

CLASS: PHY _____

STUDENT #: _____

NAME: _____

Assignment 7:

KINEMATICS 1-D and 2D Motion

Assigned: Oct 28: Due: Nov 4 6 PM

- 1 Using the two main Kinematic Equations below:

$$x(t) = x_i + v_{x_i}t + \frac{1}{2}a_x t^2 \text{ and } v(t) = v_{x_i} + a_x t$$

obtain the remaining two kinematic equations.

(use opposite side of this page for detailed calculations)

- 2 Two railroad tracks intersect at right angles at station O. At 10AM the train A, moving west with constant speed of 50 km/h, leaves the station O. One hour later train B, moving south with the constant speed of 60 km/h, passes through the station O. Find minimum distance between these trains.

Train A moves along x axis and at time t it will have position: $x_A = V_A t = 50t$

Train B moves along the y axis and at time t it will have position: $y_B = 60 - V_B t = 60 - 60t$

The distance between the two trains is given by: $D = \sqrt{x_A^2 + y_B^2} = \sqrt{(50t)^2 + (60 - 60t)^2}$

The minimum distance is given by the condition:

$$\frac{dD}{dt} = 0 \Rightarrow \frac{1}{2} \frac{1}{\sqrt{2500t^2 + (60 - 60t)^2}} \cdot [2(2500t) + 2(60 - 60t)(-60)] = 0$$

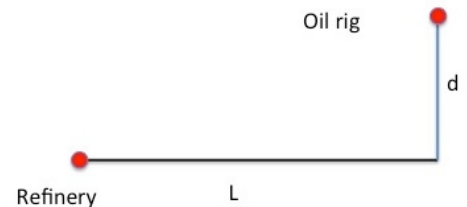
$$(2500t) + (60 - 60t)(-60) = 0 \Rightarrow (2500 + 3600)t = 3600 \Rightarrow t = \frac{3600}{6100} = \frac{36}{61} \text{ (hr)} = 35.41 \text{ min} = 35 \text{ min } 24.6 \text{ sec}$$

and so $D_{\min} = 38.94 \text{ km}$

- 3 John who is member of certain NGO has missed the meeting of his protest group at the refinery, and now needs to get to the oil rig in the shortest time to join the demonstrators who are trying to disrupt the work of the petroleum company. John can run at 10km/h but can paddle only 3km/h.

a) How far from the Refinery should John enter the water

b) What is the minimum time it will take to get to the oil rig $L=12\text{km}$, $d=4\text{km}$



$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L - x}{v_{\text{land}}} + \frac{\sqrt{x^2 + d^2}}{v_{\text{water}}}$$

$$\frac{d(\Delta t)}{dx} = -\frac{1}{v_{\text{land}}} + \frac{1}{v_{\text{water}}} \frac{1}{2}(x^2 + d^2)^{-1/2}(2x) \Rightarrow \Delta t = \min \text{ when } \frac{d(\Delta t)}{dx} = 0$$

$$-\frac{1}{v_{\text{land}}} + \frac{1}{v_{\text{water}}} \frac{1}{2}(x^2 + d^2)^{-1/2}(2x) = 0 \Rightarrow \frac{x}{\sqrt{x^2 + d^2}} = \frac{v_{\text{water}}}{v_{\text{land}}} \Rightarrow \frac{x^2}{x^2 + d^2} = \left(\frac{v_{\text{water}}}{v_{\text{land}}}\right)^2 \Rightarrow x^2 \left[1 - \left(\frac{v_{\text{water}}}{v_{\text{land}}}\right)^2\right] = d^2 \left(\frac{v_{\text{water}}}{v_{\text{land}}}\right)^2$$

$$x = \frac{d^2 \left(\frac{v_{\text{water}}}{v_{\text{land}}}\right)^2}{\sqrt{1 - \left(\frac{v_{\text{water}}}{v_{\text{land}}}\right)^2}} = \sqrt{1.58} \text{ (km)} = 1.26 \text{ km}$$

PART B Total time = 1.72hr.

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4. Cheetah can reach 107km/h in 2 s and maintain this speed for 15 s. After this time it must rest. An antelope can reach 92km/h in 2s and sustain it for a long time. Suppose they are initially separated by 80m and the antelope reacts in 0.5s. Will cheetah be able to hunt down the antelope? If not, how close will it get to it?

$$v_{ant}=25.56\text{m/s} \quad a_{ant}=12.78\text{m/s}^2 \quad v_{cheet}=29.72\text{m/s} \quad a_{cheet}=14.86\text{m/s}^2$$

Once both animals move with constant speed the problem simplifies.

This happens after initial 2.5s from the moment the cheetah starts its hunt. During this time: antelope will run away from cheetah 25.6m (during 2s acceleration and 0.5s "being frozen")

cheetah will run towards antelope 29.72m + 14.86m (during 2s a=const., and 0.5s v=const. motion)

after 2.5 seconds both animals move with constant speeds.

They are separated by $(80+25.6-44.58) \text{ m} = 61.02\text{m}$

There are numbers of ways to find out if cheetah is successful or not.

One way is to find the position of cheetah after 14.5 seconds and compare it with the position of antelope at 14.5 seconds.

$$x_{ant} = x_i + v_{ant}t = 61.02\text{m} + 25.56 \frac{\text{m}}{\text{s}} 14.5\text{s} = 431.64\text{m}$$

$$x_{cheetah} = v_{cheet}t = 29.72 \frac{\text{m}}{\text{s}} 14.5\text{s} = 430.94\text{m}$$

Cheetah misses antelope by 0.7m. This is the distance of closest approach.

- 5 A 6.0-m-diameter merry-go-round is initially turning with a 4.0 s period. It slows down and stops in 24 s.

a) Before slowing, what is the speed of a child on the rim?

b) How many revolutions does the merry-go-round make as it stops?

$$v = \frac{2\pi R}{T} = \frac{2\pi(3\text{m})}{4\text{s}} = 4.71 \frac{\text{m}}{\text{s}}$$

$$a_t = \text{const.} = \frac{v_f - v_i}{\Delta t} = \frac{0 - 4.71 \text{ m/s}}{24 \text{ s}} = -0.196 \frac{\text{m}}{\text{s}^2}$$

$$v_f^2 - v_i^2 = 2a_t \Delta s$$

$$\Delta s = \frac{-v_i^2}{2a_t} = -\frac{4.71^2}{2(-0.196)} = 56.59\text{m} = 1.50\text{turns}$$

- 6 One strategy in a snowball fight is to throw a snowball at a high angle over level ground. While your opponent is watching the first one, a second snowball is thrown at a low angle timed to arrive before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70.0° with respect to the horizontal.
- (a) At what angle should the second snowball be thrown to arrive at the same point as the first?
- (b) How many seconds later should the second snowball be thrown after the first to arrive at the same time?

Provide full solution on the opposite side of this page

SOLUTION to #6

- (a) The time of flight of the first snowball is the nonzero root of $y_f = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$

$$0 = 0 + (25.0 \text{ m/s})(\sin 70.0^\circ)t_1 - \frac{1}{2}(9.80 \text{ m/s}^2)t_1^2$$

$$t_1 = \frac{2(25.0 \text{ m/s})\sin 70.0^\circ}{9.80 \text{ m/s}^2} = 4.79 \text{ s}.$$

The distance to your target is

$$x_f - x_i = v_{xi}t_1 = (25.0 \text{ m/s})\cos 70.0^\circ(4.79 \text{ s}) = 41.0 \text{ m}.$$

Now the second snowball we describe by

$$y_f = y_i + v_{yi}t_2 + \frac{1}{2}a_y t_2^2$$

$$0 = (25.0 \text{ m/s})\sin \theta_2 t_2 - (4.90 \text{ m/s}^2)t_2^2$$

$$t_2 = (5.10 \text{ s})\sin \theta_2$$

$$x_f - x_i = v_{xi} t_2$$

$$41.0 \text{ m} = (25.0 \text{ m/s})\cos \theta_2 (5.10 \text{ s})\sin \theta_2 = (128 \text{ m})\sin \theta_2 \cos \theta_2$$

$$0.321 = \sin \theta_2 \cos \theta_2$$

Using $\sin 2\theta = 2 \sin \theta \cos \theta$ we can solve $0.321 = \frac{1}{2}\sin 2\theta_2$

$$2\theta_2 = \sin^{-1} 0.643 \text{ and } \theta_2 = \boxed{20.0^\circ}.$$

- (b) The second snowball is in the air for time

$t_2 = (5.10 \text{ s})\sin \theta_2 = (5.10 \text{ s})\sin 20^\circ = 1.75 \text{ s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79 \text{ s} - 1.75 \text{ s} = \boxed{3.05 \text{ s}}.$$