Assignment Answers: ECO 1192D (Winter 2017)

CHAPTER 2

1) **If the effective equivalent annual interest rate is 22.2%, and interest is compounded monthly, what is the corresponding nominal annual interest rate?**

The following information are given:
The effective equivalent annual interest rate, i_e = 22.2% = 0.222
Compounding period is monthly. Hence, $m = 12$
The corresponding nominal annual interest rate, r = ?

The relationship between these three are given by the equation:

$$i_e = (1 + r/m)^m - 1$$

$$\Rightarrow \quad i_e + 1 = (1 + r/m)^m$$

$$\Rightarrow \quad (i_e + 1)^{1/m} = 1 + r/m$$

$$\Rightarrow \quad r/m = (ie + 1)^{1/m} - 1$$

$$\Rightarrow \quad r = m * [(i_e + 1)^{1/m} - 1]$$

$$\Rightarrow \quad r = 12 * [(0.222 + 1)^{1/12} - 1]$$

$$\therefore \quad r = 0.2021 = 20.2\%$$

CHAPTER 2

2) **How long will it take for a $250 deposit to double in value for each of the following: i) 8% per year, compounded monthly; and ii) 11% per year, compounded weekly.**

Let the present worth, P = X (here it's $250) and Future worth, F = 2X
The relationship between the present and future worth is given by the formula:

$$F = P\,(1 + i_e)^N$$

$$\Rightarrow \quad F/P = (1 + i_e)^N$$

$$\Rightarrow \quad \ln(F/P) = N \ln(1 + i_e)$$

$$\Rightarrow \quad N = \ln(F/P) / \ln(1 + i_e)\text{, call it Eq A}$$

This equation will give us the time required to convert any present worth P into a future worth F. Note that when the future worth is twice the amount of present worth, the numerator on the right had side of the above equation reduces to $\ln(F/P) = \ln(2P/P) = \ln(2)$

Now for the above problem, we first need to calculate the equivalent annual effective interest rates for:

i) 8% per year, compounded <u>monthly</u>

Hence:

Compounding period, $m = 12$

Nominal annual interest rate, $r = 8\% = 0.08$

Equivalent effective annual interest rate, $i_e = ?$

$$\begin{aligned} i_e &= (1 + r/m)^m - 1 \\ &= (1 + 0.08/12)^{12} - 1 \\ &= 0.083 = 8.3\% \end{aligned}$$

Thus, using Eq A from above, we can calculate the time required to double the initial deposit of $250 (i.e., to accumulate $500) as:

$$\begin{aligned} N &= \ln(F/P) / \ln(1 + i_e) \\ &= \ln(500/250) / \ln(1 + 0.083) \\ &= \ln(2) / \ln(1.083) \\ &\approx 8.7 \text{ years} \end{aligned}$$

i) 11% per year, compounded <u>weekly</u>

Hence:

Compounding period, $m = 52$

Nominal annual interest rate, $r = 11\% = 0.11$

Equivalent effective annual interest rate, $i_e = ?$

$$i_e = (1 + r/m)^m - 1$$
$$= (1 + 0.11/52)^{52} - 1$$
$$= 0.11615 = 11.6\%$$

Again, using Eq A from above, we can calculate the time required to double the initial deposit of \$250 (i.e., to accumulate \$500) as:

$$N = \ln (F/P) / \ln (1 + i_e)$$
$$= \ln (2P/P) / \ln (1 + 0.116)$$
$$= \ln (2) / \ln (1.116)$$
$$\approx 6.3 \text{ years}$$

CHAPTER 3

3) **How much should you invest today at 12% interest to accumulate $1,000,000 in 20 years?**

The following information are given:
Future amount (i.e. Future worth), F = $1,000,000
Interest rate, i = 12% = 0.12
Interest period, N = 20 years
What amount should be invested, i.e. what is the present worth, P = ?

The relationship between the present and future worth is given by the formula:

$$F = P\,(1 + i)^N$$

$$\Rightarrow \quad 1{,}000{,}000 = P\,(1 + 0.12)^{20}$$

$$\Rightarrow \quad P = 1000000/(1.12)^{20}$$

$$\Rightarrow \quad P = 1000000/(1.12)^{20}$$

$$\therefore \quad P = 103{,}666.8$$

You should invest $103,666.8 today.

CHAPTER 3

4) **What is the present worth of the total 20 payments, occurring at the end of every four months (i.e. the first payment is in four months), which are $400, $500, $600, increasing by a fixed sum. Interest is 10% nominal per year, compounded weekly.**

First we need to find the effective annual interest rate in order to determine the effective interest rate for the four-month period.
Here, the following information are given:
The nominal annual interest rate is, $r = 10\% = 0.1$
Compounding period is weekly. Hence, $m = 52$
The effective equivalent annual interest rate, $i_e = ?$

Using $i_e = (1 + r/m)^m - 1$, we find
$i_e = (1+.10/52)^{52} - 1 = 0.105065$

Now we use the above effective interest rate to find the effective interest rate for every four months (where $\boldsymbol{m} = 12/4 = 3$). Hence:

$$0.105065 = (1 + \boldsymbol{i}_{4\text{-months}})^{\boldsymbol{m}} - 1$$
$$\Rightarrow \quad 0.105065 = (1 + \boldsymbol{i}_{4\text{-months}})^3 - 1$$
$$\Rightarrow \quad 1.105065 = (1 + \boldsymbol{i}_{4\text{-months}})^3$$
$$\Rightarrow \quad 1 + \boldsymbol{i}_{4\text{-months}} = (1.105065)^{1/3}$$
$$\Rightarrow \quad \boldsymbol{i}_{4\text{-months}} = (1.105065)^{1/3} - 1$$
$$\therefore \quad \boldsymbol{i}_{4\text{-months}} = 0.033862$$

So, the effective interest rate for every-four months is: 0.033862 or about 3.4%

Now note that the series of 20 payments have the form of an Annuity (A) with arithmetic gradient ($G = 100$). Hence, in order to find the present worth (P) of the whole series, we need to:

i) First use, the *arithmetic gradient to annuity conversion factor* (A/G, i, N)to convert the series into an annuity and then
ii) Use *series present worth factor* (P/A, i, N) to convert the annuity into present worth,

where, $G = 100$, $\boldsymbol{i} = \boldsymbol{i}_{4\text{-months}} = 0.033862$, $N = 20$

i) Converting the gradient series into an Annuity

A = 400 + 100 * (A/G, $\boldsymbol{i}_{\text{4-months}}$, N)

= 400 + 100 * (A/G, 3.3862%, 20)

= 400 + 100 * [(1/0.033862) – (20 / {(1 + 0.033862)^{20} – 1}]

= 400 + 100 * 8.400853

= 400 + 840.0853

= 1240.09

ii) Converting the Annuity of 20 payments into Present Worth

P = A* (P/A, $\boldsymbol{i}_{\text{4-months}}$, N)

= A* (P/A, 3.3862%, 20)

= 1240.09 * [{(1 + $\boldsymbol{i}$)^N – 1} / { $\boldsymbol{i}$ (1 + $\boldsymbol{i}$)^N}]

= 1240.09 * [{(1 +0.033862)^{20} – 1} / { 0.033862 * (1 + 0.033862)^{20}}]

= 1240.09 * 14.35987

= 17,819.16

The present worth is $17,819.16.

CHAPTER 3

5) **Octavia is looking at an investment in upgrading an inspection line at her plant. The initial cost would be $150,000 with a salvage value of $40,000 after five years. How much money must be saved every year to justify the investment at an interest rate of 15%?**

The following information are given:
The initial cost, P = $150,000
The salvage value, S = $40,000
The project life, N = 5 years
Interest rate, i = 15% = 0.15

To determine how much must be saved every year (i.e., to determine the Annuity, *A*) to justify the investment, we need to use the *Capital Recovery Formula*:

$A = (P - S)(A/P, \boldsymbol{i}, N) + S\boldsymbol{i}$

$= (150{,}000 - 40{,}000)\ [\{\ \boldsymbol{i}\ (1 + \boldsymbol{i})^{N}\} / \{(1 + \boldsymbol{i})^{N} - 1\}] + 40{,}000 * \boldsymbol{i}$

$= (150{,}000 - 40{,}000)\ [\{\ 0.15\ (1 + 0.15)^{5}\} / \{(1 + 0.15)^{5} - 1\}] + 40{,}000*(0.15)$

$= 110{,}000 * (0.298316) + 40{,}000 * (0.15)$

$= 38{,}814.76$

The investment would have to save about $38,815 per year over its 5-year life.

CHAPTER 4

6) **Margaret has a project with $28,000 first cost that returns $5,000 per year over 10-year life. It has a salvage value of $3,000 at the end of 10 years. If the MARR is 15%, what is the present worth of this project?**

The following information are given:
The first cost, P = $28,000
The return comes in the form of an annuity, A = $5,000
The salvage value, S = $3,000
The project life, N = 10 years
MARR, i = 15% = 0.15

The present worth of the project is given by:

$$\begin{aligned} PW &= -28{,}000 + 5{,}000 * (P/A, i, N) + 3{,}000 * (P/F, i, N) \\ &= -28{,}000 + 5{,}000 * (P/A, 15\%, 10) + 3{,}000 * (P/F, 15\%, 10) \\ &= -28{,}000 + 5{,}000 * [\{(1 + i)^N - 1\} / \{i(1 + i)^N\}] + 3{,}000 * [1 / (1 + i)^N] \\ &= -28{,}000 + 5{,}000 * [\{(1 + 0.15)^{10} - 1\} / \{0.15 * (1 + 0.15)^{10}\}] + 3{,}000 * [1/(1 + 0.15)^{10}] \\ &= -28{,}000 + 5{,}000 * (5.0188) + 3{,}000 * (0.24718) \\ &= -2{,}164.46 \end{aligned}$$

The present worth of the project is about −$2,164.

CHAPTER 4

7) **You want to have $750,000 in the bank when you retire. You think you can save $10,000 a year in a bank that offers you 7.5% interest. If you make your first deposit in a year's time, how many years will it be from now before you can retire?**

The following information are given:
The retirement target (i.e., the future worth), F = $750,000
The savings goal (i.e., the annuity), A = $10,000
The interest rate, i = 7.5% = 0.075
Time required (i.e. the compounding period), N = ?

Note that the question seeks to convert an annuity into a future worth over the compounding period. Hence, the *uniform series compound amount factor* – $(F/A, i, N) = [\{(1 + i)^N - 1\} / i]$ – needs to be used.

Hence we need to solve:

$$F = A * (F/A, i, N)$$
$$\Rightarrow \quad 750{,}000 = 10{,}000 * (F/A, 0.075, N)$$
$$\Rightarrow \quad 750{,}000 = 10{,}000 * [\{(1 + i)^N - 1\} / i]$$
$$\Rightarrow \quad 750{,}000 = 10{,}000 * [\{(1 + 0.075)^N - 1\} / 0.075]$$
$$\Rightarrow \quad 750{,}000 / 10{,}000 = [\{(1 + 0.075)^N - 1\} / 0.075]$$
$$\Rightarrow \quad 75 = \{(1 + 0.075)^N - 1\} / 0.075$$
$$\Rightarrow \quad 75 * 0.075 = (1.075)^N - 1$$
$$\Rightarrow \quad 5.625 = (1.075)^N - 1$$
$$\Rightarrow \quad 6.625 = (1.075)^N$$
$$\Rightarrow \quad \ln(6.625) = N \ln(1.075)$$
$$\Rightarrow \quad N = \ln(6.625) / \ln(1.075)$$
$$\therefore \quad N = 26.15 \text{ years}$$

CHAPTER 5

8) **Sam has three investment opportunities. The first one will require an initial cost of $100,000 and will return $150,000 one year from now. The second investment requires an outlay of $200,000 and will return $300,000 after one year. The third one requires an upfront cost $250,000 and will return $355,000 one year from now. The problem is that only one investment opportunity can be availed. Alternatively Sam can put his money into a savings bond that pays 20%. Which investment opportunity should Sam opt for?**

The following information are given:
For all projects, project life is, $N = 1$ year
Only one project can be undertaken; that is, projects are *mutually exclusive*
The alternative option (i.e., he do-nothing option) pays, MARR = 20% = 0.2
We need to calculate the IRR and Incremental IRR.

By definition, IRR is the interest rate () at which:
PW of all investments = PW all returns

Increment from do-nothing to 1st project:

$100{,}000 = 150{,}000\ (P/F, i^*, N)$

$\Rightarrow\quad 100{,}000 = 150{,}000\ [1 / (1 + i^*)^1]$

$\Rightarrow\quad 10 = 15\ [1 / (1 + i^*)]$

$\Rightarrow\quad (1 + i^*) = 15 / 10$

$\Rightarrow\quad i^* = 1.5 - 1$

$\therefore\quad i_1^* = 0.5 = 50\% > \text{MARR} = 20\%$

Increment is justified; we can accept the 1st project.
Note that for 1st project, IRR = Incremental IRR

IRR for 1st project:

$200{,}000 = 300{,}000\ (P/F, i^*, N)$

$\Rightarrow\quad 200{,}000 = 300{,}000\ [1 / (1 + i^*)^1]$

$\Rightarrow\quad 2 = 3\ [1 / (1 + i^*)]$

$\Rightarrow \quad (1 + i^*) = 3 / 2$

$\Rightarrow \quad i^* = 1.5 – 1$

$\therefore \quad i_2^* = 0.5 = 50\% > MARR = 20\%$

Thus, the IRR for 2nd project is 50%. Since it's higher than MARR, the project is justified as an *independent* project option. To justify the incremental investment requirement of (200,000 – 100,000 =) $100,000, however, we need to calculate the incremental IRR by comparing it to incremental return of (300,000 – 150,000 =) $150,000.

<u>Increment from 1st to 2nd project</u>:

$100{,}000 = 150{,}000 \ (P/F, i^*, N)$

$\Rightarrow \quad 100{,}000 = 150{,}000 \ [1 / (1 + i^*)^1]$

$\Rightarrow \quad 10 = 15 \ [1 / (1 + i^*)]$

$\Rightarrow \quad (1 + i^*) = 15 / 10$

$\Rightarrow \quad i^* = 1.5 – 1$

$\therefore \quad i_2^* = 0.5 = 50\% > MARR = 20\%$

So the incremental IRR is 50%. Thus the increment is justified; we can accept the 2nd project.

<u>IRR for 3rd project</u>:

$250{,}000 = 355{,}000 \ (P/F, i^*, N)$

$\Rightarrow \quad 250{,}000 = 355{,}000 \ [1 / (1 + i^*)^1]$

$\Rightarrow \quad 250 = 355 \ [1 / (1 + i^*)]$

$\Rightarrow \quad (1 + i^*) = 355 / 250$

$\Rightarrow \quad i^* = 1.42 – 1$

$\therefore \quad i_3^* = 0.42 = 42\% > MARR = 20\%$

Thus, the IRR for 3rd project is 42%. Since it's higher than MARR, the project is justified as an *independent* project option. Again, to justify the incremental investment requirement of (250,000 – 200,000 =) $50,000 over the 2nd project, however, we need to calculate the incremental IRR by comparing it to incremental return of (355,000 – 300,000 =) $55,000.

<u>Increment from 2nd to 3rd project</u>:

$50{,}000 = 55{,}000 \ (P/F, i^*, N)$

$\Rightarrow \quad 50{,}000 = 55{,}000 \ [1 / (1 + i^*)^1]$

⇒ $50 = 55\ [1 / (1 + i^*)]$

⇒ $(1 + i^*) = 55 / 50$

⇒ $i^* = 1.1 - 1$

∴ $i_3^* = 0.1 = 10\% < MARR = 20\%$

So the incremental IRR is 10%. Thus the increment from 2nd to 3rd project cannot be justified.

Project	Initial investment (i.e., first cost)	Incremental investment	Return	Incremental return	IRR	Incremental IRR
1st	$100,000	–	$150,000	–	50%	50%
2nd	$200,000	$100,000	$300,000	$150,000	50%	50%
3rd	$250,000	$50,000	$355,000	$55,000	42%	10%

Decision: Choose the 2nd project.

CHAPTER 5

9) **Patti's project has a first cost *P*, annual savings *A*, and a salvage value of $1,000 at the end of the 10-year service life. She has calculated the present worth as $20,000, the annual worth as $4,000, and the pay back period as three years. What is the IRR for this project?**

Approach 1 to solving Q9

The following information are given:
First cost = P
Annual savings = A
Project duration, N = 10 years
Salvage value, S = $1,000
Present Worth, PW = $20,000
Annual Woth, AW = $4,000
Payback period = First cost / Annual savings = P/A = 3 years

From payback period equation, we derive: $P = 3A$ (Eq i)

The equation for PW calculation is:
$PW = -P + A * (P/A, i, N) + S * (P/F, i, N)$

$\Rightarrow \quad 20,000 = -P + A * (P/A, i, N) + 1,000 * (P/F, i, N)$

$\Rightarrow \quad 20,000 = -3A + A * (P/A, i, 10) + 1,000 * (P/F, i, 10)$ (Eq ii)

The equation for AW calculation is:
$AW = -P * (A/P, i, N) + A + S * (A/F, i, N)$

$\Rightarrow \quad 4,000 = -3A * (A/P, i, N) + A + 1,000 * (A/F, i, N)$

$\Rightarrow \quad 4,000 / (A/P, i, N) = -3A + A/(A/P, i, N) + \{1,000 * (A/F, i, N)\}/ (A/P, i, N)$
That is, dividing both sides by the *Capital Recovery Factor* – $(A/P, i, N)$.

$\Rightarrow \quad 4,000 (P/A, i, N) = -3A + A * (P/A, i, N) + 1,000 * (A/F, i, N) * (P/A, i, N)$
where, we have used the fact that $\{1/(A/P, i, N)\} = (P/A, i, N)$.

$\Rightarrow \quad 4,000 (P/A, i, N) = -3A + A * (P/A, i, N) + 1,000 * (P/F, i, N)$

where, we have used the fact that $(P/A, i, N) * (P/F, i, N) = (P/F, i, N)$.

$\Rightarrow$ $4{,}000\ (P/A, i, 10) = -3A + A * (P/A, i, 10) + 1{,}000 * (P/F, i, 10)$ (Eq iii)

Note that right side of both (Eq ii) and (Eq iii) are identical, implying the left side of these equation should be equal. Hence:
$20{,}000 = 4{,}000\ (P/A, i, 10)$

$\Rightarrow$ $(P/A, i, 10) = 20{,}000 / 4{,}000$

$\Rightarrow$ $(P/A, i, 10) = 5$

$\Rightarrow$ $[\ \{(1 + i)^{10} - 1\} / \{i * (1 + i)^{10}\}\ ] = 5$

Using Excel (see the attached excel file), we can solve for the interest rate, which is 12.88%.
[Accept any interest rate within 11 – 15% range.]

Notice that this i = MARR = 12.88% = 0.1288

We use this MARR to find the value of A using (Eq ii) or (Eq iii) as $A^* = \$8{,}035.22$

We the use (Eq i) with A^* to find $P^* = \$24{,}105.22$

Now, the definition of IRR requires:
PW of Costs = *PW* of Benefits

Using this MARR and using the definition of IRR we derive:

$P^* = A^*\ (P/A, IRR, N) + S\ (P/F, IRR, N)$

$\Rightarrow$ $P^* = A^*\ (P/A, IRR, 10) + S\ (P/F, IRR, 10)$

$\Rightarrow$ $24{,}105.22 = 8{,}035.22\ \ [\ \{IRR * (1 + IRR)^{10}\} / \{(1 + IRR)^{10} - 1\}\ ] + 1{,}000/(1 + i)^{10}$
This is again a system of one equation in one unknown, namely the IRR. Using excel (see the attached excel file), we can solve for the IRR = 31.22%.

Approach 2 to solving Q9

The following information are given:
First cost = P
Annual savings = A
Project duration, N = 10 years
Salvage value, S = \$1,000
Present Worth, PW = \$20,000
Annual Woth, AW = \$4,000
Payback period = First cost / Annual savings = P/A = 3 years

From payback period equation, we derive: $P = 3A$ (Eq i)

The equation for PW calculation is:
$PW = -P + A * (P/A, i, N) + S * (P/F, i, N)$

$\Rightarrow \quad 20{,}000 = -P + A * (P/A, i, 10) + 1{,}000 * (P/F, i, 10)$

$\Rightarrow \quad 20{,}000 = -3A + A * [\ \{(1+i)^{10} - 1\} / \{i * (1+i)^{10}\}\ ] + 1{,}000 * [1 / (1+i)^{10}]$

where, we've used the full expression of *Series Present Worth Factor* – $(P/A, i, N)$ – and the *Present Worth Factor* – $(P/F, i, N)$:

$(P/A, i, 10) = [\ \{(1+i)^{10} - 1\} / \{i * (1+i)^{10}\}\ ]$, and

$(P/F, i, 10) = [1 / (1+i)^{10}]$

$\Rightarrow \quad 20{,}000 = A * \{\ [\ \{(1+i)^{10} - 1\} / \{i * (1+i)^{10}\}\ ] - 3\ \} + [1{,}000 / (1+i)^{10}]$

$\Rightarrow \quad 20{,}000 - [1{,}000 / (1+i)^{10}] = A * \{\ [\ \{(1+i)^{10} - 1\} / \{i * (1+i)^{10}\}\ ] - 3\ \}$

$\Rightarrow \quad A = \{\ 20{,}000 - [1{,}000 / (1+i)^{10}]\ \} / \{\ [\ \{(1+i)^{10} - 1\} / \{i * (1+i)^{10}\}\ ] - 3\ \}$ (Eq ii)

The equation for AW calculation is:

Method 1

$AW = -P * (A/P, i, N) + A + S * (A/F, i, N)$

$\Rightarrow \quad 4{,}000 = -3A * (A/P, i, 10) + A + 1{,}000 * (A/F, i, 10)$

⇒ $4{,}000 = -3A * [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ] + A + 1{,}000 * [\ i / \{(1+i)^{10} - 1\}\ ]$

where, we've used the full expression of *Capital Recovery Factor* – $(A/P, i, N)$ – and the *Sinking Fund Factor* – $(A/F, i, N)$:

$(A/P, i, 10) = [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ]$, and

$(A/F, i, 10) = [\ i / \{(1+i)^{10} - 1\}\ ]$

⇒ $4{,}000 = A * \{\ 1 - 3 * [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ]\ \} + 1{,}000 * [\ i / \{(1+i)^{10} - 1\}\ ]$

⇒ $A = 4{,}000\ / [\ \{\ 1 - 3 * [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ]\ \} + 1{,}000 * [\ i / \{(1+i)^{10} - 1\}\ ]]$ (Eq iii)

Or,

Method 2 (Using the *Capital Recovery Formula*)

$A = (P - S) * (A/P, i, N) + S * i$

⇒ $4{,}000 = (P - 1{,}000) * (A/P, i, 10) + 1{,}000 * i$

⇒ $4{,}000 = (3A - 1{,}000) * (A/P, i, 10) + 1{,}000 * i$

⇒ $4{,}000 = (3A - 1{,}000) * [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ] + 1{,}000 * i$

where, we've used the full expression of *Capital Recovery Factor* – $(A/P, i, N)$

⇒ $(4{,}000 - 1{,}000 * i) = (3A - 1{,}000) * [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ]$

⇒ $(3A - 1{,}000) = \{\ (4{,}000 - 1{,}000 * i) / [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ]\ \}$

⇒ $3A = 1{,}000 + \{\ [\ (4{,}000 - 1{,}000 * i) / [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ]\ \}$

⇒ $A = (1/3) * [\ \{\ 1{,}000 + (4{,}000 - 1{,}000 * i) / [\ \{i * (1+i)^{10}\} / \{(1+i)^{10} - 1\}\ ]\}\ ]$ (Eq iii)

Notice that by equating the expression for A from (Eq ii) and (Eq iii), we arrive at a system of *one equation in one unknown*, namely the interest rate, i. Using excel (see the attached excel file), we can solve for the interest rate, which is 12.88%.

Notice that this i = MARR = 12.88% = 0.1288

We use this MARR to find the value of A using (Eq ii) or (Eq iii) as A^* = \$8,035.22

We the use (Eq i) with A^* to find P^* = \$24,105.22

Now, the definition of IRR requires:

PW of Costs = PW of Benefits

Using this MARR and using the definition of IRR we derive:

$P^* = A^* (P/A, IRR, N) + S (P/F, IRR, N)$

$\Rightarrow \quad P^* = A^* (P/A, IRR, 10) + S (P/F, IRR, 10)$

$\Rightarrow \quad 24{,}105.22 = 8{,}035.22\ [\ \{IRR * (1 + IRR)^{10}\} / \{(1 + IRR)^{10} - 1\}\ ] + 1{,}000/(1 + i)^{10}$

This is again a system of one equation in one unknown, namely the IRR. Using excel (see the attached excel file), we can solve for the IRR = 31.22%.

CHAPTER 6

10) **A five-year-old gadget has a current book value of $203,000. It has been observed that the gadget loses it value over time in a manner that can be best captured through the use of the declining-balance method. The depreciation rate was estimated to be 0.31. What was the original price? What will be the book value in two years from now?**

The following information are given:
Usage period of the gadget, $n = 5$ years

Five-year-old gadget's book value, $BV(n = 5) = 203{,}000 = S$
Notice that this $BV(5)$ is same as the salvage value, S, now if this is sold today.

The depreciation rate, $d = 0.31$

The purchase price, $P = ?$

Formula for the declining-balance depreciation method is given by:

$d = 1 - (S/P)^{1/n}$

$\Rightarrow \quad 0.31 = 1 - (203{,}000 / P)^{1/n}$

$\Rightarrow \quad (203{,}000 / P)^{1/n} = 0.1 - 0.31$

$\Rightarrow \quad 203{,}000 / P = (0.69)^{n}$

$\Rightarrow \quad P = 203{,}000 / (0.69)^5$

$\therefore \quad P = \$1{,}297{,}927.95$

Therefore, the purchase price was $1,297,927.95.

To calculate the book value in next two years, $BV(n = 7)$, we use the formula:

$BV(n) = P(1 - d)^n$

$\Rightarrow \quad BV(7) = 1{,}297{,}927.95\,(1 - 0.31)^7$

$\Rightarrow \quad BV(7) = 1{,}297{,}927.95\,(0.69)^7$

$\therefore \quad BV(7) = \$96{,}648.30$

So, the book value after seven year will be $96,648.30.

CHAPTER 6

11) **Candi has just purchased a car for $7,500. She expects that the value of this car will decline by 13% each year. Eventually Candi wants to sell this car for $2,000 and buy a new one. How many years should Chandi use this car before she can sell it?**

The following information are given:
Purchase price, P = $7,500
The expected selling price or the salvage value, S = $2,000
Depreciation rate, $d = 13\% = 0.13$
The usage period, $n = ?$

Note also that the question specifies that the value of the car depreciates by the fixed proportion. Hence, the appropriate model to use is the declining-balance depreciation model.

Formula for the declining-balance depreciation method is given by:

$d = 1 - (S/P)^{1/n}$

$\Rightarrow \quad 0.13 = 1 - (2{,}000 / 7{,}500)^{1/n}$

$\Rightarrow \quad (20 / 75)^{1/n} = 1 - .13$

$\Rightarrow \quad (1/n)\ ln\ (4/15) = ln\ (0.87)$

$\Rightarrow \quad (1/n) = ln\ (0.87) / ln\ (0.267)$

$\Rightarrow \quad n = ln\ (0.267) / ln\ (0.87)$

$\therefore \quad n = 9.49$ years

So, Candi should hold on to the car for approximately 9.5 years.

CHAPTER 8

12) **A chemical recovery system costs $30,000 and saves $5,280 each year of its seven-year life. The salvage value is estimated at $7,500. The after-tax MARR is 9%, the CCA rate is 20% and taxes are at 45%. What is the net after-tax annual benefit or cost of purchasing the chemical recovery system?**

The following information are given:
Initial cost, P = $30,000
Annual savings, A = $5,280
Salvage value, S = $7,500
After-tax MARR, $i = 9\% = 0.09$
CCA rate, $d = 20\% = 0.2$
Tax rate, $t = 45\% = 0.45$
The life of the chemical recovery system, $N = 7$ years
Net annual cost, C = ?

To find out the net after-tax annual benefit or cost of purchasing the chemical recovery system, we first need to find the *Capital Tax Factor* (CTF) and the *Capital Salvage Factor* (CSF).

Using the formula for CTF, we derive:

$$\text{CTF} = 1 - [\{(td)*(1 + i/2)\} / \{(i + d)*(1 + i)\}]$$
$$= 1 - [\{(0.45*0.2)*(1 + 0.09/2)\} / \{(0.09 + 0.2)*(1 + 0.09)\}]$$
$$= 1 - 0.297532$$
$$\therefore \text{CTF} = 0.0725$$

Using the formula for CSF, we derive:

$$\text{CSF} = 1 - \{(td) / (i + d)\}$$
$$= 1 - \{0.09 / 0.29\}$$
$$= 1 - 0.310345$$
$$\therefore \text{CSF} = 0.6897$$

The annual cost of the chemical recovery system is then given by:

$$C = -P * (A/P, i, N) * \text{CTF} + A * (1 - t) + S * (A/F, i, N) * \text{CSF}$$
$$= -30{,}000 * (A/P, 9\%, 7) * \text{CTF} + 5{,}280 * (1 - 0.45) + 7,500 * (A/F, 9\%, 7) * \text{CSF}$$

$$= -30{,}000*(0.19869)*(0.7025) + 5{,}280*(0.55) + 7{,}500*(0.10869)*(0.6897)$$
$$= -721.17$$

So, the chemical recovery system has an after-tax annual cost of about $721.

CHAPTER 8

13) What is the total after-tax annual cost of a machine with a first cost of $45,000 and operating and maintenance cost of $0.22 per unit produced? It will be sold for $4,500 at the end of five years. Production is 750 units per day; 250 days per year. The CCA rate is 30%, the after-tax MARR is 20%, and the corporation income tax rate is 40%.

The following information are given:
Initial cost, P = $45,000
Annual operating & maintenance cost, M = per unit cost * production per day* no. of days of operation in a year
= $0.22 * 750 * 250 = $41,250
Salvage value, S = $4,500
After-tax MARR, i = 20% = 0.2
CCA rate, d = 30% = 0.3
Tax rate, t = 40% = 0.4
The life of the chemical recovery system, N = 5 years

To find out the total after-tax the annual cost of a machine, we first need to find the *Capital Tax Factor* (CTF) and the *Capital Salvage Factor* (CSF).

Using the formula for CTF, we derive:

$$\text{CTF} = 1 - [\{(td)*(1 + i/2)\} / \{(i + d)*(1 + i)\}]$$

$$= 1 - [\{(0.4*0.3)*(1 + 0.2/2)\} / \{(0.2 + 0.3)*(1 + 0.2)\}]$$

$$= 1 - 0.22$$

$$\therefore \text{CTF} = 0.78$$

Using the formula for CSF, we derive:

$$\text{CSF} = 1 - \{(td) / (i + d)\}$$

$$= 1 - \{0.12 / 0.5\}$$

$$= 1 - 0.24$$

$$\therefore \text{CSF} = 0.76$$

The total after-tax annual cost of a machine is then given by:

$$A = -P * (A/P, i, N) * \text{CTF} + M * (1 - t) + S * (A/F, i, N) * \text{CSF}$$

$= -45{,}000 * (A/P, 20\%, 5) * \text{CTF} - 41{,}250 * (1 - 0.4) + 4{,}500 * (A/F, 20\%, 5) * \text{CSF}$
$= -45{,}000*(0.33438)*(0.78) + 41{,}250*(0.6) + 4{,}500*(0.13438)*(0.76)$
$= -36{,}027.2$

Therefore, the total after-tax annual cost is about $36,027.

CHAPTER 9

14) Inflation is expected to average 4 percent over the next 50 years. How much would you expect to pay 50 years from now for a burger costing $1.59 today?

The following information are given:
Average expected inflation rate, $f = 4\% = 0.04$
The duration of time, $n = 50$ years
Current burger price, $P = \$1.59$
Future burger price, $P_f = ?$

Formula for calculating future price with inflationary impact is given by:

$$P_f = P\,(1 + f)^{n}$$

$$\Rightarrow \quad P_f = 1.59 * (1 + 0.04)^{50}$$

$$\Rightarrow \quad P_f = 1.59 * 7.106683$$

$$\therefore \quad P_f = \$11.29$$

Therefore, with an average inflation rate of 4 percent over 50 years, the burger that costs now $1.59 will cost $11.29.

CHAPTER 9

15) **Ken will receive a $5,000 annual payment from a family trust. This will continue until Ken in 30; he is now 20. Inflation averages 4% and Ken's real MARR is 8%. If the first payment is a year from now and a total of 10 payments are to be made, what is the present worth of his remaining income from the trust?**

The following information are given:
Average expected inflation rate, $f = 4\% = 0.04$
Real MARR, $MARR_R = 8\% = 0.08$
Equal annuity payment, $A = \$5{,}000$
Total number of payments, $N = 10$
Present worth of income, $PW = ?$

From real MARR, we need to find the current MARR using the equation:

$$MARR_C = MARR_R + f + MARR_R * f$$
$$= 0.08 + 0.04 + 0.08 * 0.04$$
$$= 0.1232$$

We then find the present worth of the remaining income as:

$$PW = A * (P/A, MARR_C, N)$$
$$= 5{,}000 * (P/A, 0.1232, 10)$$
$$= 5{,}000 * 5.577$$
$$= \$27{,}884.87$$

Therefore, the present worth of Ken's remaining stream of income is $27,884.87.

CHAPTER 9

16) **The widget industry maintains a price index for a standard collection of widgets. The base year was 2002 until 2012, when the indeed was recomputed with 2012 as the base year. The following data concerning prices for the years 2010 to 2013 are available:**

Year	Price Index 2002 Base	Price Index 2012 Base
2010	125	N/A
2011	127	N/A
2012	130	100
2013	N/A	110

What was the percentage increase in prices of widgets between 2010 and 2013?

Given the information, we can solve the problem either by calculating 2010 price with 2012 base year or by calculating 2013 price with 2002 base year.

Let:
i) the 2010 price with 2012 base year be x
ii) the 2013 price with 2002 base year be y

Calculation with i)
Given the above information, the following ration must hold:

(2010 price with 2002 base year) / (2012 price with 2002 base year)
= (2010 price with 2012 base year) / (2012 price with 2012 base year)

$\Rightarrow$ 125 / 130 = x / 100

$\Rightarrow$ x = (125 *100) / 130

$\therefore$ x = 96.15

Hence, the percentage increase in prices of widgets between 2010 and 2013 with base price of 2012 is given by:

(2013 price with 2012 base year – 2010 price with 2012 base year) / (2010 price with 2012 base year)

= (110 – 96.15) / 96.15

= 0.144 = 14.4%

Or, **<u>Calculation with ii)</u>**

Given the above information, the following ration must hold:

(2012 price with 2002 base year) / (2013 price with 2002 base year)
= (2012 price with 2012 base year) / (2013 price with 2012 base year)

$\Rightarrow$ 130 / y = 100 / 110

$\Rightarrow$ y = (130 *110) / 100

$\therefore$ y = 143

Hence, the percentage increase in prices of widgets between 2010 and 2013 with base price of 2002 is given by:

(2013 price with 2002 base year – 2010 price with 2002 base year) / (2010 price with 2002 base year)

= (143 – 125) / 125

= 0.144 = 14.4%

Therefore, between 2010 and 2013 in real terms the widgets prices have increased by 14.4%.

CHAPTER 9

17) **Lifeware, a manufacturer of women's sports clothes, is considering adding a line of skirts and jackets. The production would take place in a part of its factory that is now not being used. The first output would be available in time for the 2015 fall season. The following information is available:**

New Product Line Information	
First cost in 2014 ($)	15,500,000
Planned output (units/year)	325,000
Observed, current dollar MARR before tax	0.25
Study period	6 years
Year 2014 Prices ($/unit)	
Materials	12
Labour	7.75
Output	35

(a) What is the real internal rate of return?

(b) What inflation rate will make the real MARR equal to the real internal rate of return?

(c) Calculate the present worth of the project under three possible future inflation rates: 1%, 2% or 3% per year.

(d) Decide if Lifeware should add this new line of skirts and jackets. Provide your explanation.

The following information are given:
First cost, P = $15,500,000
Current MARR, $MARR_C$ = 0.25 = 25%
Planner output per year = 325,000
Study period, or life of the project = 6 years
Profit per year from the project = Output * unit price – material cost – wage cost
= 325,000 * ($35 – $12 – $7.75) = 325,000 * $15.25
= $4,956,250

(a) Note that the question provides information on first cost, profit and MARR – all in real (i.e. in 2014) dollars. Hence, present worth (*PW*) can be calculated in real terms.

Let, IRR_R be the real IRR (i.e., IRR in 2014 terms). By definition, for the IRR_R:

$PW = 0$

$\Rightarrow \quad -15{,}500{,}000 + 4{,}956{,}250 *(P/A, IRR_R, N) = 0$

$\Rightarrow \quad -15{,}500{,}000 + 4{,}956{,}250 *(P/A, IRR_R, 6) = 0$

$\Rightarrow \quad 15{,}500{,}000 = 4{,}956{,}250 *(P/A, IRR_R, 6)$

$\Rightarrow \quad (P/A, IRR_R, 6) = 15{,}500{,}000/4{,}956{,}250$

$\Rightarrow \quad [\ \{(1 + IRR_R)^6 - 1\} / \{\ IRR_R * (1 + IRR_R)^6\}\ ] = 3.127364$

Using Excel (see the attached excel file), we can solve for $IRR_R = 22.52\%$.

(b) Here, we need to find the inflation rate that will make the $MARR_R$ equal to IRR_R (= 22.52%).

From the equation relating current and real MARR (i.e., $MARR_C$ and $MARR_R$) with the inflation rate (f), we get:

$(1 + MARR_R) = (1 + MARR_C) / (1 + f)$

$\Rightarrow \quad (1 + f) = (1 + MARR_C) / (1 + MARR_R)$

$\Rightarrow \quad f = \{\ (1 + MARR_C) / (1 + MARR_R)\ \} - 1$

$\Rightarrow \quad f = \{\ (1 + MARR_C) / (1 + IRR_R)\ \} - 1$

$\Rightarrow \quad f = \{\ (1 + 0.25) / (1 + 0.2252)\ \} - 1$

$\Rightarrow \quad f = (1.25/1.2252) - 1$

$\Rightarrow \quad f = 1.020242 - 1$

$\therefore \quad f = 0.020242 \approx 2.02\%$

Thus, in order for $MARR_R$ to be equal to IRR_R, the inflation rate has to be around 2.02%. That is, the project would break even if the inflation rate is 2.02%.

In other words, the project would only be worth undertaking if the average inflation rate exceeds 2.02% over the life of the project.

(c) To find out the project's present worth (*PW*) at the given inflation rates of 1%, 2% amd 3%, we first observe that the cash flows presented in the question are all provided in real terms (i.e., cash flows are given in the year when they are to be received over the life of the project in each of the next 6 years). Hence, to calculate the *PW*, we need to find out the real MARR ($MARR_R$), which is given by:

$(1 + MARR_R) = (1 + MARR_C) / (1 + f)$

$\Rightarrow MARR_R = \{ (1 + MARR_C) / (1 + f) \} - 1$

$\Rightarrow MARR_R = \{ (1+0.25) / (1 + f) \} - 1$

Thus,

with $f = 1\%$, $MARR_R = \{ (1.25) / (1 + 0.01) \} - 1 = (1.25/1.01) - 1 = 1.237624 - 1 = 0.237624 \approx 23.76\%$

with $f = 2\%$, $MARR_R = \{ (1.25) / (1 + 0.02) \} - 1 = (1.25/1.02) - 1 = 1.22549 - 1 = 0.22549 \approx 22.55\%$

with $f = 3\%$, $MARR_R = \{ (1.25) / (1 + 0.03) \} - 1 = (1.25/1.03) - 1 = 1.213592 - 1 = 0.213592 \approx 21.36\%$

We also know that, given $MARR_R$, the present worth (*PW*) can be calculated as:

$PW = -15{,}500{,}000 + 4{,}956{,}250 * (P/A, MARR_R, N)$

with $MARR_R = 23.76\%$:

$PW = -15{,}500{,}000 + 4{,}956{,}250 * (P/A, 23.76\%, 6)$

$= -15{,}500{,}000 + 4{,}956{,}250 * (P/A, 23.76\%, 6)$

$= -15{,}500{,}000 + 4{,}956{,}250 * [\{(1 + 0.2376)^6 - 1\} / \{ 0.2376 * (1 + 0.2376)^6\}]$

$= -15{,}500{,}000 + 4{,}956{,}250 * (3.037444)$

$= -15{,}500{,}000 + 15{,}054334$

$= -445{,}666$

That is since $PW < 0$, the project is not economically viable at $MARR_R = 23.76\%$.

with $MARR_R = 22.55\%$:

$$PW = -15{,}500{,}000 + 4{,}956{,}250 *(P/A, 22.55\%, 6)$$
$$= -15{,}500{,}000 + 4{,}956{,}250 * (P/A, 22.55\%, 6)$$
$$= -15{,}500{,}000 + 4{,}956{,}250 * [\ \{(1 + 0.2255)^6 - 1\} / \{ 0.2255 * (1 + 0.2255)^6\}\ ]$$
$$= -15{,}500{,}000 + 4{,}956{,}250 * (3.125488)$$
$$= -15{,}500{,}000 + 15{,}490{,}698$$
$$= -9{,}302.19$$

That is since $PW < 0$, the project is not economically viable at $MARR_R = 22.55\%$.

with $MARR_R = 21.36\%$:

$$PW = -15{,}500{,}000 + 4{,}956{,}250 *(P/A, 21.36\%, 6)$$
$$= -15{,}500{,}000 + 4{,}956{,}250 * (P/A, 21.36\%, 6)$$
$$= -15{,}500{,}000 + 4{,}956{,}250 * [\ \{(1 + 0.2136)^6 - 1\} / \{ 0.2136 * (1 + 0.2136)^6\}\ ]$$
$$= -15{,}500{,}000 + 4{,}956{,}250 * (3.216285)$$
$$= -15{,}500{,}000 + 15{,}940{,}710$$
$$= 440{,}710.20$$

That is since $PW > 0$, the project becomes economically viable at $MARR_R = 21.36\%$.

(d) This is a close call. We see that we need to have inflation over 2.02% per year for the project to break even. This is on the low side historically for inflation rates. But there have been countries with periods when inflation has been this low.

CHAPTER 12

18) **Power Tech builds power-surge protection devices. One f the components, a plastic moulded cover, can be produced by two automated machines, A1 and A2. Each machine produces a number of defects with probabilities shown in the following table.**

A1		A2	
No. of Defects (out of 100)	**Probability**	**No. of Defects (out of 100)**	**Probability**
0	0.3	0	0.25
1	0.28	1	0.33
2	0.15	2	0.26
3	0.15	3	0.1
4	0.1	4	0.05
5	0.02	5	0.01

Which machine is better with regard to the expected number of defective products?

Given the probability distribution of defects for both machien, we can calculate the expected number of defects as:

Expected (number of defects, A1)
= 0.3*(0) + 0.28*(1) + 0.15*(2) + 0.15*(3) + 0.1*(4) + 0.02*(5) = 1.53/100 units

Expected (number of defects, A2)
= 0.25*(0) + 0.33*(1) + 0.26*(2) + 0.1*(3) + 0.05*(4) + 0.01*(5) = 1.4/100 units

According to the expected number of defects, A2 seems to be slightly better than A1.

CHAPTER 12

19) **Regional Express is a small courier service company. By introducing a new computerized tracking device, it anticipates some increase in revenue, currently estimated at $2.75 per parcel. The possible new revenue ranges from $2.95 to $5.00 per parcel, with probabilities shown in the table below. Assuming that Regional's monthly capacity is 60,000 parcels and the monthly operating and maintenance costs are $8,000, what is the present worth of the expected revenue over 12 months? Regional's MARR is 12%, compounded monthly.**

Revenue per parcel	$2.95	$3.25	$3.50	$4.00	$5.00
Probability	0.1	0.35	0.3	0.15	0.1

The following information are given:
Monthly capacity = 60,000
Monthly operating & maintenance costs, M = $8,000
MARR (compounded monthly) = 12%
Present worth of income, PW = ?

Given the distribution of revenues and the associated probability distribution, we can calculate the expected revenue as:
Expected(Revenue) = 0.1*(2.95) + 0.35*(3.25) + 0.3*(3.50) + 0.15*(4.00) + 0.1*(5.00) = $3.58

Using Expected (Revenue), the present worth of the expected revenue over 12 months is:

PW = (3.58 * 60,000 – 8,000) * (P/A, 12%, 12)

= (206,800) * (6.1944) = $1,281,002

<u>CHAPTER 10</u>

20) **A province in Canada is considering the construction of a bridge. The bridge would cross a narrow part of a lake near a provincial park. The major benefit of the bridge would be reduced travel time to a campsite from a nearby urban centre. This lowers th cost of camping trips at the park. As well, an increase in the number of visits resulting from the lower cost per visit is expected.**

Data concerning the number of week-long visits and their costs are shown below:

Inputs	**Number of Visits and Average Cost per Visit to Park**	
	Without Bridge	**With Bridge**
Travel cost ($)	140	87.5
Use of equipment ($)	50	50
Food ($)	100	100
Total ($)	290	237.5
Number of visits/year	8,000	11,000

The following data are available as well:

(a) The bridge will take one year to build.

(b) The bridge will have a 25-year life once it is completed. This means that the time horizon for computations is 26 years.

(c) Construction cost for the bridge is $3,750,000. Assume that this cost is incurred at the beginning of year 1.

(d) Annual operating and maintenance costs for the bridge are given by:
$7,500 + 0.25$q$
Where $7,500 is the fixed operating and maintenance cost per year and q is the number of crossings.

(e) Operating and maintenance costs are incurred at the end of each year over which the bridge is in operation. This is at the ends of years 2, 3, …., 26.

(f) The MARR is 10%.

(g) The annual benefits for this project is estimated to be $498,750.

i) Compute the present worth of the project.

ii) Compute the benefit-cost ratio.

iii) Compute the modified benefit-cost ratio.

The following information are given:
Firs tcost, P = $3,750,000
Number of visits with bridge, q = 11,000
Operating & maintenance cost, M = 7,500 + 0.25q = 7,500+ 0.25*11,000 = $10,250

Useful life of the bridge, N = 25 years
$MARR$ = 10% = 0.1
Annual benefits of the bridge, A = $498,750

We first need to compute the present worth of costs, *PW*(Costs), given by:

PW(Costs) = *PW*(First cost) + *PW*(Operating & maintenance cost)

= *PW*(*P*) + *PW*(*M*)

where, *PW*(*P*) = *P* = $3,750,000, and

PW(*M*) = *M* * (*P*/*A*, *MARR*, *N*) = [10,250 * (*P*/*A*, 10%, 25)] * (*P*/*F*, 10%, 1)
since [10,250 * (*P*/*A*, 10%, 25)] is *PW* of operating & maintenance cost as of the end of year 1, which we then bring back to the start of year 1 by multiplying it with the *Present Worth Factor* – (*P*/*F*, 10%, 1).

So, *PW*(*M*) = [10,250 * (*P*/*A*, 10%, 25)] * (*P*/*F*, 10%, 1)

= 10,250 * (9.077) * (0.90909)

= $84,581.05

Therefore, total *PW*(Costs) = $3,750,000 + $84,581.05 = $3,834,581.05

We then need to compute the present worth of benefits, *PW*(Benefits), given by:

PW(Benefits) = *A* * (*P*/*A*, *MARR*, *N*) = [498,750 * (*P*/*A*, 10%, 25)] * (*P*/*F*, 10%, 1)

same reason as above

So, *PW*(Benefits) = [498,750 * (*P*/*A*, 10%, 25)] * (*P*/*F*, 10%, 1)

= 498,750 * (9.077) * (0.90909)

= $4,115,590

Therefore the net present worth is:

i) *PW*(Bridge) = *PW*(Benefits) – *PW*(Costs) = 4,115,590 – 3,834,581 = $281,009

ii) The *Benefit-Cost-Ratio* (*BCR*) is given by:

BCR = *PW*(Benefits) / *PW*(Costs) = 4,115,590 / 3,834,581 = 1.073 > 1
Since *BCR* is greater than 1, the project is worth under-taking.

ii) The *Modified Benefit-Cost-Ratio* (*BCRM*) is given by:

BCRM = { *PW*(Benefits) – *PW*(*M*) }/ *PW*(Capital Cost)

= { 4,115,590 – 84,581 }/ 3,750,000 = 4,031,009 / 3,750,000 = 1.075 > 1
Again, since *BCRM* is greater than 1, the project is worth under-taking.