^3. For this packing calculate the following:

(a) Volume of a single face centred cubic unit cell.

(b) Mass of one cubic meter of the porous bed.

Answer:

(a) Diagonal of the face of unit cell = 5 x 2 = 10 mm.

Side of the cube = $10/\sqrt{2} = 7.071\,\text{mm}$

Volume of unit cell = $(7.071\,\text{mm})^3 = 353.6\,\text{mm}^3 = 3.53\times10^{-7}\,\text{m}^3$

(b) Number of spheres per unit cell = 4

Number of spheres per m^3 of packing = $\dfrac{4}{3.53\times10^{-7}} = 1.13\times10^{7}$

Volume of one sphere = $\dfrac{\pi}{6}D^3 = \dfrac{\pi\times5^3}{6} = 65.45\,\text{mm}^3 = 6.545\times10^{-8}\,\text{m}^3$

Mass of one sphere = 5.545 x 2650 x 10^{-8} = 1.7344 x 10^{-4} kg

Mass of one cubic meter of packing = 1.7344 x 10^{-4} kg x 1.13 x 10^{7} = **1.96 x 10^{3} kg**

Problem IV:

Consider a vessel that contains hydrogen gas (M=2kg/kmol). It is found that $3x10^{24}$ hydrogen molecules per minute strike $1.0cm^2$ of the vessel wall. Determine the pressure of the gas in the container knowing that the gas molecules move with an average speed of 10^3m/s.

<u>Answer:</u>

The rate of collisions per unit area, $Z=3x10^{24}/[60(s)x10^{-4}(m^2)]=5x10^{26}$ collisions/m^2.s

$Z= \rho_N*c/4$ ➔ $\rho_N =4*Z/c=2x10^{24}\ m^{-3}$

$\rho_N =P/kT$ ➔ $P= \rho_N*k*T$

But $\bar{c}=\sqrt{8.R.T/\pi.M}$ ➔ $T=\pi.M.c^2/8*R= \pi *2*10^6/(8*8314)=94.47K$

Therefore, $P= \rho_N*k*T =2x10^{24}*1.3805x10^{-23}*94.47=$**<u>2608Pa.</u>**

Problem V:
A pipe is carrying natural gas that contains 85 mole% methane, 10 mole% ethane and 5 mole% propane. The volumetric flow rate, measured at 10MPa pressure and 12°C temperature is 0.534m^3/s. Some of the pure component properties are provided in the table below.

Calculate the mass flow rate of gas (Mass of gas flowing per second in units of kg/s) using the generalized compressibility chart.

Component	Molar Mass (kg/kmol)	T_c (K)	P_c (atm)
Methane	16	190.6	45.4
Ethane	30	305.4	48.2
Propane	44	369.8	41.9

Answer:

Component	Molar Mass (kg/kmol)	Mole%	Tc (K)	Pc (atm)	y_iM_i	y_iT_{ci}	y_iP_{ci}
Methane	16	85	190.6	45.4	13.6	162.01	38.59
Ethane	30	10	305.4	48.2	3.	30.54	4.82
Propane	44	5	369.8	41.9	2.2	18.49	2.10
					18.8	211.04	45.51

Average molar mass = 18.8 kg/kmol

P_{pc} = 45.5 atm

T_{pc} = 211 K

T_r = 285.15/211 = 1.35

$P_r = 10^7/(101325 \times 45.5) = 2.17$

Value of Z from the compressibility chart = 0.75

$$v_m = \frac{ZRT}{P} = \frac{0.75 \times 8314 \times 285.15}{10^7} = 0.178 \frac{m^3}{kmol}$$

Use basis of one second of time.

Volume flowing = 0.534 m^3

Number of moles = 0.534 m^3/(0.178 m^3/kmol) = 3 kmol

Mass = 3 kmol x 18.8 kg/kmol = 56.46 kg

The mass flow rate is 56.46 kg/s

Problem VI:

A pressure vessel of 5 litre internal volume contains a mixture of n-pentane and n-hexane. The following information on the pure component properties is provided.

Component	Molar Mass (kg/kmol)	Normal Boiling Point (^oC)	Latent Heat of Vaporization (MJ/kmol)
n-Pentane	72	36.1	25.77
n-Hexane	86	68.7	29.04

The pressure and temperature in the vessel are 200 kPa and 68.7 ^oC respectively and it is known that both liquid and vapour phases are present in the vessel and the system obeys Raoult's law.

Calculate the Molar composition of the liquid phase present in the vessel.

<u>Answer:</u>

The first step is to calculate the vapour pressure of both components at the given vessel temperature. The vapour pressure of n-hexane is 1 atm, since the temperature is equal to the normal boiling point of n-hexane. For pentane we need to calculate the vapour pressure using

$$\ln(P_v) = \frac{-\Delta H_v}{RT} + C\ .$$

The vapour pressure is one atmosphere at the normal boiling point.

$$\ln(101.325) = \frac{-25.77\times10^6}{8314\times(36.1+273.15)} + C$$

C = 14.64

At 68.7 ^oC:

$$\ln(P_v) = \frac{-25.77\times10^6}{8314\times(68.7+273.15)} + 14.64$$, which gives P_v = 263.2 kPa.

Let the mole fraction of pentane in the liquid be x. Then the total pressure is given by

$$P = x\times263.2 + (1-x)\times101.325 = 200$$

x = 0.61.

Hence the molar composition of the liquid is :

n-Pentane = 61 mole%
n-Hexane = 39 mole%

Problem VII:

This problem deals with a technique that can be used to determine the isobaric coefficient of volume expansion of materials,

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

Consider a solid of density ρ_s which is floating in a liquid of density ρ_L. It is found that when both the liquid and the solid are at a temperature of 0^oC, 82.1% of the volume of the solid is submerged in the liquid. When the temperature is raised to 50^oC, 86.6% of the volume of the solid is now submerged in the liquid.

Use the above information to determine the isobaric coefficient of volume expansion of the solid, knowing that that of the liquid is equal to $1.2x10^{-3}$ K^{-1}.

N.B. *The buoyancy force acting on a solid floating in a liquid is* $F_b = \rho.g.V_s$, *where* ρ *is the density of the liquid, g is gravitational acceleration and* V_s *is the volume of the solid submerged in the liquid.*

Answer:

We need to write the balance of forces for the floating solid, i.e. the weight must be balanced by the buoyancy force:

$$\text{Weight of solid} = F_b$$

$$\rho_s .v.g = \rho_L.v_{sub}.g$$

$$\rho_s = \rho_L .(v_{sub}/v)$$

where v is the total volume of the solid and v_{sub} is the volume of the submerged solid.

Case1: at $T=0^oC$: $\rho_{s,0} = \rho_{L,0}.(v_{sub}/v)_0$

Case2: at $T=50^oC$: $\rho_{s,50} = \rho_{L,50}.(v_{sub}/v)_{50}$

Therefore,

$$\frac{\rho_{s,0}}{\rho_{s,50}} = \frac{\rho_{L,0}}{\rho_{L,50}}.\frac{\left(v_{sub}/v\right)_0}{\left(v_{sub}/v\right)_{50}}$$

But, $$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V_0}\left(\frac{V - V_0}{T-T_0}\right) = \rho_0\left(\frac{1/\rho - 1/\rho_0}{T-T_0}\right) = \left(\frac{\rho_0/\rho - 1}{T-T_0}\right) \Rightarrow \frac{\rho_0}{\rho} = 1+\alpha_p(T-T_0)$$

Therefore,

$$\frac{\rho_{s,0}}{\rho_{s,50}} = \frac{\rho_{L,0}}{\rho_{L,50}}.\frac{\left(v_{sub}/v\right)_0}{\left(v_{sub}/v\right)_{50}} = [1+\alpha_{PL}.(T-T_0)]\frac{\left(v_{sub}/v\right)_0}{\left(v_{sub}/v\right)_{50}}$$

$$= [1+1.2x10^{-3}.(50-0)]x\frac{0.821}{0.866} = 1.00492$$

Now we can get the value of α_{ps},

$$\alpha_{ps} = \left(\frac{\rho_{s0}/\rho_{s,50} - 1}{T-T_0}\right) = \frac{1.00492-1}{50} = \mathbf{\underline{9.8x10^{-5}\ K^{-1}.}}$$

The students can also answer the problem this way

The volume expansion of liquid by heating is given by

V = Vo (1 + α ΔT), therefore

$$\frac{V_{50}}{V_o} = 1 + 1.2\times10^{-3}\times(50-0) = 1.06$$

The change in density of liquid by heating is inverse of the change in volume, i.e.

$$\frac{\rho_0}{\rho_{5o}} = 1.06$$

The density of solid at 0 ^oC is related to liquid density at 50 ^oC by the fraction submerged,

$$\rho_{s,50} = 0.866\times\rho_{l,50}$$

At 0 ^oC:

$\rho_{s,0} = 0.821\times\rho_{l,0}$, and combining these two we get.

$$\frac{\rho_{s,0}}{\rho_{s,50}} = \frac{0.821}{0.866}\times\frac{\rho_{l,0}}{\rho_{l,50}} = \frac{0.821\times1.06}{0.866} = 1.00492$$

Change in the volume of solid is inverse of the change in density.

$$\frac{V_{s,50}}{V_{s,o}} = \frac{\rho_{s,0}}{\rho_{s,50}} = 1.00492$$

The volume change is related to α for solid as: $V_{s,50} = V_{s,0}(1 + \alpha\,\Delta T) = V_{s,0}(1 + 50\alpha)$

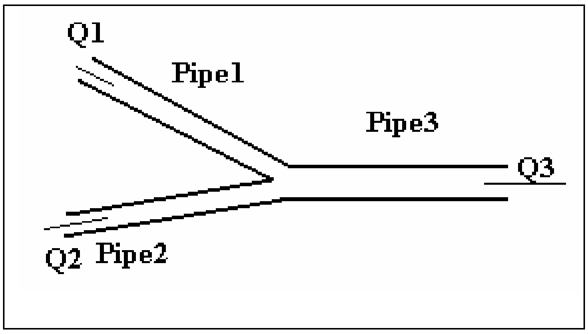
Comparing the above two relationships:

$50\alpha = 0.00492$

α = 0.00492/50 = 9.8 x 10^{-5} K^{-1}

Problem VIII:

Consider two liquids of densities ρ_1=1000kg/m^3 and ρ_2=800kg/m^3 that are initially flowing in pipe1 and pipe2, respectively. The two flow streams are mixed when they merge into the horizontal pipe3 as shown in the figure.



The flow rates in the three pipes are Q_1=10^{-2}m^3/s, Q_2=2.5x10^{-3}m^3/s and Q_3=1.25x10^{-2}m^3/s and the viscosities of all three fluids are equal:
$\mu_1 = \mu_2 = \mu_3 = 5.1\text{x}10^{-4}$Pa.s,

(a) Determine the density ρ_3 of the liquid mixture in pipe3.
(b) Determine the pressure drop per unit length (-ΔP/L) in pipe3 if the pipe is assumed to be smooth and to have a diameter equal to 20cm.

Answer:

(a) Conservation of mass: $\rho_1.Q_1 + \rho_2.Q_2 = \rho_3.Q_3$, therefore:

$\rho_3 = (\rho_1.Q_1 + \rho_2.Q_2)/Q_3$

$= (1000\text{x}10^{-2} + 800\text{x}2.510^{-3})/1.25\text{x}10^{-2}\text{m}^3 = \underline{960\text{kg/m}^3}$

(b) The viscosities of the fluids in the three pipes are the same, $\mu = 8\text{x}10^{-4}$ Pa.s.

The average velocity in pipe3 is, $u = 4.Q_3/\pi.D^2 = 4\text{x}1.25\text{x}10^{-2}\text{m}^3/\ \pi\text{x}(0.2)^2 = 0.398$ m/s

The Reynolds number in pipe3 is $Re = (\rho.v.D)/\mu = (960\text{x}0.398\text{x}0.2)/5.1\text{x}10^{-4} = 1.5\text{x } 10^5$

The flow in pipe3 is therefore turbulent, and from the moody chart, we get $f \approx 0.004$

Therefore, for the horizontal pipe3, we have:

$$(-\Delta P/L) = 2.\rho.f.u^2/D = 2\text{x}960\text{x}0.004\text{x}(0.398)^2/0.2 = \underline{6.08\text{ Pa/m}}$$

Problem IX:
A 10 m long steel pipe carrying steam has a wall thickness of 0.5 cm and an inside radius of 5 cm. The temperature of the outer surface of pipe is 198^{o}C and the temperature of the internal surface is at the steam temperature of 200 ^{o}C. The mass flow rate of steam through the pipe is 0.68 kg/s. The steam enters the pipe as 100% saturated vapour at 200^{o}C and some of it condenses due to the heat loss through the pipe.

Calculate the mass fraction of liquid in the flowing stream at the downstream end of the pipe.

Given Additional Information:

Thermal conductivity of steel	= 50 W/(m K)
Latent heat of vaporization of water at 200 ^{o}C	= 1940.6 kJ/kg

Answer:

$$Q = \frac{2\pi L\kappa(T_1 - T_2)}{\ln(r_2 / r_1)} = \frac{2\pi \times 10 \times 50 \times (200 - 198)}{\ln(5.5/5)} = 65923\,\mathrm{W} = 65.923\,\mathrm{kW}$$

Mass of steam condensed per second = $\dfrac{65.923\,\mathrm{kW}}{1940.6\,\mathrm{kJ/kg}} = 0.034\,\mathrm{kg/s}$

Fraction of liquid at the downstream end of the pipe = 0.034/0.68 = 5%

Problem X:

A rod made of Aluminum has an initial length of 1m, and a square section (2cmx2cm). The rod is compressed isothermally along its length by a force F=100kN.

For Aluminum we give: Young modulus, E= 68.9 x 10^9 Pa

Poisson ratio, ν =0.33

(a) Determine the new length of the rod.
(b) Determine the new size of the square section of the rod
(c) Determine the relative change in the density of the rod.

Answer:

(a)

$$\sigma_y = \frac{F}{A} = \frac{F}{a^2} = \frac{-10^5}{(2.10^{-2})^2} = -0.25x10^9 \text{ Pa}$$

$$\varepsilon_y = \frac{\sigma_y}{E} = \frac{-0.25x10^9}{68.9x10^9} = -3.628x10^{-3}$$

$$L = L_o(1+\varepsilon_y) = 1.(1-3.628x10^{-3}) = 0.9964\text{m}$$

(b)

$$\varepsilon_x = \varepsilon_z = -\upsilon.\varepsilon_y = -0.33x(-3.628x10^{-3}) = 1.197x10^{-3}$$

$$L_x = L_y = L_{ox}(1+\varepsilon_x) = 2x(1+1.197x10^{-3}) = 2.0024\text{cm}$$

The new area is A = $(2.0024)^2$=4.0096cm^2

(c) Before the compression, the volume is: $v_o = L_o x A_o = 1x(0.02)^2 = 4.0000x10^{-4}$ m^2

After the compression, the volume is: $v = L\,xA = 0.9964x(0.020024)^2 = 3.9952x10^{-4}$ m^2

$$\frac{\Delta\rho}{\rho} = \frac{\rho-\rho_o}{\rho_o} = \frac{\frac{m}{v}-\frac{m}{vo}}{\frac{m}{vo}} = \frac{vo}{v} - 1 = 1.2x10^{-3}$$ or <u>0.12%</u>

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