Class Feb 28th

Tuesday, February 28, 2017 1:00 PM

Office hours
- Monday 11-1230
- Thursday: after class until 310

7-10 chap + earlier skill > DGD RK/SS

is ~~this sub~~?

REVIEW TEST #2

Class Feb 28th

Audio recording started: 1:02 PM Tuesday, February 28, 2017

Recap 6-10 + Matrix multiplication

Recall

vector spaces

practical way of finding vector space → span

every subspace = span by some vector

(all vector spaces appear in this way)

- $\{\vec{v}_1, \ldots, \vec{v}_n\}$ $\underbrace{\text{is linearly independant}}_{\text{written on test}}$

if only solution to $a_1\vec{v}_1 + a_2\vec{v}_2 .. + a_n\vec{v}_n = \vec{0}$

is $a_1 = a_2 = a_n = 0$ *audio explanation here

- a basis for V is a linearly independant spanning set

⟺ a largest LI set in V

⟺ a smallest spanning set of V

ex) $W = \text{span}\ \begin{matrix}1 & 0\\ 0 & 1\end{matrix}, \begin{matrix}0 & 1\\ 0 & 0\end{matrix}$ $\left\{ a \begin{matrix}1 & 0\\ 0 & 1\end{matrix} + b \begin{matrix}0 & 1\\ 0 & 0\end{matrix} \right\}$ $a, b \in \mathbb{R}$

$= \begin{matrix}a & b\\ 0 & a\end{matrix}$ $a, b \in \mathbb{R}$

$W = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$ $a, b \in \mathbb{R}$

2 vectors

∴ LI, if not multiples of each other

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is a basis of W

dimension of U = number of vectors in a basis of U

eg) dimension of $W = 2$

Given a basis, we can write vectors in U coordinates with respect to the basis

eg) basis $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$ element in W $\quad \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 3 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \in W$ $\quad$ coordinate vector $(2, 3)$

*questions to answer audio

*Examples on the theme of Ch 7-10

$U = \text{span} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

Is this set linearly independent?

↳ Solve $a[\ ] + b[\ ] + c[\ ] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 3 & 1 & 0 \end{array}\right] \xrightarrow{RR} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$a + 2c = 0$

$b - c = 0$

let $c = t$

$a = -2t$

$b = t$

$c = t$

not LI

audio exp

$-2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

3rd vector redundant

$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} - 1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \in \text{span} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

LI ✓

$W = \text{span} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

LI

∴ is a basis for W

$\dim W = 2$

Theorem: If you have a LI set in U, then you can extend it to a basis of U

Theorem: If $U \subseteq V$ is a subspace, then the $0 \leq \dim U \leq \dim W$

Also: $\dim U = \dim V$ then $U = V$

Also: $\dim U = \dim V$ then $U = V$

__Theorem:__ The number of vectors in any LI set in U

$\leq \dim U \leq$ number of vectors in any spanning set for U

eg) $U = \text{span}\left\{\begin{matrix}1 & 1\\ 2 & -1\end{matrix}, \begin{matrix}4 & 1\\ 1 & 1\end{matrix}\right\}$

$\begin{matrix}1 & 1\\ 2 & -1\end{matrix}, \begin{matrix}4 & 1\\ 1 & 1\end{matrix}$ is a basis for U

a random element of U looks like

$$\begin{bmatrix} a+4b & a+b \\ 2a+b & -a+b \end{bmatrix}$$

a coordinate vector is

(a, b)

$(1,2) \leftrightarrow \begin{bmatrix} 9 & 3 \\ 4 & 1 \end{bmatrix}$ $\quad (1,2)+(3,1) = (4,3)$

$(3,1) \leftrightarrow \begin{bmatrix} 7 & 4 \\ 7 & -2 \end{bmatrix}$ $\xrightarrow{\text{add}} \begin{bmatrix} 16 & 7 \\ 11 & -1 \end{bmatrix} = 4\left(\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\right) + 3\begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}$

also true: $5(1,2) = (5,10)$

$= 5\vec{u} + 10\vec{v}$

We say that U is __isomorphic__ to $\mathbb{R}^2$

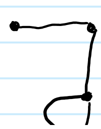
* audio for questions during the test *

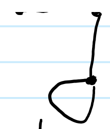
__Matrix Multiplication__

Def: A matrix is a table of numbers

If it has m rows & n columns, we call it a $m \times n$ matrix

eg) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ 3 rows 2 columns $\underset{r}{3} \times \underset{c}{2}$ matrix

examples:
- tables of data
- coefficients of a linear system
- a network ("a graph") → eg)  adjacency matrix
- a dynamical system for a

- a network (a graph) $G(V)$  adjacency matrix

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- a dynamical system for a probabilistic state machine (Markov processes/chains)

- geometric transformations

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

claim rotation by θ in $\mathbb{R}^2$

Different ways to "see" a matrix

- a table of numbers
- a collection of column vectors
- a collection of row vectors
- as a generalized number (something we can add & multiply, NOT divide)

How to multiply matrices

① Why should we multiply matrices?

eg) total cost $= \frac{\text{cost}}{\text{unit}} \times$ units

	water A	HCl B	sugar C
alpha	5	1	3
beta	1	2	1

to produce 1 unit of beta, I need 2 units of B

Production	cost $	Liters
A	1	3
B	10	1
C	2	10

2\$ per unit of C

each unit of C is 10 liters

$$P = \begin{matrix} 5 & 1 & 3 \\ 1 & 2 & 1 \end{matrix} \qquad C = \begin{matrix} 1 & 3 \\ 10 & 1 \\ 2 & 10 \end{matrix}$$

2 10

How much does 1 unit of alpha cost?

$$\begin{array}{ccc} 5 & 1 & 3 \\ A & B & C \end{array} \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 1 \\ 10 \\ 2 \end{bmatrix} = 5 \times 1 + 1 \times 10 + 3 \times 2 = 21$$

So to multiply is the dot product of row i and column j of C to make the entry (i,j)

$$\begin{matrix} 5 & 1 & 3 \\ 1 & 2 & 1 \end{matrix} \quad \begin{matrix} 1 & 3 \\ 10 & 1 \\ 2 & 10 \end{matrix} = \begin{vmatrix} 21 & \\ 23 & \end{vmatrix}$$

prod row 1 $\times$ col 2

$1 \times 3 + 2 \times 1 + 1 \times 10$

$$= \begin{bmatrix} 21 & 46 \\ 23 & 15 \end{bmatrix}$$

eg) $\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}}_{3 \times 2} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{2 \times 1} = \begin{bmatrix} x + 2y \\ 3x + 4y \\ 5x + 6y \end{bmatrix}$

eg) $\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}}_{3 \times 2} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{3 \times 1}$ $\times$ not allowed

$2 \neq 3$

eg) $\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{2 \times 3} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{3 \times 1} = \begin{bmatrix} x + 2y + 3z \\ 4x + 5y + 6z \end{bmatrix}$

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rows

even worse:

$A = \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$ $B = \begin{matrix} 1 & 2 \\ 1 & 1 \end{matrix}$

$AB = \begin{bmatrix} 3 & 4 \\ 7 & 10 \end{bmatrix}$

$BA = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 4 & 6 \end{bmatrix}$

$\boxed{AB \neq BA}$

eg) $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ think of these as 3x1 matrices

$\underset{3\times1}{\vec{u}} \underset{3\times1}{\vec{v}}$ X

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = 0$

transpose

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^T =$

swap rows & columns

$v^T v = \vec{u} \cdot \vec{v}$ ↪ dot product

$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

$\Leftrightarrow \vec{u}^T \vec{v} = \vec{v}^T \vec{u}$ ✓ check

$u v^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 3 & 3 & -3 \end{bmatrix}$

has nothing to do with the dot product

write down sizes and realize

eg) more: $C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

$CD = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

It is possible to have $CD = 0$ when neither C or $D = 0$ or has 0's