ASSIGNMENT 4:

UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/31 Fall 2016

STUDENT #: ____________________

NAME: ___________________________

Heat Engines,

Released: Oct 7, **Due: Oct 14 6PM Sharp!**

1. a) Follow the discussion presented during the lecture to derive the Law of Atmospheres.
   b) Using Maxwell=Boltzmann Distribution of speeds for Ideal Gas obtain the Boltzmann Distribution of Energies for Ideal Gas. (Follow Lecture Discussions)
/Present your work on the opposite site of this page/

2A refrigerator has a coefficient of performance of 3.00. The ice tray compartment is at –20.0°C, and the room temperature is 22.0°C. The refrigerator can convert 30.0 g of water at 22.0°C to 30.0 g of ice at –20.0°C each minute. What input power is required? Give your answer in watts.

$\mathrm{COP} = 3.00 = \frac{Q_c}{W}$. Therefore, $W = \frac{Q_c}{3.00}$. The heat removed each minute is

$$\frac{Q_C}{t} = (0.030\,0\ \mathrm{kg})(4\,186\ \mathrm{J/kg\,^\circ C})(22.0^\circ\mathrm{C}) + (0.030\,0\ \mathrm{kg})(3.33\times10^5\ \mathrm{J/kg}) + (0.030\,0\ \mathrm{kg})(2\,090\ \mathrm{J/kg\,^\circ C})(20.0^\circ\mathrm{C}) = 1.40\times10^4\ \mathrm{J/min}$$

or, $\frac{Q_c}{t} = 233\ \mathrm{J/s}$.

Thus, the work done per sec $= P = \frac{233\ \mathrm{J/s}}{3.00} = \boxed{77.8\ \mathrm{W}}$.

3 A heat engine operating between 200°C and 80.0°C achieves 20.0% of the maximum possible efficiency. What energy input will enable the engine to perform
10.0 kJ of work?

The Carnot efficiency of the engine is $e_c = \frac{\Delta T}{T_h} = \frac{120\ \mathrm{K}}{473\ \mathrm{K}} = 0.253$

At 20.0% of this maximum efficiency, $e = 0.200(0.253) = 0.050\,6$

From the definition of efficiency $W_{eng} = |Q_h| e$ and $|Q_h| = \frac{W_{eng}}{e} = \frac{10.0\ \mathrm{kJ}}{0.050\,6} = \boxed{197\ \mathrm{kJ}}$

4 A 20.0%-efficient real engine is used to speed up a train from rest to 5.00 m/s. It is known that an ideal (Carnot) engine using the same cold and hot reservoirs would accelerate the same train from rest to a speed of 6.50 m/s using the same amount of fuel. The engines use air at 300 K as a cold reservoir. Find the temperature of the steam serving as the hot reservoir.

The work output is $W_{eng} = \frac{1}{2} m_{train} (5.00\ \mathrm{m/s})^2$. We are told $e = \frac{W_{eng}}{Q_h}$; so $0.200 = \frac{1}{2} m_t \frac{(5.00\ \mathrm{m/s})^2}{Q_h}$

and $e_C = 1 - \frac{300\ \mathrm{K}}{T_h} = \frac{1}{2} m_t \frac{(6.50\ \mathrm{m/s})^2}{Q_h}$. Substitute $Q_h = \frac{1}{2} m_t \frac{(5.00\ \mathrm{m/s})^2}{0.200}$.

Then, $1 - \frac{300\ \mathrm{K}}{T_h} = 0.200 \left(\frac{\frac{1}{2} m_t (6.50\ \mathrm{m/s})^2}{\frac{1}{2} m_t (5.00\ \mathrm{m/s})^2} \right)$

Finally: $1 - \frac{300\ \mathrm{K}}{T_h} = 0.338$

$T_h = \frac{300\ \mathrm{K}}{0.662} = \boxed{453\ \mathrm{K}}$

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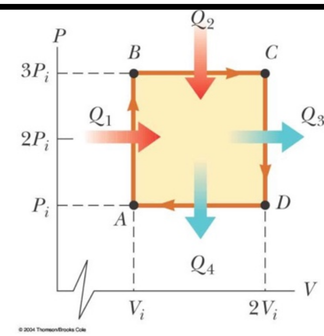
A 1.00-mol sample of a monatomic ideal gas is taken through the cycle shown in Figure P22.65. At point *A*, the pressure, volume, and temperature are P_i, V_i, and T_i, respectively. In terms of R and T_i, find (a) the total energy entering the system by heat per cycle, (b) the total energy leaving the system by heat per cycle, (c) the efficiency of an engine operating in this cycle, and (d) the efficiency of an engine operating in a Carnot cycle between the same temperature extremes.

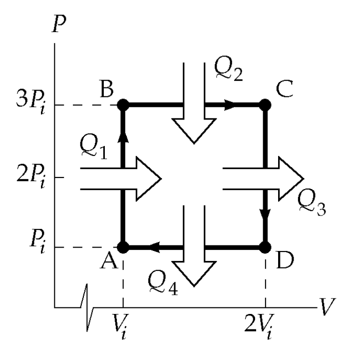


At point *A*, $P_iV_i = nRT_i$ and $n = 1.00$ mol

At point *B*, $3P_iV_i = nRT_B$ so $T_B = 3T_i$

At point *C*, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point *D*, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$



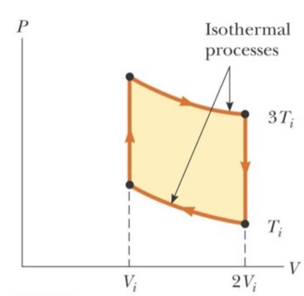
The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a)Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$ (b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c)Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$ (d)Carnot efficiency, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{T_i}{6T_i} = \boxed{0.833}$

8 In 1816 Robert Stirling, a Scottish clergyman, patented the *Stirling engine*, which has found a wide variety of applications ever since. Fuel is burned externally to warm one of the engine's two cylinders. A fixed quantity of inert gas moves cyclically between the cylinders, expanding in the hot one and contracting in the cold one. Figure below represents a model for its thermodynamic cycle. Consider *n* mol of an ideal monatomic gas being taken once through the cycle, consisting of two isothermal processes at temperatures 3 T_i and T_i and two constant-volume processes. Determine, in terms of *n*, *R*, and T_i, (a) the net energy transferred by heat to the gas. (b) its efficiency.



A Stirling engine is easier to manufacture than an internal combustion engine or a turbine. It can run on burning garbage. It can run on the energy of sunlight and produce no material exhaust.

(a) For an isothermal process, $Q = nRT\ln\left(\frac{V_2}{V_1}\right)$

Therefore, $Q_1 = nR(3T_i)\ln 2$

and $Q_3 = nR(T_i)\ln\left(\frac{1}{2}\right)$

For the constant volume processes, $Q_2 = \Delta E_{\text{int},\,2} = \frac{3}{2}nR(T_i - 3T_i)$

and $Q_4 = \Delta E_{\text{int},\,4} = \frac{3}{2}nR(3T_i - T_i)$

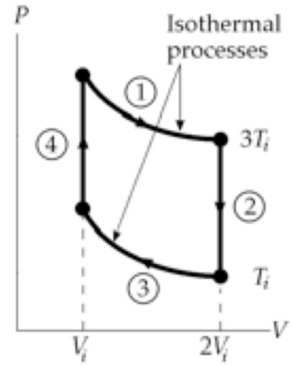


FIG. P22.57

The net energy by heat transferred is then

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

or $Q = \boxed{2nRT_i \ln 2}$.

(b) A positive value for heat represents energy transferred into the system.

Therefore, $|Q_h| = Q_1 + Q_4 = 3nRT_i(1 + \ln 2)$

Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}} = 0 \text{ and } W_{\text{eng}} = Q$$

Therefore, the efficiency is $e_c = \frac{W_{\text{eng}}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2\ln 2}{3(1+\ln 2)} = \boxed{0.273}$