

**ASSIGNMENT 3:**  
**First Law of**  
**Thermodynamics,**  
**Heat and Work in Gas Processes**  
**Kinetic Theory of Gases**

UNIVERSITY OF OTTAWA  
 Principles of Physics  
 PHY1321/31 Fall 2016  
 Dr. A. Czajkowski

STUDENT #: \_\_\_\_\_

NAME: \_\_\_\_\_

GROUP: \_\_\_\_\_

Released: Sept 30,

Due: Oct 7 6PM

**1** A sample of an ideal gas is in a vertical cylinder fitted with a piston. As 5.79 kJ of energy is transferred to the gas by heat to raise its temperature, the weight on the piston is adjusted so that the state of the gas changes from point A to point B along the semicircle shown. Find the change in internal energy of the gas.

SOLUTION:

The area of a true semicircle is  $\frac{1}{2} \pi r^2$ . The arrow in figure may look like a semicircle but in reality we are dealing with half of the ellipsoid with  $a=r_v = 1.2 \text{ l}$  and  $b=r_p = 100 \text{ kPa}$ . The area of the half ellipsoid is  $\frac{1}{2} \pi r_v r_p$ .

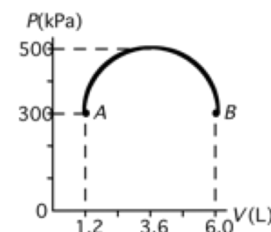
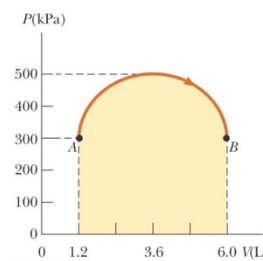
$$\frac{1}{2} \pi (2.4 \text{ L})(200 \text{ kPa}) = \frac{1}{2} \pi (2.4 \times 10^{-3} \text{ m}^3)(2 \times 10^5 \text{ N/m}^2).$$

$$W = -\int_A^B P dV = -\text{area under the arch shown in the graph}$$

$$= -\left(\frac{1}{2} \pi (2.4)(200) \text{ J} + 3 \times 10^5 \text{ N/m}^2 \cdot 4.8 \times 10^{-3} \text{ m}^3\right)$$

$$= -(754 \text{ J} + 1440 \text{ J}) = -2190 \text{ J}$$

$$\Delta E_{\text{int}} = Q + W = 5790 \text{ J} - 2190 \text{ J} = \boxed{3.60 \text{ kJ}}$$



**2** **A)** One mole of an ideal gas does 3 000 J of work on its surroundings as it expands isothermally to a final pressure of 1.00 atm and volume of 25.0 L. Determine (i) the initial volume and (ii) the temperature of the gas.

$$(i) W = -nRT \ln\left(\frac{V_f}{V_i}\right) = -P_f V_f \ln\left(\frac{V_f}{V_i}\right) \Rightarrow V_i = V_f \exp\left(\frac{W}{P_f V_f}\right) = (0.025 \text{ m}^3) \exp\left[\frac{-3000}{0.025 \text{ m}^3 (1.013 \times 10^5)}\right] = \boxed{0.00765 \text{ m}^3}$$

$$(ii) T_f = \frac{P_f V_f}{nR} = \frac{1.013 \times 10^5 \text{ Pa} (0.025 \text{ m}^3)}{1.00 \text{ mol} (8.314 \text{ J/K} \cdot \text{mol})} = \boxed{305 \text{ K}}$$

**B)** As a 1.00-mol sample of a monatomic ideal gas expands adiabatically, the work done on it is -2 500 J. The initial temperature and pressure of the gas are 500 K and 3.60 atm. Calculate (iii) the final temperature, and (iv) the final pressure.

$$(iii) W = nC_V(T_f - T_i) \text{ so that } -2500 \text{ J} = 1 \text{ mol} \cdot \frac{3}{2} \cdot 8.314 \text{ J/mol} \cdot \text{K} (T_f - 500 \text{ K}) \text{ and } T_f = \boxed{300 \text{ K}}$$

$$(iv) P_i V_i^\gamma = P_f V_f^\gamma \text{ and thus } P_i \left(\frac{nRT_i}{P_i}\right)^\gamma = P_f \left(\frac{nRT_f}{P_f}\right)^\gamma \Rightarrow T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\frac{T_i^{\gamma/(1-\gamma)}}{P_i} = \frac{T_f^{\gamma/(1-\gamma)}}{P_f} \text{ and } P_f = P_i \left(\frac{T_i}{T_f}\right)^{\gamma/(1-\gamma)} = P_i \left(\frac{T_i}{T_f}\right)^{(5/3)/(2/3)} = 3.60 \text{ atm} \left(\frac{500}{300}\right)^{5/2} = \boxed{1.00 \text{ atm}}$$

**4** Fill the table below:

	Degrees of Freedom	AVG Energy of single molecule	Cv	Cp	Gamma
A	1	1/2 kT	$\frac{1}{2} R$	$\frac{3}{2} R$	3
B	3	3/2 kT	$\frac{3}{2} R$	$\frac{5}{2} R$	5/3
C	5	5/2 kT	$\frac{5}{2} R$	$\frac{7}{2} R$	7/5
D	11	11/2 kT	$\frac{11}{2} R$	$\frac{13}{2} R$	13/11
E	2	kT	R	2R	2

**5** Using the approach demonstrated during the lecture show that for  $pV^\gamma = \text{const.}$  for adiabatic gas process. (Present your derivation on the opposite side of this page). DETAILS OF THIS CALCULATION WERE GIVEN IN LECTURE.

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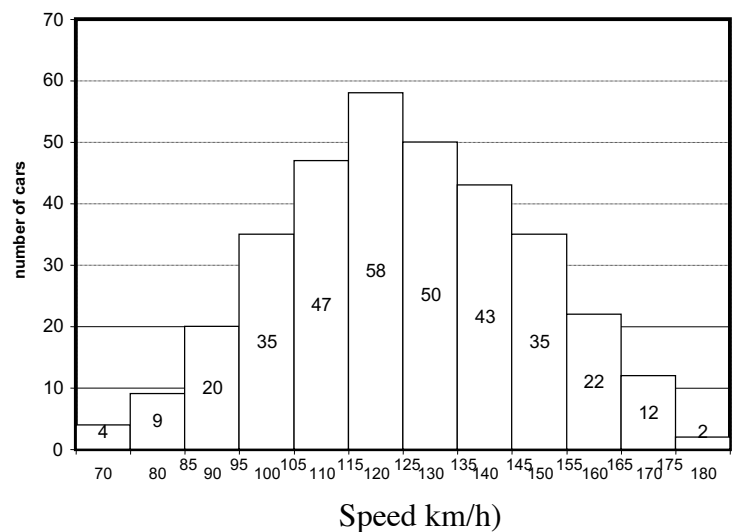
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- 6 Given is distribution of speeds of cars at 417 Highway as measured by OPP.
- Is this a discrete or continuous distribution?
  - Find the  $V_{mp}$ ,  $V_{rms}$ ,  $V_{avg}$ .
  - Find the probability that a randomly picked car will have speed larger than 120km/h.
  - Find the probability that a randomly picked car will have speed larger than 90km/h and less than 110km/h.

Answers:

- discrete
- $V_{mp}=120\text{km/h}$ ;  $V_{rms}=127\text{km/h}$ ;  $V_{avg}=125\text{km/h}$
- 0.49
- 0.16



- 7 Given is 1mole of nitrogen molecules at atmospheric pressure and temperature of 20°C.
- Find the number of molecules having their speed between 200m/s and 201m/s

$$N = 5.95 \times 10^{20}$$

$$\text{Since: } P(v)dv = 4\pi \left[ \frac{1}{2\pi} \frac{m}{kT} \right]^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv \quad \text{and} \quad P(v) = \frac{N}{N_A} \Rightarrow N_A = NP(v)$$

$$N = N_A 4\pi \left[ \frac{1}{2\pi} \frac{M}{RT} \right]^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}} dv = N_A [3.11153 \cdot 10^{-8}] \cdot v^2 e^{-\frac{Mv^2}{2RT}} [1m/s]$$

Answers to part a may slightly vary depending on which value was taken as the  $v$  in the formula above (centre value as 200.5m/s or  $v_{left} = 200\text{m/s}$  or  $v_{right} = 201\text{m/s}$ )

- Find the most probable speed:
- Find the number of molecules with the most probable speed (within 1 m/s from it)
- Find the number of molecules with the speed of 1000m/s
- Find the number of molecules with the speed of 2000m/s

ANS: 402m/s

ANS:  $11.99 \times 10^{20}$

ANS:  $0.60 \times 10^{20}$

ANS:  $7.69 \times 10^{12}$