You only need to finish the first 10 questions to get the full mark.

1. Assume the wage is \$1000, and the cost of capital is \$2. Fill out the following table. Is the marginal product of labor diminishing?

L	Q	K	VC	TC	AVC	ATC	AFC
0	0	1000	0	2000			
2	1800	1000	2000	4000	1.1	2.2	1.1
4	2800	1000	4000	6000	1.4	2.1	0.7
5	3000	1000	5000	7000	1.7	2.3	0.67

MP_L	MC
900	1.1
500	2
200	5

Yes, the marginal product of labor is diminishing, and MC is increasing in Q.

2. The firm's short-run cost function $C(q) = 10 + \frac{1}{2}q^2 + q$, $MC(q) = q + 1$.
   a. The price $p = 6$. What is the optimal quantity and profit? Should the firm shut down in the short-run? Why?
   b. If $p = 2$, what is the optimal quantity and profit? Should the firm shut down in the short-run? Why?
   c. At which price does the price just cover the firm's variable cost ($p = AVC$)?
   d. At which price the firm's profit is 0?
   e. What is the firm's supply curve in the short-run?

   Suppose there are 4 identical firms in the market, and the demand function is $p = 30 - 2Q_d$.

   f. What is the market supply curve? Compare the slope of the market supply curve with the slope of the supply curve of a single firm. If there are 10 firms in the market, answer these questions again.
   g. What is the market equilibrium (market price and quantity), profit and output of each firm?

   In the long-run suppose the demand function is $p = 30 - 2Q_d$.

   h. What is the long-run equilibrium price, market output, output of each firm, and number of firms?
   i. Suppose the cost function becomes $C(q) = 10 + \frac{1}{2}q^2 + 2q$, $MC(q) = q + 2$. What is the long-run equilibrium price, market output, output of each firm, and number of firms?

Answer:

a. At the optimal, we have $p = MC$, so $6 = q + 1 \Rightarrow q = 5$. The profit $\pi = R - C = pq - C(q) = pq - \left(10 + \frac{1}{2}q^2 + q\right) = 6 * 5 - \left(10 + \frac{1}{2}5^2 + 5\right) = \frac{5}{2}$. Since the profit is positive, we don't need to compare the price with the AVC. The firm should not shut down. Of course we can always check if the price can cover the AVC: VC=$\frac{1}{2}q^2 + q$, so AVC=$\frac{1}{2}q + 1 = \frac{1}{2} * 5 + 1 = \frac{7}{2}$. So we have $p = 6 > \frac{7}{2}$=AVC. So the firm should not shut down.

b. Now $p = 2$. At the optimal we have $p = MC = q + 1 \Rightarrow 2 = q + 1 \Rightarrow q = 1$. The profit $\pi = pq - \left(10 + \frac{1}{2}q^2 + q\right) = 2 * 1 - \left(10 + \frac{1}{2}1^2 + 1\right) = -9.5$. Now let's compare the price with the AVC: AVC=$\frac{1}{2}q + 1 = \frac{1}{2} * 1 + 1 = 1.5$, so $p >$ AVC. So the firm should not shut down even though the profit is negative because the price can cover the average variable cost.

c. When $p = AVC$, because at optimal $p = MC$ we have $MC = AVC$. So $\frac{1}{2}q + 1 = q + 1 \Rightarrow q = 0$. At $q = 0$, $p = \frac{1}{2}q + 1 = 1$. So the price is equal to 1, we have $p = AVC$.

d. When the profit is 0, we have $p = ATC(q)$. Also we have $p = MC(q)$, so we have $MC(q) = ATC(q) \Rightarrow q + 1 = \frac{10}{q} + \frac{1}{2}q + 1 \Rightarrow q = \sqrt{20}$. Then $p = MC = q + 1 = \sqrt{20} + 1$.

e. Each firm's supply curve is: $p = MC = q + 1$. Of course here when p<1, q=0.

f. n=4. From e., each firm's supply curve is $q = p - 1$. $Q_s = nq = n(p - 1)$. If we write p as a function of Q, we have $p = \frac{Q_s}{n} + 1$. So the slope of the market supply curve is $\frac{1}{n}$, and the slope of each firm's supply curve is 1. When n=4, $p = \frac{Q_s}{4} + 1$; when n=10, $p = \frac{Q_s}{10} + 1$. It is easy to see that the market supply curve is flatter when there are 10 firms in the market.

g. We can now get the market equilibrium by using the demand and supply functions. The demand function is given: $p = 30 - 2Q_d$, and the supply function is derived by above: $p = \frac{Q_s}{4} + 1$. In equilibrium we have $Q_d = Q_s \equiv Q$. So $30 - 2Q = \frac{Q}{4} + 1 \Rightarrow Q = \frac{116}{9}, p = \frac{29}{9} + 1$. The output of each firm $q = \frac{Q}{n} = \frac{\frac{116}{9}}{4} = \frac{29}{9}$. $\pi = pq - C(q) = \left(\frac{29}{9} + 1\right)\left(\frac{29}{9}\right) - \left(10 + \frac{1}{2}\left(\frac{29}{9}\right)^2 + \frac{29}{9}\right) = \frac{1}{2}\left(\frac{29}{9}\right)^2 - 10$.

h. In the long-run the equilibrium price $p = \min(AC)$. The AC is minimized at the quantity at which $AC = MC \Leftrightarrow \frac{10}{q} + \frac{1}{2}q + 1 = q + 1 \Rightarrow q = \sqrt{20}$ (actually we already have it in d. above). So $p = \min(AC) = q + 1 = \sqrt{20} + 1$. So the market supply curve is $p = \sqrt{20} + 1$. Since the demand function is is $p = 30 - 2Q_d$, we can get the market output $Q_d = \frac{1}{2}(30 - p) = \frac{1}{2}\left(30 - \sqrt{20} - 1\right) = \frac{1}{2}\left(29 - \sqrt{20}\right)$. Number of firms $n = \frac{Q_d}{q} = \frac{\frac{1}{2}\left(29 - \sqrt{20}\right)}{\sqrt{20}}$ (Please note that n doesn't have to be an integer).

i. As in h. above we start from $p = \min(AC)$. When AC is minimized, we have $AC = MC \Leftrightarrow \frac{10 + \frac{1}{2}q^2 + 2q}{q} = q + 2 \Leftrightarrow q = \sqrt{20}$. So $p = \min(AC) = q + 2 = \sqrt{20} + 2$. So the market supply curve is $p = \sqrt{20} + 2$. Since the demand function is is $p = 30 - 2Q_d$, we can get the market output $Q_d = \frac{1}{2}(30 - p) = \frac{1}{2}\left(30 - \sqrt{20} - 2\right) = \frac{1}{2}\left(28 - \sqrt{20}\right)$. Number of firms $n = \frac{Q_d}{q} = \frac{\frac{1}{2}\left(28 - \sqrt{20}\right)}{\sqrt{20}}$.

3. Suppose the production function is $q = K^{0.6}L^{0.7}$. Does it have IRS, CRS, or DRS?
   Answer:
   Suppose the multiplier is d>1. $f(dK, dL) = (dK)^{0.6}(dL)^{0.7} = d^{1.3}K^{0.6}L^{0.7} > dK^{0.6}L^{0.7} = df(K, L)$. So it has IRS.

4. Suppose the cost function is $C(q) = 20 + 0.5q^2 + q$. What are the FC, VC, ATC, AFC, AVC? The MC= $q + 1$. Find out the quantity level at which ATC is minimized, the quantity at which AVC is minimized. At which quantity is AFC minimized?
   Answer:
   $FC = 20, VC = 0.5q^2 + q, ATC = \frac{20}{q} + 0.5q + 1, AFC = \frac{20}{q}, AVC = 0.5q + 1.$
   When ATC is minimized, $MC = ATC$. So $q + 1 = \frac{20}{q} + 0.5q + 1 \Rightarrow q = \sqrt{20}$.
   When AVC is minimized, $MC = AVC$. So $q + 1 = 0.5q + 1 \Rightarrow q = 0$.
   $AFC = \frac{20}{q}$. So the larger the q, the smaller the AFC.

5. Suppose the production function is $q = \sqrt{KL}$. In the short-run capital is $\bar{K}$, the wage is w, and the cost of capital is r. What is the cost function in the short-run? What is the MC function? Draw the diagram. (Hint 1: in both of the cost function and MC function we write the costs as a function of output q; Hint 2: $MP_L = \frac{\sqrt{K}}{2\sqrt{L}}$)
   Answer:
   We need to write the cost as a function of output q. First we can write the cost as a function of K and L: $C = wL + rK$. Then we replace the variables with functions of q.
   In the short run, $C = wL + r\bar{K}$. The only variable in the short run is L. Since $q = \sqrt{\bar{K}L} \Rightarrow L = \frac{q^2}{\bar{K}}$. So $C(q) = r\bar{K} + \frac{w}{\bar{K}}q^2$. We can get the MC function from MP_L. From the notes,
   $MC = \frac{w}{MP_L} = \frac{w}{\frac{\sqrt{\bar{K}}}{2\sqrt{L}}} = \frac{2w\sqrt{L}}{\sqrt{\bar{K}}} = \frac{2w}{\bar{K}}q.$

6. $w = 5, r = 2, q = K^{0.5}L^{0.5}$, $MP_L = \frac{\sqrt{K}}{2\sqrt{L}}$, $MP_K = \frac{\sqrt{L}}{2\sqrt{K}}$. What are the optimal K and L that produces an output of 20, 40, and 60? Draw the graph to illustrate your solution, and draw the expansion path in the graph. Draw the total cost function in a new diagram.
   Answer:
   The optimal bundle of K and L has two properties:
   a) Slope of isocost line = slope of isoquant at the optimal bundle. That is, $\frac{w}{r} = MRTS = \frac{MP_L}{MP_K} = \frac{\frac{\sqrt{K}}{2\sqrt{L}}}{\frac{\sqrt{L}}{2\sqrt{K}}} = \frac{K}{L} \Rightarrow K = \frac{w}{r}L$ (1)
   b) The optimal bundle must be on the isoquant curve. So we have $q = K^{0.5}L^{0.5}$ (2)

   From equations (1) and (2): $q = K^{0.5}L^{0.5} = \left(\frac{w}{r}L\right)^{0.5} L^{0.5} = \left(\frac{w}{r}\right)^{0.5} L \Rightarrow L = \left(\frac{r}{w}\right)^{0.5} q$

From (1): $K = \left(\frac{w}{r}\right) L = \left(\frac{w}{r}\right)\left[\left(\frac{r}{w}\right)^{0.5} q\right] = \left(\frac{w}{r}\right)^{0.5} q$

The cost function is $C(q) = rK + wL = r\left[\left(\frac{w}{r}\right)^{0.5} q\right] + w\left[\left(\frac{r}{w}\right)^{0.5} q\right] = (rw)^{0.5} q + (rw)^{0.5} q = 2(rw)^{0.5} q$

Above is a general solution. You can plug in the numbers for w, r, and q to get the optimal bundles and the cost. For example, when q=20, the optimal $L = \left(\frac{r}{w}\right)^{0.5} q = \left(\frac{2}{5}\right)^{0.5} 20$, $K = \left(\frac{w}{r}\right)^{0.5} q = \left(\frac{5}{2}\right)^{0.5} 20$, and the cost is $C = 2(rw)^{0.5} q = 2(2*5)^{0.5} 20 = 40\sqrt{10}$.

7. Suppose $q = 2K + 3L$, w=5, r=1. What is the long-run cost function? Now suppose w=1 and r=5, what is the long-run cost function?
Answer:
Now the isoquant curves are straight lines because the production function is linear. In Figure 7a, the red lines are isoquents with slope = $-\frac{3}{2}$. The black lines are isocost lines with slope = -5. Suppose now at the quantity of q the isoquant is the red line passing through point B. It is easy to see that among all isocost lines the one that passes through B (line AB) gives the lowest cost. This is true because all isocost lines above AB represent higher cost, and all isocost lines below AB have no interaction point with the isoquant at q. Therefore the optimal bundle is at the point B, at which $L = 0, K = \frac{q}{2}$. So the cost function $C(q) = wL + rK = \frac{r}{2} q = \frac{1}{2} q$.

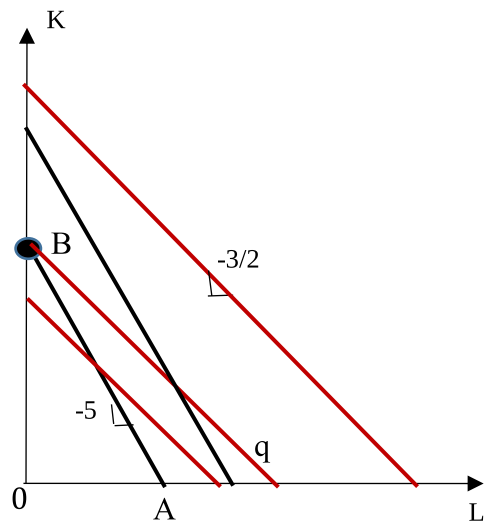


Figure 7a

When w=1 and r=5, see Figure 7b. The isoquents are the same red lines as in Figure 7a, with slope=-3/2. But now the isocost lines are flatter with slope = -1/5=-0.2. For the same reason as shown above, now the optimal bundle is A, at which $K = 0, L = q/3$. So the cost $C(q) = wL + rK = \frac{w}{3} q = \frac{1}{3} q$.

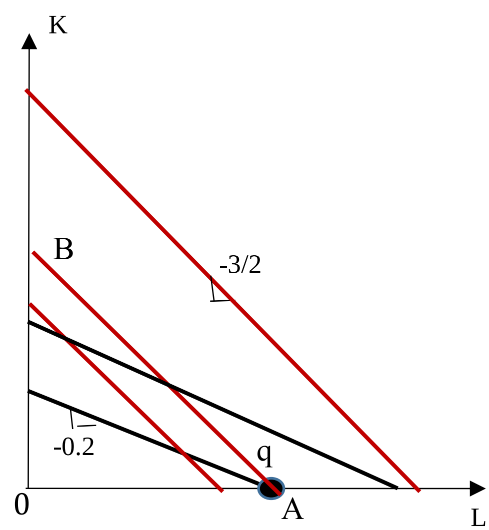
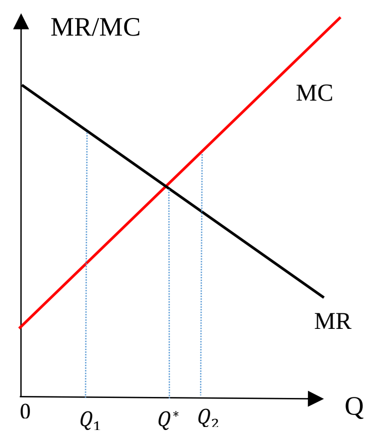


Figure 7b

8. Explain why in a perfectly competitive market each firm is a price taker (It wouldn't set the price higher or lower than the market price)?
Answer:
First each firm wouldn't set the price higher than the market price. Because all firms produce identical product, if a firm set the price higher than the market price, it will lose all its customers. Secondly, the demand curve faced by each firm is horizontal. That means a firm can sell all its output at the market price. So the firm has no incentive to lower its price. Or you can explain it without using the horizontal demand curve of each firm: Suppose now the market is in a long-run equilibrium so that the profit of all firms is zero. If one firm sets its price lower than the market price, all other firms will be forced to do the same thing. Then at a lower price all firms will have a negative profit. And then they have to raise the price to the equilibrium level. Therefore no firms would set the price lower than the market price.

9. When MC is increasing, and MR is constant or decreasing, explain why at the optimal quantity MR=MC. (Use a graph if it is helpful.)
If when Q=0, MC>=MR, then output Q=0. Now assume at Q=0, MR>MC. Let MC crosses MR at Q^*. Since MC is increasing and MR is decreasing here, it is easy to see in the diagram that for any unit Q_1 before Q^*, the firm has a positive profit because MR > MC at Q_1. So the firm should produce the unit at Q_1. Similarly for any unit Q_2 after Q^*, MR < MC at Q_2, so the firm should not product the unit at Q_2. Otherwise it will has a loss from that unit. Therefore the firm should produce all units before Q^*, and not product any unit after Q^*. So the optimal quantity is Q^*, the quantity at which MR = MC.



10. $w = 2, r = 3, q = \sqrt{KL},\ MP_L = \frac{\sqrt{K}}{2\sqrt{L}},\ MP_K = \frac{\sqrt{L}}{2\sqrt{K}}$. What are the optimal K and L that produces an output of 10? Now suppose the wage increases to 4, get the optimal bundle of K and L. Draw the graph to illustrate your solution.

Answer:

A general solution is provided in 6, and you can just plug in the numbers. Or see below for a specific answer:

Two conditions must be satisfied at the optimal bundle. First, the slope of isocost line = slope of isoquant. That is, $\frac{w}{r} = \frac{MP_L}{MP_K} = \frac{\frac{\sqrt{K}}{2\sqrt{L}}}{\frac{\sqrt{L}}{2\sqrt{K}}} = \frac{K}{L} \Rightarrow K = \frac{w}{r}L = \frac{2}{3}L.$ (1)

Secondly, the optimal bundle must be on the isoquant line. That is, $q = \sqrt{KL}$ (2)

Use (1) to replace K in (2): $q = \sqrt{\left(\frac{2}{3}L\right)L} = \sqrt{\frac{2}{3}}L$. So $L = \sqrt{\frac{3}{2}}q = 10\sqrt{\frac{3}{2}}$, and use (1) we can get $K = \frac{2}{3}L = \frac{2}{3}\sqrt{\frac{3}{2}}q = \sqrt{\frac{2}{3}}q = 10\sqrt{\frac{2}{3}}$.

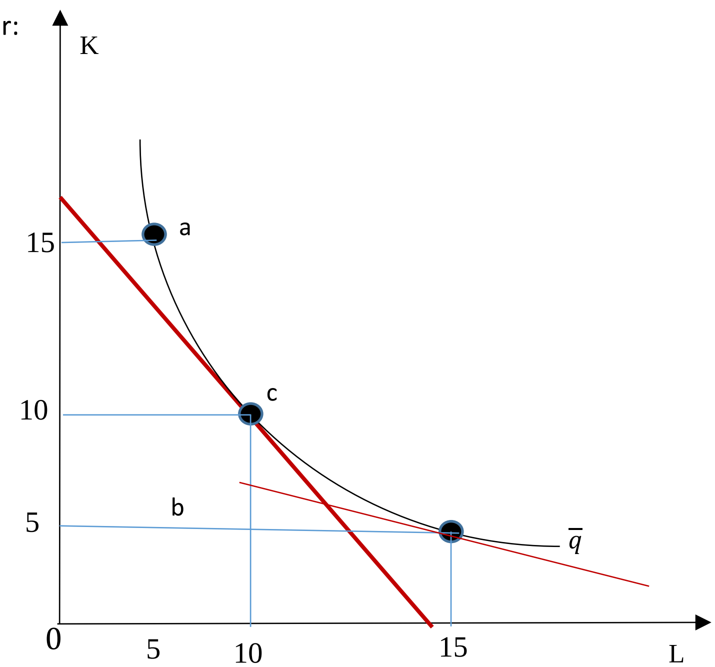
When the wage w increases to 4, you can solve for the optimal K and L similarly. Details is skipped here.

11. We usually assume the MP_L is diminishing. Explain why this assumption makes sense, or does not make sense.
Answer:
In the product function the output q depends on both K and L. When we calculate MP_L, we hold K constant. So MP_L refers to the marginal product of labor in the short-run. When L is higher, we generally need a higher K to maintain the same output level. For example, when the firm has more workers sharing the same number of machines, if the firm does not purchase new machines, the productivity of each worker will decrease. That is, MP_L is diminishing.

12. Remember Malthus theory? Explain why it worked three hundred years ago but doesn't work today. (If you don't remember it, google it on the internet.)
Answer:
If you don't remember, please google "Malthus theory" on line.
Three hundred years ago, the main industry was the agriculture industry and the main capital was land. Even in the long run, there was a limit to the land. When the population size increased, there were more farmers but the capital: land, had reached its limit. Because of the diminishing MP_L, the output of each farmer kept decreasing as the number of farmers increased. When the population was large enough, the output of each farmer was so small that farmers could not support themselves, then starvation was unavoidable.

13. Suppose the production function is $q = \sqrt{KL}$. Draw the isoquant at which $q = 10$. Find out the slope of the curve at the point of (K=10,L=10) graphically and mathematically? Is this slope larger or smaller than the slope at the point of (K=5, L=20)? (Hint: $MP_L = \frac{\sqrt{K}}{2\sqrt{L}}, MP_K = \frac{\sqrt{L}}{2\sqrt{K}}$)

Answer:



We need to find at least three bundles of K and L to draw the isoquant at q=10. One point can be (K=10, L=10). Another two points can be (K=20, L=5) and (K=5, L=20). Then we can draw the curve.

Assume K is on vertical axis. Graphically draw a line that passes through (K=10, L=10) and that is tangent to the isoquant at (K=10, L=10). The slope at (K=10, L=10) is equal to the slope of the line we just draw.

At (K=10, L=10), slope of the isoquant = -MRTS = $-\frac{MP_L}{MP_K} = -\frac{\frac{\sqrt{K}}{2\sqrt{L}}}{\frac{\sqrt{L}}{2\sqrt{K}}} = -\frac{K}{L} = -\frac{10}{10} = -1$. At (K=5, L=20), slope of the isoquant = -MRTS = $-\frac{MP_L}{MP_K} = -\frac{K}{L} = -\frac{5}{20} = -\frac{1}{4}$. So at (K=5, L=20) the slope is larger (but its absolute value is smaller).

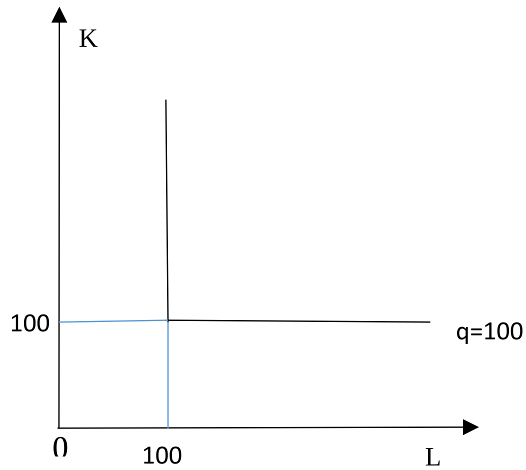
14. Suppose the production function is $q = 3K + 2L$. Draw the isoquant at which $q = 100$. What is the slope of the line?

Answer:

The isoquant is a straight line with two ending points $(K = \frac{100}{3}, L = 0)$ and $(K = 0, L = 50)$. Assume K is on the vertical axis. The slope of the isoquant is $-\frac{2}{3}$.

(Please note that the slope has a sign.)

15. Suppose the production function is $q = \min(K, L)$. Draw the isoquant at which $q = 100$.
Answer:



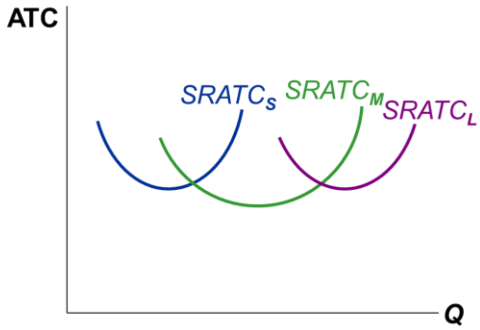
16. Suppose the production function is $q = \sqrt{KL}$. Does it have IRS, CRS, or DRS?
Answer:
We want to compare $f(dK, dL)$ with $df(K, L)$, where d is the multiplier.
$f(dK, dL) = \sqrt{(dK)(dL)} = d\sqrt{KL} = df(K, L)$. So it has CRS.

17. Suppose the production function is $q = K^{0.3}L^{0.7}$. Does it have IRS, CRS, or DRS?
Answer:
$f(dK, dL) = (dK)^{0.3}(dL)^{0.7} = d^{0.3+0.7}K^{0.3}L^{0.7} = dK^{0.3}L^{0.7} = df(K, L)$. So it has CRS.

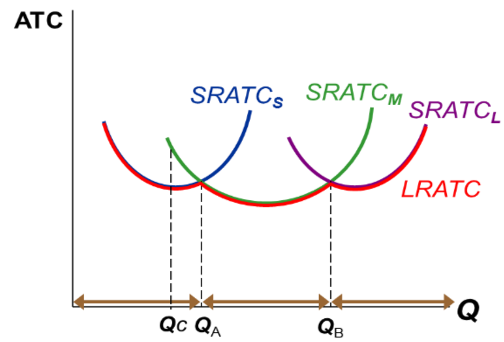
18. In the long-run all firms in a perfectly competitive market earn a 0 profit. Why are they still in business if their profit is 0?
Answer:
Because they have no other better alternatives. The accounting profit of the firm may not be 0 though, because here when calculating the profit we use opportunity cost. For example, a firm's accounting profit margin may be 5%, which is the best rate of return among all its other investment alternatives.

19. If we assume that MP_L is decreasing. Show that MC is increasing intuitively and mathematically.
Answer:
Intuitively, MP_L shows how much extra output we get by increasing the labor hour L by one unit. A decreasing MP_L means the extra output from the next unit of L is decreasing in L (the higher L, the lower the extra output from the next unit of L). That means, in order to get the extra one unit of output we need a larger L when L is high. Since the capital is fixed, a larger L means a larger total cost in order to produce the next unit of output. Therefore the MC, the cost of producing the next unit of output, is larger when L is high.
Mathematically, $MC = \frac{w}{MP_L}$, where w is the wage. Since the wage is fixed, a decreasing MP_L means an increasing MC.

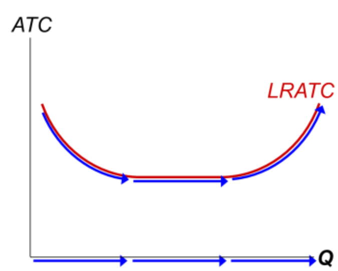
20. Suppose we have the following short-run ATC curves. Draw the long-run ATC curve.



Answer:
The long-run ATC curve is the lower envelop of the SRATCs. That is, at any Q, the LRATC is equal to the lowest SRATC. See the graph on the right, the LRATC is shown by the red curve.



21. Explain why when the LRATC is decreasing we have IRS, when the LRATC is stable, we have CRS, and when the LRATC is increasing we have DRS?



Answer:
When LRATC is decreasing, if the firm increases its output by d>1 times, its cost of labor and capital increases by less than d times. In another word, if the firm increases its cost by d times, its output will be more than d times higher. Therefore the firm has IRS.
When LRATC is stable, if the firm increases its output by d>1 times, its cost increases by d times too. So the firm has CRS.
When LRATC is increasing, if the firm increases its output by d>1 times, its cost increases by more than d times. Therefore if the firm increases its cost by d times, its output increases by less than d times. So the firm has DRS.

22. Can you give some reasons why a firm may have IRS? DRS? What is the optimal number of firms in the market if the firms have IRS, DRS, or CRS?
(I've changed the second question to make it more interesting.)

One reason for IRS is the improved specialization. When the firm has a larger scale, it may rearrange the tasks and labors so that the specialization is improved. For example in a sole proprietorship the owner must do everything. If he finds a partner, then one person can be responsible for the production, while another focuses on selling. The output is likely to be more than the sum of those of the two sole proprietorships. (There are many other reasons for IRS, and you just need to give one here.)

One reason for DRS is the increased difficulty in managing a larger company. The difficulty in managing a firm with 100 employees is more than doubled than managing a firm with 50 employees.

If the firms have IRS, the larger the scale, the better. So the firms will merge. If the new firms still have IRS, then merger will continue until there is only one firm left.

If the firms have DRS, then the smaller the scale, the better. So large firms will split into smaller firms. If the smaller firms still have DRS, then the process will continue, until the new firms no longer have DRS or cannot split.

If the firms have CRS, the scale does not matter, and the CRS property has no influence on the number of firms in the market.

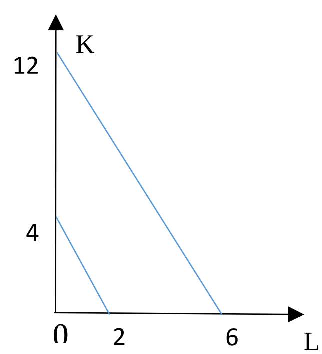
23. Suppose the wage is 10 and the cost of capital is 5. Draw the isocost line at which the cost is equal to 20. Draw another at which the cost is 60. What are the slopes of these two lines?

Answer:

When the cost is 20, the equation represented by the isocost line is: $20 = 10L + 5K$.

When the cost is 20, the equation represented by the isocost line is: $60 = 10L + 5K$.

If we put K on the vertical axis, the slope of both lines are -2.

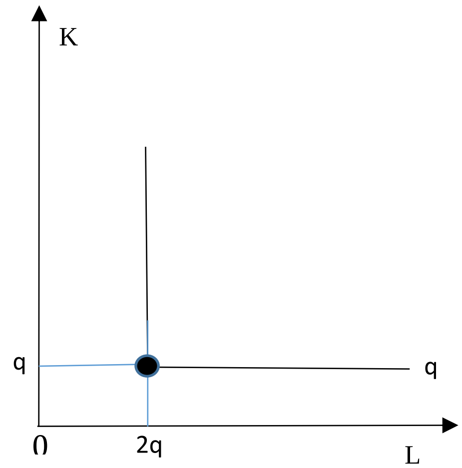


24. Suppose the wage is w=4 and the cost of capital is r=2. It takes 2 units of labor and 1 unit of capital to produce 1 unit of output. What is the long-run cost function? (If you can't get the general cost function, then calculate the costs at three different quantity levels.). Now suppose w=5 and r=1. What is the long-run cost function?

Answer:

The isoquant is as shown in the graph. It is easy to see that for any isocost line, the optimal bundle is at the kink. So the cost is determined by the K and L at the kink.

Because it takes 2 units of labor and 1 unit of capital to produce 1 unit of output, it takes 2q units of labor and q units of capital to produce q units of output. That is, $L = 2q, K = q$. So when the output is q, at the optimal bundle (the kink) the cost $C(q) = wL + rK = w(2q) + r(q) = (2w + r)q = (2 * 4 + 2)q = 10q$



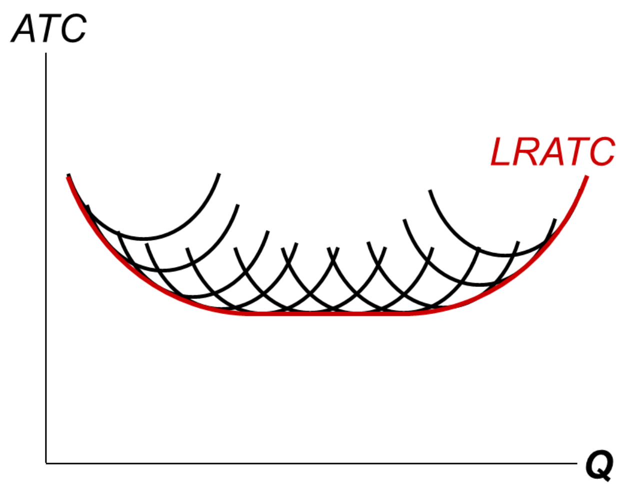
25. When there are many firms in the market, explain why the demand curve faced by each firm is horizontal while the market demand curve is still downward sloping.
Answer:
When all firms set the same price, they share the market quantity demanded. If one firm increases the price, its sales will become 0 because its customers will switch to other firms which produce the identical product. So the firm would not increase the price. On the other hand the firm has no incentive to lower the price. Because if one firm lowers the price, all the other firms will do the same thing, and thus lower the profit of all firms. So each firm just takes the market price as it is, and adjust its output according to the demand. That is, the demand curve faced by each firm in a competitive market is horizontal.

26. In a same graph draw a typical MC, AVC, and ATC curves. Explain why the MC curve crosses the other two curves at their minimums. Can you prove it mathematically?
Answer:
Check your notes, there are many graphs that have the typical MC, AVC, and ATC curves. Now let us prove that the MC curve crosses the other two curves at their minimums. Because we assume the MC curve is increasing, we only need to show that when MC is larger than AVC (ATC), AVC (ATC) increases; when MC is smaller than AVC (ATC), AVC (ATC) decreases. In the following I am using AVC in the proof. The proof for ATC is similar. Suppose now the quantity is q, and the average variable cost is AVC. By definition, MC is the cost of producing the next unit. So when quantity increases from q to $q + 1$, variable cost increases from VC to $VC + MC = AVC * q + MC$, so average variable cost increases to $AVC' = \frac{AVC*q+MC}{q+1}$. Is this new average variable cost larger than AVC? Let us compare the two: $AVC' = \frac{AVC*q+MC}{q+1} \sim AVC \Leftrightarrow AVC * q + MC \sim AVC(q + 1) = AVC * q + AVC \Leftrightarrow MC \sim AVC$, where $\sim$ is the comparison sign, and may take the value of " $>, \geq, =, <, \leq$ ". So if $MC > AVC$, we have $AVC' > AVC$, and if $MC < AVC$, we have $AVC' < AVC$. That is, when $MC > AVC$, AVC increases; when $MC < AVC$, AVC decreases.

27. Explain why a LRATC curve is below all SRATC curves? Draw the graph and show the change in the scale to return.

Answer:

Different SRATC curves have different capital levels. For each SRATC, the capital level is fixed because in the short run the firm cannot adjust its capital level. But in the long run the firm can always pick the capital level such that the ATC is the lowest.



Initially LRATC decreases in Q and the firm has IRS (increasing return to scale). In the middle LRATC becomes stable, and the firm has CRS (constant return to scale). In the end, LRATC increases, and the firm has DRS (decreasing return to scale).

28. In a perfectly competitive market what are the conditions a firm uses to decide on the optimal quantity and whether to shutdown in the short run?

Answer:

If the firm produces, output quantity is determined by $MC(q) = p$.

If $p > \min(AVC)$, the firm should operate. Otherwise the firm should shut down.

29. Suppose FC=2000, VC=1500, q=100, p=20. What is the short-run profit? Should the firm shutdown? What if the price is 10 now?

Answer:

If the firm operates, its profit $\pi = TR - TC = pq - (FC + VC) = 100 * 20 - (2000 + 1500) = 2000 - 3500 = -1500$.

If the firm shuts down, its profit $\pi_s = TR - TC = pq - (FC + VC) = 0 - (2000 + 0) = -2000$.

Since $\pi_s < \pi$, the firm should operate.

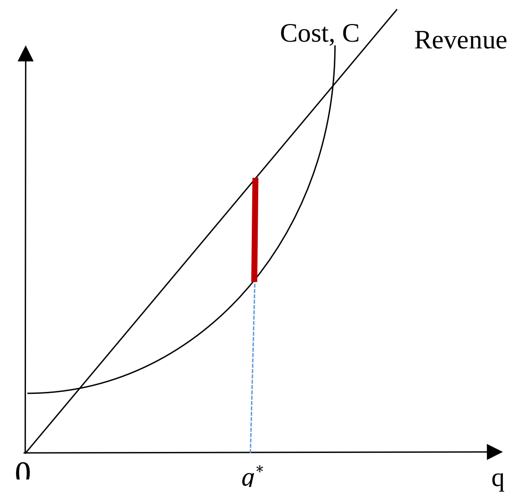
When price p=10, if the firm operates, its profit $\pi = pq - (FC + VC) = 100 * 10 - (2000 + 1500) = -2500$. If the firm shuts down, its profit $\pi_s = -FC = -2000$. Since $\pi_s > \pi$, the firm should shut down.

You can also compare the revenue TR with the variable cost VC (or to compare the price p with AVC) to quickly check whether the firm should shut down.

When p=20, $TR = pq = 20 * 100 = 2000$. So $TR > VC$, so the firm should operate.

When p=10, $TR = pq = 10 * 100 = 1000$. So $TR < VC$, so the firm should shut down.

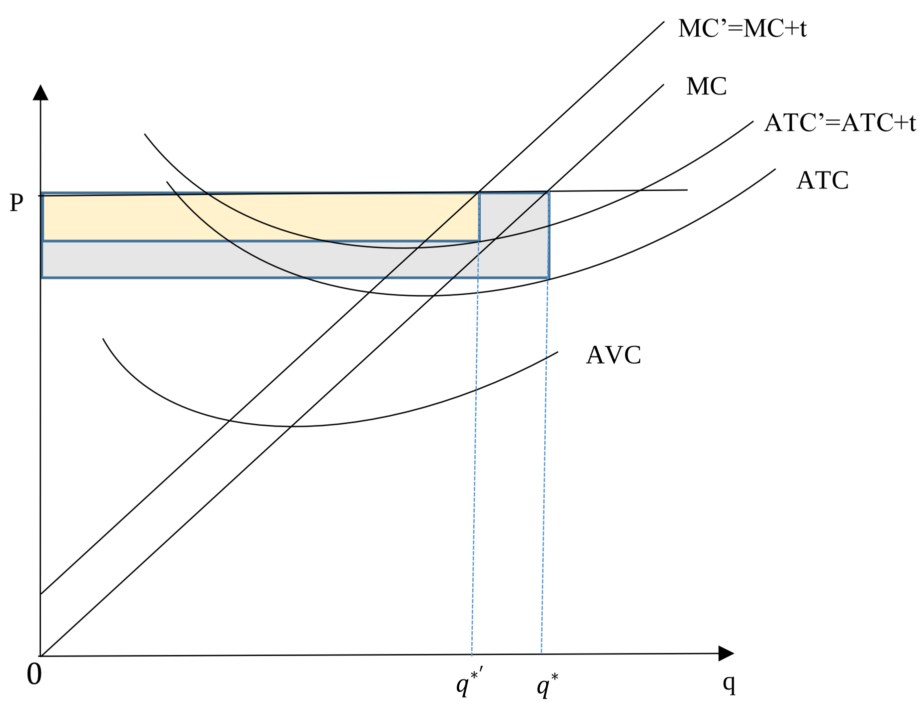
30. $C(q) = 1 + q^2$, $MC = 2q$, and the price $p = 6$. In the same graph, draw the cost curve and the revenue curve. Show the optimal quantity and the profit in the graph.



31. In the same graph, draw the typical MC and ATC curves. Suppose now the price is higher than the minimum of ATC. Show the optimal quantity q and the profit in the graph. Now suppose MC increases by 1 everywhere. In the same graph draw the new MC, ATC curves. Show the optimal quantity q and the profit. (You can assume the price is still higher than the minimum of the new ATC curve.) Please label your graph clearly.

Answer:

See below.



Please note that the MC (MC') curve must cross the ATC (ATC') curve at its minimum. The profit is represented by the area of the two rectangles.

32. In the graph, draw the typical MC, AVC, and ATC curves. Do all the following in the same graph:
    a. Label the price at which the profit of the firm is 0;
    b. Show the price range in which the profit is positive.
    c. Show the price range in which the profit is negative but the firm should not shut down in the short run.
    d. Show the price range in which the firm should shut down in the short run.
    e. Show the price range in which the firm should exit in the long run.

    Answer:

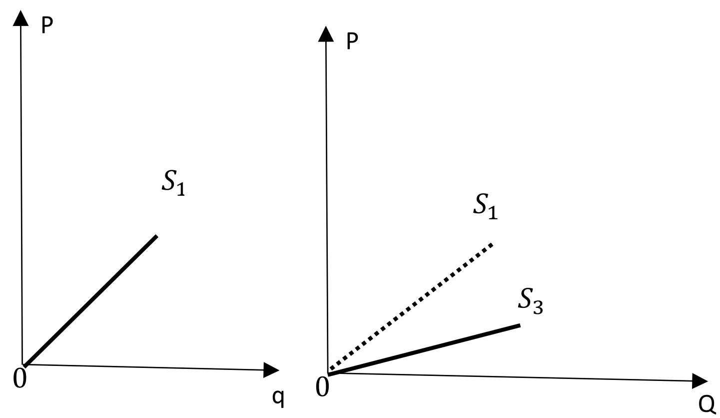
    a. When the price is equal to the minimum of ATC, the profit is 0.
    b. When the price is larger than the minimum of ATC, the profit is positive.
    c. When the price is lower than the minimum of ATC but higher than the minimum of AVC, the profit is negative but the firm should not shut down in the short run.
    d. When the price is lower than the minimum of AVC, the firm should shut down.
    e. When the price is lower than the minimum of ATC, the firm should exit in the long run because the profit is negative.

33. $C(q) = 10 + \frac{1}{2}q^2$, $MC = q$, and the market price $p = 2$. What is the firm's supply function in the short run? Suppose there are 3 identical firms in the market, what is the market supply function? Draw the firm's supply curve and the market supply curve in the same graph.

Answer:

The firm's supply function is identical to the part of its MC curve that is above the AVC curve. So we first need to get the firm's shutdown price: $p = \min(AVC)$. When the AVC is minimized, we have $MC = AVC \Leftrightarrow q = \frac{1}{2}q \Rightarrow q = 0$. So $\min(AVC) = 0$. So the firm's supply function is: $p = MC = q$.

The market output $Q_s = 3q = 3p$. So the market supply function is $Q_s = 3p$.



34. $C(q) = 4 + \frac{3}{2}q^2 + 2q, MC = 3q + 2$. The demand function is $Q_D = 20 - p$. There are 5 identical firms in the market. Characterize the short-run equilibrium. What is the profit of each firm?

Answer:

a) Let's get the firm's supply function:

When AVC is minimized, $MC = AVC \Leftrightarrow 3q + 2 = \frac{3}{2}q + 2 \Rightarrow q = 0$.

$p = MC = 3q + 2 \Rightarrow q = \frac{p}{3} - \frac{2}{3}$.

b) Let's get the market supply function: $Q_s = nq = 5q = 5\left(\frac{p}{3} - \frac{2}{3}\right) = \frac{5p}{3} - \frac{10}{3}$.

c) Let's get the market equilibrium using the market demand and supply function:

$Q_D = 20 - p$ (1)

$Q_s = \frac{5p}{3} - \frac{10}{3}$ (2)

Since at the equilibrium, $Q_s = Q_d \equiv Q$, from (1) and (2) above, we get $p = \frac{35}{4}, Q_s = Q_d = \frac{45}{4}$. So $q = \frac{Q}{5} = \frac{9}{4}$.

d) Let's get the profit of each firm:

$\pi = pq - C(q) = \frac{35}{4} * \frac{9}{4} - \left[4 + \frac{3}{2}\left(\frac{9}{4}\right)^2 + 2\left(\frac{9}{4}\right)\right] = \frac{115}{32}$.

35. The long-run cost function is $C(q) = 4 + q^2$, $MC = 2q$. The demand function is $Q_d = 28 - p$.
    a. What is the long-run supply function?
    b. What is the long-run equilibrium price, quantity, number of firms, output of each firm, and profit of each firm?
    c. Suppose the above cost function is also the short-run cost function. There are 12 identical firms in the market. What is the short-run market supply function? If your answer of the number of firms in part b. is also 12, compare the short-run market supply function with the long-run supply function you get in part a., are they different? If yes, explain why they are different.

Answer:

a. The long supply curve is horizontal with the price equal to the minimum of the AC. When AC is minimized, $MC = AC \Leftrightarrow 2q = \frac{4}{q} + q \Rightarrow q = 2$. At q=2, MC = 4 = AC. So the price p=4. So the long run supply function is $p = 4$.
b. The long-run equilibrium price is $p = 4$. The equilibrium quantity $Q = 28 - p = 24$. Output of each firm is given by $q = 2$ above in a. The number of firms $n = \frac{Q}{q} = \frac{24}{2} = 12$. The profit of each firm is 0.
c. In the short run, the supply function is identical to MC function is the firm operates. We need to study the firm's shutdown decision first. When AVC is minimized, $AVC = MC \Leftrightarrow q = 2q \Rightarrow q = 0$. So the firm never shut down in the short run. Then its supply function is $p = MC = 2q$. The market supply function is $Q_s = nq = 12\left(\frac{p}{2}\right) = 6p$. So the short-run market supply function is $Q_s = 6p$, and it is different from the above long-run supply function. They are different because in the long run only one price is possible, but in the short run all prices are possible.

36. If the cost function is $C(q) = 10q - q^2 + \frac{1}{3}q^3$ and $MC(q) = 10 - 2q + q^2$.
    a. What is the long-run market supply function?
    b. Do you need the demand function to get the output of each firm in the long-run equilibrium? If no, what is it?
    c. Do you need the demand function to get the profit of each firm in the long-run equilibrium? If no, what is it?

Answer:

a. When AC is minimized, we have $AC = MC \Leftrightarrow 10 - q + \frac{1}{3}q^2 = 10 - 2q + q^2 \Leftrightarrow q - \frac{2}{3q} = 0 \Rightarrow q = 0 \; or \; q = \frac{3}{2}$. We want to find the quantity at which AC is minimized. When $q = 0, AC = 10$; when $q = \frac{3}{2}, AC = 10 - \frac{3}{2} + \frac{1}{3}\left(\frac{3}{2}\right)^2 = 10 - \frac{3}{2} + \frac{3}{4} = \frac{37}{4} < 10$, so at $q =$

$\frac{3}{2}$, AC is minimized and $AC = \frac{37}{4}$. So the long run equilibrium price $p = \min(AC) = \frac{37}{4}$. It is also the long-run market supply function.

b. No. In the long run equilibrium the output of each firm is given by $q = \frac{3}{2}$ above.

c. No. In the long run equilibrium the profit of each firm is 0.

37. If the cost function is $C(q) = 9 + 10q + q^2$ and $MC(q) = 10 + 2q$. The demand function is $p = 60 - 2Q_d$.

a. What is the number of firms in the long-run equilibrium? (The number of firms does not have to be an integer.)

b. If the number of firms must be an integer. What is the number of firms in the long run? What is the profit of each firm?

c. If a firm needs a license to enter the market, and the government gives out at most 5 licenses. What is the profit of each firm in the long run? What if the government gives out 30 licenses at most?

Answer:

a. When AC is minimized, $AC = MC \Leftrightarrow \frac{9}{q} + 10 + q = 10 + 2q \Rightarrow q = 3$. So the price $p = \min(AC) = MC = 10 + 2 * 3 = 16$. The long run market supply function is $p = 16$. From the demand function $Q = 30 - \frac{1}{2}p = 22$. So number of firms $n = \frac{Q}{q} = \frac{22}{3} \approx 7.3$.
If n does not have to be an integer, then n=7.3, and the profit of each firm is 0.

b. If n must be an integer, n=7. The supply function of each firm is $p = MC = 10 + 2q$. (We don't need to check the shutdown decision because in the end the profit will be positive.) . So $q = \frac{p-10}{2}$. The market supply function is $Q_s = 7q = \frac{7p}{2} - 35$. Use this supply function and the market demand function we can solve for the equilibrium price and quantity:$p = \frac{65}{4}, Q = \frac{175}{8}, q = \frac{p}{2} - 5 = \frac{25}{8}$. So the profit of each firm is $\pi = pq - (9 + 10q + q^2) = (10 + 2q)q - (9 + 10q + q^2) = q^2 - 9 = \left(\frac{25}{8}\right)^2 - 9 \approx 0.77$.

c. Now there are at most 5 firms in the market. In order to get the profit of each firm, let first ignore this constraint and find out the number of firms n when the profit of each firm is 0. If n>5, then there will be only 5 firms in the market and the profit of each firm will be positive; If n<5, then there will be only n firms in the market and the profit of each firm is 0. From a. $n = \frac{22}{3} > 5$. So there will be only 5 firms in the market.
Now let's get the profit of each firm. Now the market supply function is no longer a horizontal line in the long run equilibrium, but instead is the aggregation of the supply functions of all firms. Each firm's supply function is identical to its MC function: $p = MC = 10 + 2q \Rightarrow q = (p - 10)/2$. The market supply function is $Q_s = 5q = \frac{5p}{2} - 25$. From the market supply and demand function we have $p = \frac{55}{3},\ q = \frac{25}{6}$. The profit of each firm is $\pi = pq - (9 + 10q + q^2) = (10 + 2q)q - (9 + 10q + q^2) = q^2 - 9 = \left(\frac{25}{6}\right)^2 - 9 \approx 8.36$.
If the government gives out 30 licenses, since n<30, the profit of each firm is 0 (assuming the number of firms does not have to be an integer).